Banco de México
Documentos de Investigación

Banco de México
Working Papers

N° 2020-16

**On the Drivers of Inflation in Different Monetary
Regimes**

Daniel Garcés Díaz
Banco de México

December 2020

La serie de Documentos de Investigación del Banco de México divulga resultados preliminares de trabajos de investigación económica realizados en el Banco de México con la finalidad de propiciar el intercambio y debate de ideas. El contenido de los Documentos de Investigación, así como las conclusiones que de ellos se derivan, son responsabilidad exclusiva de los autores y no reflejan necesariamente las del Banco de México.

The Working Papers series of Banco de México disseminates preliminary results of economic research conducted at Banco de México in order to promote the exchange and debate of ideas. The views and conclusions presented in the Working Papers are exclusively the responsibility of the authors and do not necessarily reflect those of Banco de México.

On the Drivers of Inflation in Different Monetary Regimes*

Daniel Garcés Díaz[†]
Banco de México

Abstract: This document proposes a general macroeconomic framework to analyze the behavior of inflation. This approach has two characteristics. The first is the distinction of monetary regimes based on the number of shocks that have a permanent effect on the price level. When all shocks have a permanent impact, the regime determines the inflation rate, as in inflation targeting. On the other hand, when there is only one shock with permanent effects, the regime determines the price level. An example of this is a regime with a fixed exchange rate. Even if there is no explicit target for the domestic price level, this becomes determined by the operation of a regime of this type. The second characteristic comes from the factors that Granger cause the rate of inflation or the price level. With this, a new perspective on four different historical cases emerges. One is the German hyperinflation; the second is that of the United States for a very long sample. For Brazil and Mexico, the analysis demonstrates that their inflationary processes' complexity arises from the regime changes they have gone through.

Keywords: Pricing Equation, Money, Exchange Rate, Inflation Predictability.

JEL Classification: C32, E41, E42, E52.

Resumen: El artículo desarrolla un marco macroeconómico general para analizar el comportamiento de la inflación. Este enfoque tiene dos características. La primera es la diferenciación de regímenes monetarios en base al número de choques que tienen un efecto permanente sobre el nivel de precios. Cuando todos los choques tienen un efecto permanente, se determina a la tasa de inflación. Este es el caso por ejemplo de regímenes de objetivos de inflación. Por otra parte, cuando solamente un choque tiene efectos permanentes, el régimen determina al nivel de precios. Un ejemplo es el de regímenes de tipo de cambio fijo. Es decir, si bien la autoridad monetaria no tiene un objetivo explícito para el nivel de precios, en la práctica, este se determina por el funcionamiento de un régimen de este tipo. La segunda característica es la identificación de los factores que causan, en el sentido de Granger, al nivel de precios o a la tasa de inflación. Con estos elementos se proponen nuevas perspectivas sobre cuatro ejemplos históricos: el primero es el de la hiperinflación alemana, el segundo es el de Estados Unidos para una muestra muy larga. Los casos de Brasil y México se utilizan para ilustrar cómo la complejidad de sus procesos inflacionarios puede entenderse identificando los cambios de régimen que estos han tenido.

Palabras Clave: Ecuación de Precios, Dinero, Tipo de Cambio, Predictibilidad de la Inflación.

*Comments by two anonymous referees are acknowledged. The views of this article are the author's only.

[†] Dirección General de Investigación Económica. Email: dgarcés@banxico.org.mx.

1 Introduction

Inflation has displayed some distinctive characteristics since the mid eighties in developed countries and since the nineties in most emerging market economies. First, it has been low and not very volatile, at least compared with the period known as the “Great Inflation” (mid sixties to early eighties). Second, its correlations at all frequencies with standard monetary aggregates and the exchange rate have gone down sharply. Third, its relationship with the output gap and other slack measures has become undependable. As a consequence, inflation forecasts, conditional on causes identified by well-known economic models have not been much better or have been even inferior to those produced by simple univariate time-series models (Faust and Wright, 2013). More recent models that claim to perform better are not really of the traditional kind and perhaps could be more suitable for modern economies (for example, Bobeica and Jarociński, 2017).

This paper sheds light onto these and other related matters by examining the experience of some countries that have changed their monetary regimes at some point in time. When an economy moves from one monetary regime into another, the inflation process changes in particular ways to be explained later. Although the basic idea behind this study is not new and it is somehow implicitly contained in many papers, there are several novel aspects in the analysis developed below. A key objective is to explain what happens to Granger causality of inflation and other variables within each regime. To justify the need for this view, it might be useful to discuss some basic aspects on the modeling of inflation. This will also serve to explain the contributions of this paper.

In a monetary model, inflation dynamics is summarized by a postulated macroeconomic pricing equation for a representative bundle of goods and services. A good example, but by no means the only one, is the New Keynesian Phillips curve (NKPC), the dominant contemporary approach to inflation modeling. The use of that particular pricing equation makes sense only when price stability is the central bank’s main mandate and, therefore, it might not be so useful in other circumstances. For example, when serious fiscal problems arise, another pricing setting could be at work. Those situations in modern times have become relatively rare, but they certainly have been present in the histories of many countries.

A pricing equation specifies the response of either the price level or the inflation rate to their determinants. At least in theory, the main determinants can be affected directly or indi-

rectly by the monetary authority through a chosen policy instrument such as the money supply, the exchange rate or the interest rate. The pricing equation could, in principle, determine not only the magnitude of the impact of a shock but the particular structure of Granger causality among the variables involved. Thus, when the monetary authority modifies its objective, its instrument or both, the dynamics of inflation might change.

This approach has been applied in Garcés-Díaz (2016 and 2017) and this paper extends that work in two main aspects. First, it provides an explicit theoretical framework where many inflation models can be embedded to explain changes in inflation dynamics. In those papers, the inflation models had been obtained as implications of the classical monetary model but this approach cannot account for a regime with inflation affected by economic slack, for example. Therefore, a more general approach based on changing pricing equations is proposed here. Another contribution is to show that the model applies more generally than in Garcés-Díaz (2016), where only the cases of some Latin American countries were analyzed. It adds two very different and important cases to show how the regime changes in causality and predictability in inflation are widespread events across countries and epochs. This has the potential to help to better understand the behavior of inflation at large.

A pricing equation implies a particular set of theoretical causality relations among monetary variables although such properties can be hidden to the observer sometimes by the dynamics of forward-looking expectations (see, for example, Killian and Lütkepohl, 2017). For simplicity, in this paper it will be assumed that the statistical causality can be related to the one stated by theory. Also, it is considered that there can be systematic, temporarily systematic and near-random causes of inflation. Sometimes there can be only one systematic cause of inflation understood as a good predictor for inflation that can change of identity at some point in time, as will be shown below. Some other times, the causes of inflation can be multiple and its influence temporary or random. This is the case when the inflation rate is the one that is determined, as opposed to price level determination. It is very important from the outset not to confuse the expression “price level determination” used here with frameworks of “price level targeting.” The difference will be clearer below. The study of the inflationary history of a country would require the identification of the pricing equation for each sample point which is made through a historical review, statistical tests or perhaps both. It is worth pointing out that in monetary regimes with price level determination, the central bank does not necessarily have an explicit target for the price level. For example, in an economy where there is a precise

objective for the exchange rate, there is only one kind of shocks that have a permanent effect on the price level in practice.

The empirical analysis of inflation obviously requires that there is enough information in the data to study any possible changes. This is typically accomplished through two possibilities: 1) long spans of data or; 2) short periods of sharp movements of the nominal variables. The empirical analysis of this paper uses the first alternative of many decades of data to study the United States, Mexico and Brazil cases. The example of the second possibility to obtain enough information from the sample is the few months of German hyperinflation.

However, inflation studies with the first alternative, very long samples, have been rare. The best known examples of this kind are Hendry (2001, 2015), which model UK inflation with annual data first from 1875 to 1991 and then extending it to 2011. A shorter version of the similar approach is Castle and Hendry (2007), which uses quarterly data but only from 1965 to 2003. This kind of studies is even harder to find if more than a single country is examined. One example is Sargent et al. (2009). The scarceness of such studies might have a good reason: it is not easy to find a unified framework to study long periods and different countries. A plausible explanation for that is that there are “structural changes,” although such changes are rarely explicitly identified. This paper suggests that at least some of such changes can be usefully described as shifts in the underlying pricing equations.

As mentioned above, the other possibility to get enough information in a sample to study the behavior of nominal variables is that of hyperinflation episodes. The reason to analyze here the most-studied event of this kind, the German hyperinflation in the early 1920s, is to demonstrate that changes in causality are determined by the choices of the monetary authority regardless of the levels of inflation.

The rest of the paper contains the following. Section 2 describes the main characteristic of how inflationary factors are treated here. Section 3 describes the role of pricing equations in explaining inflation dynamics. Section 4 presents the empirical cases where changes in pricing equations and Granger causality occurred. Section 5 contains the conclusions and final remarks.

2 The Causes of Inflation

There are several views on what causes inflation. The best known is that inflation is the view famously spoused by Friedman (1987) “...inflation is always and everywhere a monetary phenomenon in the sense that it is and can be produced only by a more rapid increase in the quantity of money than in output. Many phenomena can produce temporary fluctuations in the rate of inflation, but they can have lasting effects only insofar as they affect the rate of monetary growth.” This hypothesis has evolved toward some kind of consensus where, as expressed in Romer (2011): “Thus there are many potential sources of inflation. Negative technology shocks, downward shifts in labor supply, upwardly skewed relative-cost shocks and other factors that shift the aggregate supply curve to the left” and “factors that shift the aggregate demand curve to the right.” It agrees with Friedman in that “Nonetheless, when it comes to understanding inflation over the longer term, economists typically emphasize just one factor: growth of the money supply. The reason for this emphasis is that no other factor is likely to lead to persistent increases in the price level.”

Despite being the dominant model to study inflation, there are some facts about the Phillips curve that are little known or mentioned. The first is that the paper that made it part of the Keynesian model (Samuelson and Solow, 1960) states the curve was “roughly estimated” (in the note of Figure 2, p. 192). This actually meant that it was drawn over the scatter plot of 25 years of annual data. When the curve was estimated according to the statistical techniques available at the time, the curve was actually very different to the one displayed in that article even if the functional form is restricted to be a hyperbola (De Long, 1996 and Hall and Hart, 2012). Actually, it is extremely hard to obtain something close to the ideal Phillips curve with US data unless one picks purposely the years. Second, even if one can get a convincing Phillips curve out of raw data, this is not necessarily a proof of a relationship between both variables that goes from unemployment to inflation. For example, the Tobin effect indicates that higher inflation shifts funds from government securities into private investment and this lowers (increases) unemployment (output). The failures of the last few decades to forecast inflation (see, for example Dotsey et al. 2017, among many others) should be taken as a warning that the Phillips curve is not an unblemished model for inflation despite its current ascendancy.

Another approach to model inflation is to consider it as the result of many systematic

causes at once. A well-known study that uses many decades of data for one country is Hendry (2001), who estimates an “eclectic model of price inflation in the UK over the 1875-1991 span of time.” In subsequent related papers, Clemens and Hendry (2008) found the forecasts from that model to be not good when there were structural breaks.

It is worthwhile to emphasize how this article departs from Hendry (2001) approach. His model has a very large number of explanatory variables and dummies to control for problematic data points and outliers. Such profligacy comes from the specific aim to show that money is not the only cause of inflation. The analysis in this paper does not reject this view, but it considers the possibility that not all those causes are acting at the same time. This simple conjecture allows very parsimonious models, with three or less explanatory variables and a near absence of outliers. This is based on the hypothesis that only one type of pricing equation is working at any time and this implies few, or perhaps only one systematic cause of inflation instead of many and not necessarily money. As a result, most inflationary factors might show their impact through the error terms in regressions and, therefore, they are often of little use in inflation forecasting.

Sometimes it is quite difficult to identify when an economy changes its pricing equation. This article identifies such changes based on the hypothesis that when an economy switches its pricing equation, the Granger structure among monetary variables gets modified as well. This causes problems in models that were working well in a previous regime but that need to be replaced by another based on a different pricing equation for the new state of the world. The most evident failure is a degradation in the out-of-sample forecasting performance.

More recently, Bobeica and Jarociński (2017) apply the idea that inflation has many causes and not only the domestic output gap. They use many variables and lags of them within a flexible VAR framework. They include eight variables to represent domestic activity, eight global variables and six financial ones. They are able to capture the so called “missing” disinflation and inflation that other more scant or rigid models, such as New Keynesian ones, were unable to explain. It is a very compelling result if turns out to be robust enough. That result is consistent with the proposition that when the inflation rate is determined there can be many variables that can drive it and to attach too much weight to only a few can lead to bad forecasts. Notice also that they exclude any monetary aggregates, which confirm the observation that, for a long time money has, not been useful to forecast inflation in advanced economies.

3 Pricing Equations

A monetary model has as a key ingredient a mechanism to determine the evolution of the price level sometimes called a pricing equation. It describes the evolution of the price of a bundle of goods and services that are considered representative for the typical consumer of an economy. In that sense, it is similar to models for the prices of financial assets and shares some characteristics with them that can be exploited to study the behavior of the price level or the inflation rate in different monetary regimes.

Monetary regimes are determined by the actions of the corresponding authority, which it can try to pin down either the price level or the inflation rate that considers appropriate. The best known pricing equations at present are variations of the New Keynesian type and they have become essential parts of modern dynamic macroeconomic models. Another well-known pricing equation is the one used by Sargent and Wallace (1973) to study hyperinflation episodes. This was used by Sargent et al. (2009) also to study hyperinflation episodes in Latin America even though most of their samples cover periods not considered of hyperinflation and even some periods of low inflation. As will be seen, they are unlikely to be useful in forecasting.

Both pricing equations are similar in that they contain an expectations term for the next period's dependent variable (inflation in one case and the price level in the other). Actually, Gordon (2013) argued that Sargent's studies on hyperinflation were the origin of the modern NKPC. Nevertheless, this was derived explicitly first by Roberts (1995) from different price settings based on sticky prices or adjustment costs. In a NKPC, a slack measure of real activity, presumed to be a good proxy of the marginal cost of a representative producer, replaces money as the forcing variable for inflation but it keeps a forward-looking term related to the one present in Sargent's equations.

Despite that commonality, studies where more than one appears are very rare. There are, however, studies where the parameters of the equation shift or time-vary but the dependent and explanatory variables remain unchanged (for example Sims and Zha, 2006), which is not the case in this study. One can get some insights considering the possibility that the New Keynesian, the monetarist or other pricing equations had been used at some point and then were substituted for others, when the circumstances of the economy changed.

In those situations, one can expect that the substitution of a pricing equation, which reflects

a change in the monetary authority's framework, impacts the behavior of inflation and other variables, particularly their Granger causality structure. For example, if a country controls an hyperinflation process, where money or the exchange rate were the driving variables, into a regime where price stability were the main objective, then economic activity or other factors could begin to play a role in the determination of inflation that did not have before. At the same time, inflation might not follow so closely the movements of either money or the exchange rate. Thus, the variables that help to predict inflation are not always necessarily the same. They can change and they often have.

A basic characteristic of a pricing equation comes from the nature of the monetary authority's objective with respect to the nominal sector. The monetary authority actions can either influence the behavior of the nominal variables in ways this is better described by equations where the price level or the inflation rate are the alternative dependent variables. The choice of actions taken is fundamental to explain the effect of shocks on nominal variables. Historically, price level determination has been tied to either money or the exchange rate but, at least in the cases here examined, not both simultaneously. For inflation rate determination, the central bank can opt for the control of the money supply, the exchange rate depreciation, the movements of the output gap or some combination of them.

Again, it is important not to confuse the determination of the price level through a specific pricing equation where the price level is on the left-hand side, instead of the inflation rate, with price level targeting. Modern price level target for a central bank committed to price stability is not necessarily tied to any of those variables but there are still not known lasting empirical examples of it even though the Federal Reserve announced a variation of it, which was called "average inflation targeting." It is too soon to evaluate it. The gold standard is a form of implicit price level targeting (Barro, 1979 and Citu, 2002) but it is of the passive type in that the national stock of the metal determines the price level and it is not under the government's control (Hall, 2002).

Sweden from 1931 to 1937 adopted a price level targeting system after it abandoned the gold standard. This, however, should not be considered similar to modern proposals as the one adopted by the Federal Reserve because this consider the use of the interest rate as the policy instrument in the context of a floating exchange rate. Although the discount rate was regarded one of the policy instruments in the Swedish experience, the Riksbank also tied the value of the krona to the the British pound in 1933. Moreover, even though monetary aggregates were

not explicitly considered neither as intermediate objectives neither as indicators, the money stock was kept nearly constant for most of this period (Jonung 1979, Figure 2). Thus, the Swedish case is more similar to the Mexican and Brazilian ones for several decades until the seventies. The big difference lies in the central objective. While the Riksbank was trying to maintain price level stability, those other two countries at times faced economic problems that made price stability not the primary concern and they faced high levels of inflation. This leads to a key implication: when the concern is price stability, price level determination is not necessarily forward-looking as is the case of inflation targeting where “bygones are bygones.” As in the cases here analyzed there was no commitment to price stability, the forward-looking element remains.

3.1 Differences in the Implications of Price Level and Inflation Rate Based Pricing Equations

A crucial difference between regimes with a price level and inflation rate determination is the persistence of inflationary shocks on the price level. When the price level is determined, only the driving variable matters in the long run and the effects of any other shocks are always transitory. In inflation rate determination, the price level is not necessarily driven by a single variable and, as the central bank is not committed to rectify previous deviations from its target, all shocks have a permanent effect on the price level.

In principle, because of the different impact of inflationary shocks, both regimes cannot coexist permanently although one of them can be a transient episode embedded into the other. There are a few examples of when that could happen. The most famous of them all is the case of the wide-spread view that in the short run inflation has many causes, but that in the long run the money supply is the only determinant of the price level (see for example, Romer, 2011).

Although very popular, that view has some unsolved issues. The first one is that the transition from multiple inflation factors to a unique one (money) takes place is rarely modeled, if ever. For example, a common practice is to use a Phillips curve or other cashless models to determine medium and short term inflation without any reference to money as an inflationary cause. Despite attempts or suggestions of giving money a role in more general New Keynesian models of inflation, current practice has not abandoned cashless models as its main paradigm.

Nelson (2003) argues that money could be introduced in cashless models of inflation “as a proxy for yields that matter for aggregate demand” but that observation has not been seen as very useful in inflation forecasting at least.

Another problem is that not only the short-run but also the long-run correlation of money and the price levels has weakened since at least the 80s. If that correlation will ever be restored in the future, it is a question that has no answer right now. Recent work (Benati *et al.* 2016) showing that for very long samples there exists a long-run money demand does not by itself prove that money is the ultimate cause of inflation, as pointed out by Hendry and Ericsson (1991) and Hendry (2017). Moreover, regular cointegration techniques, which in fact can be interpreted as tests for long-run correlation between money and the price level controlling for other variables (Levy, 2002), might hide the presence of regime changes, as those discussed in this paper. It is shown here that even though one of the cases studied in Benati *et al.* (2016), Mexico, the stable money demand that had been present for a long period broke down since the year 2000. This event cannot be detected with regular cointegration tests for the whole sample because the money demand was stable during the period with the highest variability, which tends to hide what happened after 2000. Third, even though there is a widely spread belief that money only matters when there is high inflation, Granger causality does not necessarily run from money to prices. Thus, even in those cases that relationship, if it exists, cannot be exploited for forecasting purposes, as the current dominance of cashless inflation models suggests.

Specific cases where one monetary regime gets embedded in the other are those of Germany and Sweden. Germany had a hyperinflation and price level determination period although price stability was not the central bank’s main objective during the early 1920s. However, most of the time there has been an inflation determination regime in that country. In Sweden, as discussed before, during the 1930s there was a price level determination regime where price stability was the chief objective of the central bank, therefore it is considered as an experiment of sorts for the modern proposal of substituting inflation targeting for price level targeting.

Another key difference between price level and inflation rate price equations lies in the form of their respective forward-looking solution. That form is relevant to obtain the final inflation models. When a pricing equation is defined for the price level, one has to deal with a relationship among nonstationary variables that under suitable assumptions produce

cointegration relationships. When the pricing equation is instead defined for the inflation rate dependent on a stationary variable, the solution ties current inflation to all the future discounted values of the driving variable. This will be explored next.

3.2 Some Types of Pricing Equations and Their Solutions

Among many possibilities, the following five pricing equations are enough to analyze the empirical cases of this paper. The first two correspond to price level determination and the rest to inflation rate determination. In all cases, the variables are in natural logarithm form.

3.2.1 Price Level Determination Through a Monetary Aggregate

This pricing equation corresponds to the simplified case where there is no interaction between the real and nominal sectors of the economy. This was used by Sargent and Wallace (1973). It is solved under rational expectations:

$$p_t = \gamma_m E_t[p_{t+1}] + (m - y)_t \quad (1)$$

where $(m - y)_t$ is the inflationary part of money supply (“inflationary money”) that the central bank controls to meet its desired price level and $0 < \gamma_m < 1$.

Something important for the approach of this paper is that, for consistency and in order to obtain some general results, equation (1) requires that money growth is Granger-causing inflation. This is not necessarily the practice. For example, it is well-known that during the German hyperinflation, Granger causality did not run from money to prices but the other way around. Equation (1) can be solved forward by successive substitution applying the law of iterated expectations to obtain:

$$p_t = \frac{1}{1 + \gamma_m} \sum_{\tau=0}^T \left(\frac{\gamma_m}{1 + \gamma_m} \right)^\tau E_t(m - y)_{t+\tau} + \left(\frac{\gamma_m}{1 + \gamma_m} \right)^{T+1} E_t(m - y)_{t+T+1} \quad (2)$$

To rule out an explosive (speculative bubble) solution a transversality condition must be applied:

$$\lim_{T \rightarrow \infty} \left(\frac{\gamma_m}{1 + \gamma_m} \right)^{T+1} E_t(m - y)_{t+T+1} = 0 \quad (3)$$

With this, a present value relationship arises:

$$p_t = \frac{1}{1 + \gamma_m} \sum_{\tau=0}^{\infty} \left(\frac{\gamma_m}{1 + \gamma_m} \right)^{\tau} E_t(m - y)_{t+\tau} \quad (4)$$

I.e., the current price level is equal to the sum of the discounted future values of the fundamentals. For the purposes of obtaining an empirically tractable model of inflation, the assumption on the stochastic behavior of the price level p_t and the fundamental $(m - y)_t$ are $I(1)$ variables, following Campbell and Schiller (1987) and MacDonald and Taylor (1993), one can subtract $(m - y)_t$ from both sides of the equation and obtain:

$$p_t - (m - y)_t = \frac{1}{1 + \gamma_m} \sum_{\tau=0}^{\infty} \left(\frac{\gamma_m}{1 + \gamma_m} \right)^{\tau} E_t \Delta(m - y)_{t+\tau} \quad (5)$$

Under rational expectations, the forecasting errors of the right-hand side are stationary, thus the left-hand one is also stationary, therefore p_t and $(m - y)_t$ must have a cointegration relationship. Naming the right-hand as v_t , one can obtain the well-known relationship known as the quantitative equation of money (QEM):

$$p_t = (m - y)_t + v_t \quad (6)$$

In other words, if the central bank targets a price level controlling the money supply then the quantitative equation of money is a cointegration relationship. Moreover, the long-run deviation v_t can be interpreted as the velocity of money. In this case the quantitative equation of money is not a tautology because of the requirement that money velocity is mean-reverting.

If this pricing equation applies, then in the long run only money matters to determine the price level. Other variables can have only a transitory impact on inflation. Two of the empirical examples presented below (Brazil and Mexico before the eighties) are completely in agreement with this but they are exceptions rather than the rule. In fact, in many countries the term velocity within the quantitative equation of money does not show mean reversion (see Juselius, 2006).

Even if the quantitative equation of money holds in a nontautological way, it is not necessarily true that it can be used to obtain a forecasting inflation model. It could be the case that the associated error-correcting variable is not the price level but money, as happens in many countries (for example, in the United States). In that case, one cannot say that money is the cause of inflation, at least not in the Granger sense, even if the quantitative equation of

money holds. However, under the assumption that the central bank targets the price level and uses money as its instrument, the error-correcting variable must be the price level and money should be useful to forecast inflation. This is the case in other examples shown below but not the rule in modern economies.

3.2.2 Price Level Determination Tied to the Exchange Rate

This situation is more often analyzed in international economics research and it has been very important in countries that have had problems with their current account balances. A different take on this relationship is that of the monetary approach to the exchange rate with rational expectations, as in MacDonald and Taylor (1993), where the dependent variable is not the price level but the exchange rate. In this case, the forward-looking pricing equation takes the following form:

$$p_t = \gamma_e E_t[p_{t+1} - p_t] + (e + p^f)_t \quad (7)$$

where e_t is the nominal exchange rate (defined as number of domestic currency units per a foreign one) that the central bank controls and $0 < \gamma_e < 1$. The central bank could be controlling the nominal exchange rate for a given foreign price level p^f_t to set a trajectory for the real exchange rate (for example, to keep trade competitiveness). However, if PPP holds then this policy will eventually determine the domestic price level only. As in the previous case, the parameter γ_e is between zero and one. Following the same steps as in the case for money above and also assuming that the fundamental $(e + p^f)_t$ is an $I(1)$ variable, one obtains the expression:

$$p_t = (e + p^f)_t + rer_t \quad (8)$$

where rer_t , the real exchange rate, is a stationary process. Thus, if the central bank determines a price level controlling the nominal exchange rate, then the real exchange rate rer_t is the residual of a cointegration relationship where the price level is the error-correcting variable. More often, a similar equation has been used as a description of the determination of the nominal exchange rate but it is always difficult to obtain an error correction mechanism or any type of predicting model for that variable (see for example, Engels and West, 2005).

3.2.3 Inflation Rate Determination Tied to Economic Activity

Modern inflation rate targeting can be explicit, as in many economies, or implicit (for example, in the United States before 2012). The target is achieved throughout the determination of an interest rate that affects aggregate demand through different channels. In that approach to monetary policy, the dominant model of inflation is the Phillips curve, particularly in its forward-looking version, which is a fundamental ingredient of the New Keynesian theory of business cycles. The forward-looking pricing equation takes the following form:

$$\Delta p_t = \gamma_y E_t[\Delta p_{t+1}] + h_y(y_t - y_t^*) \quad (9)$$

where y_t and y_t^* are the levels of actual and potential output, thus their difference is the output gap while $0 < \gamma_y < 1$ and h_y are constants. The central bank cannot determine directly the output gap or other measures of economic slack. It must do it indirectly through the policy interest rate, which impacts the spending decisions of firms and consumers. This is known as conventional monetary policy but in some circumstances, as when the short-run interest rate is at or near the zero lower bound, the central bank might use unconventional measures such as quantitative easing and credit easing. The (rational) expectations term allows a short-run, but not a long-run trade-off between the output gap and inflation. Other variables stationary might be included. The forward solution of equation (9) is the following:

$$\Delta p_t = h_y \sum_{\tau=0}^{\infty} \gamma_y^{\tau} E_t(y - y^*)_{t+\tau} \quad (10)$$

This means that current inflation depends on the stream of future output gaps, which are proxies for marginal costs, as has often been pointed out in modern literature (Goodfriend and King, 2009). These authors consider a more general model than the one used by Woodford (2008). They assume that inflation contains a stochastic trend. This is necessary for the US economy, where the price level appears to be an $I(2)$ process, at least before the beginning of the Great Moderation, which started more or less since the mid eighties. With trend inflation, the solution is:

$$\Delta p_t = \overline{\Delta p_t} + h_y \sum_{\tau=0}^{\infty} \gamma_y^{\tau} E_t(y - y^*)_{t+\tau} \quad (11)$$

where $\overline{\Delta p_t}$ is trend inflation. Of course, a more complete model has to specify a monetary rule, an Euler equation and some other components as in Woodford (2007) and Goodfriend

and King (2009). Such elements, although necessary for a more complete picture of the economy, are not indispensable to examine the inflation forecasting performance of models based on slack measures of economic activity (see, for example, Stock and Watson, 2007 and Faust and Wright, 2013).

3.2.4 Inflation Rate Determination Tied to Money Growth

This monetary equation differs from that of section 3.2.1 mainly in that money growth does not necessarily translates in the long run into inflation at a 1-to-1 rate. The reason for this is that in an inflation rate determination regime the central bank has no pledge to amend past deviations from the objective. Thus, there can be, and usually there are, other determinants for the trajectory of the price level in the long run. Money growth as an inflation driver could even be combined with something like the output gap, as possibly happened in the United States for some time.

The pricing equation is similar to that of section 3.2.3 except in that the economic slack measure is substituted by money growth, a stationary variable:

$$\Delta p_t = \overline{\Delta p_t} + h_{\Delta m} \sum_{\tau=0}^{\infty} \gamma_{\Delta m}^{\tau} E_t \Delta m_{t+\tau} \quad (12)$$

where $h_{\Delta m}$ and $0 < \gamma_{\Delta m} < 1$ are corresponding constants.

3.2.5 Inflation Target Tied to the Depreciation Rate

In this case, the pricing equation is similar to that of section 3.2.3 but with the exchange rate depreciation as the macroeconomic fundamental:

$$\Delta p_t = \overline{\Delta p_t} + h_{\Delta e} \sum_{\tau=0}^{\infty} \gamma_{\Delta e}^{\tau} E_t \Delta e_{t+\tau} \quad (13)$$

where $h_{\Delta e}$ and $0 < \gamma_{\Delta e} < 1$ are constants.

4 Empirical Examples of Pricing Equation Changes

The use of more than one pricing equation allows the study of inflation in short and long data samples alike, provided there is enough information in the available data. At the same

time, it simplifies the construction of forecasting models by selecting variables that are the most adequate for each period. This section analyzes four cases, one for a short sample, the German hyperinflation (1921-1923) and three for long samples. First, the analysis is applied to the cases of Mexico (1932-2013) and Brazil (1960-2013) not only because they are also examples of the work of price level determination systems but because they used different instruments to achieve their goals and finally adopting an inflation target system. The case of the United States (1880-2007) is the best known, but there are some aspects about it that are not frequently stressed. One of them is the changes in Granger causality, that is here documented.

4.1 The German Hyperinflation (1921-1923)

This is an event where the sample is short but, given the very large rates of growth of monetary variables and the speed of adjustment to shocks, it contains a lot of information about monetary relationships and it has been a widely used case to study different aspects of money demand, inflation and the exchange rate.

Hyperinflation has typically been seen as a convincing proof that inflation is ultimately a monetary phenomenon in the sense that money drives the price level. However, Sargent and Wallace (1973), among others, noted that money growth did not Granger-cause inflation during the hyperinflations after World War I (WWI) but, instead, it was inflation the variable that Granger-caused monetary growth. To make the monetary aggregate the economic cause of inflation, they proposed a mechanism that involves increasing government spending which demands growing amounts of money, and hence higher inflation, to finance it. If the price setters have rational expectations then inflation leads money growth. As a follow up to this idea, Makinen and Woodward (1988) found that after the price stabilization period that followed six cases of after-WWI hyperinflations, among them the German one, the Granger causality from prices to money vanished. Because of the Sargent and Wallace's arguments, changes in Granger causality have had a secondary or irrelevant role in the validation of inflation models.

The approach of this paper is different in that it considers those changes a relevant part of what a monetary regime switch brings and not only in hyperinflation episodes. Thus, an alternative explanation to the causality among monetary variables during the German hyperinflation is implicit in Bresciani-Turroni (1968). He divided the period in several stages, related

Figure 1: Exchange Rate Depreciation, Inflation and Money Growth In German Hyperinflation

to the German government's demand of foreign currency to meet war reparations. In his diagram I, "volume of circulation" (currency) and "domestic prices" had a closer relationship than that between domestic prices and the paper-marks-per-dollar exchange rate. In fact, until just before the beginning of the hyperinflation period (July 1921), the exchange rate only appeared correlated with the "prices of import goods" (diagrams III and IV). From May 1921 to the end of the hyperinflation in December 1923 (diagrams VI and VII), the relationship between domestic prices and the exchange rate became very tight, as Figure 1 shows.

Thus, Bresciani-Turroni was in fact chronicling the substitution of one pricing equation for another in which the driving cause was the need for foreign currency. This is actually a balance of payment problem and it matches a situation that had been recognized as early as the XVIII century, during the Swedish bullion controversy (Moosa and Towadros, 1999) and has been present in many other cases, two of them examined below.

Figure 1 shows the time path of inflation, money growth and the depreciation rate for this episode. It can be seen that since June 1921 on, money growth is led by both inflation and the depreciation rate. It is also seen that inflation and the exchange rate depreciation are more tightly correlated. The correlation between the first difference of these two variables is 0.92 while those of money growth with either of them is 0.5. This suggests that the exchange rate and the price level were determined more or less simultaneously. This will be made more precise later.

These impressions are confirmed by the following econometric analysis based on the ap-

proach of Garces-Diaz (2016), who applied it to a group of Latin American economies. First, it must be said that the approach followed for the case of the German hyperinflation differs from the other three cases because it is a typical case where cointegration must be applied. For Brazil and Mexico a similar analysis is carried out in Garcés-D'az (2016). For the United States, such approach cannot be applied because there is no evidence that country ever followed a policy of price level determination. Therefore, a simpler Granger causality VAR analysis is carried out for the last three cases.

Some definitions are needed here before discussing the econometric results. When there are cointegration relationships in a model with nonstationary variables one can distinguish two kinds of Granger causality. If the short-term shocks, for example lags of the changes of some variables or any other stationary terms in the model help to forecast a different variable then there is short-run Granger causality. However, in a cointegration system there is another way in which one variable can be forecast on the basis of others. This is through the impact of the error correction term where those other variables might be present. In this case, there is long-run Granger causality, also known sometimes as “cointegrating causality,” going from the nonstationary variables in the error correction term to the variable being forecast (Hatanaka, 1996, pp. 237-8). Long-run Granger causality is closely related to weak exogeneity, a concept introduced by Engle et al. (1983):

“...a variable z , in a model is defined to be weakly exogenous for estimating a set of parameters λ if inference on λ conditional on z_t involves no loss of information. Heuristically, given that the joint density of random variables (y_t, z_t) always can be written as the product of y_t conditional on z_t , times the marginal of z_t , the weak exogeneity of z_t , entails that the precise specification of the latter density is irrelevant to the analysis, and, in particular that all parameters which appear in this marginal density are nuisance parameters.”

This is a general definition of weak exogeneity that when applied to the case of a cointegrated system is stated as follows: “...a variable is weakly exogenous for the cointegrating parameters if none of the cointegration relations enter the equation for that variable.” (Lütkepohl, 2004). Thus, in a cointegrated system a variable is weakly exogenous if it is not Granger-caused in the long run by the other nonstationary variables. In other words, the long-run causality must exist in only one direction, from the weakly exogenous variable to the

others that do not have this property. Notice that the property is maintained even if short-run Granger causality runs in the other direction. Moreover, Engle et al. (1983) define a variable as strongly exogenous if it is weakly exogenous (or, equivalently, not Granger-caused in the long run) and it is not Granger-caused in the short run either. One should remember that when both concepts, weak and strong exogeneity, are applied to linear models they become equivalent to the more traditional ones of predeterminess and strict exogeneity (Killian and Lütkepohl, 2017, ch. 7).

With this in mind one can analyze the German hyperinflation episode. The following error-correction mechanisms show that the exchange rate and not the money supply was the forcing variable during this period. Also, unbalanced regressions derived from the monetary model show that the circulant was the only variable that was Granger-caused by the others by it did not caused them, i.e., it was determined by the price level which in turn was determined by the exchange rate.

Given that the number of observations is relatively small and, more to the point, it spans less than three years, there are some issues that must be addressed related to the validity of the results. It turns out, that neither of this characteristics of the sample are a matter of concern. The reasons are two. First, there is not a number of observations that can be regarded as minimum for the test to be valid. The reason is that, as discussed before, for the parameters to be rightly estimated is necessary that there is enough information within the sample. This is similar to the case discussed in Campos and Ericsson (1999), who use only 16 years of annual data to estimate a model with five parameters for consumption expenditures in Venezuela for the period 1970-1985. They argue that each observation is highly informative because the variability is very high. In the case of the hyperinflation episodes the situation is similar and the speed of adjustment is so high that the around 30 months used are enough to estimate the models below. This is in fact a way to define a long-run relationship: the period that a system takes to absorb a shock. If the speed of adjustment is high enough, as it is in the case here analyzed, then the long-run happens in a short span of calendar time.

The first hypotheses to test are related to the role of the exchange rate in the determination of the other nominal variables. As discussed above, the demand for foreign currency to pay the war reparations was one of the most pressing matters for the Weimar Republic budget. Thus, one would expect that this fact was reflected in the behavior of the nominal sector. In particular, the exchange rate should be a weakly exogenous variable with respect to the price

level and the amount of currency, according to what Bresciani-Turroni (1968) observed. In this case, in an equation for the exchange rate with dependent variable Δe_{t-1} , there would be no feed back from the price level. For this to happen, the variable e_{t-1} should be statistically nonsignificant in an unrestricted error correction mechanism. Thus, the null hypothesis in the equation is that the coefficient for e_{t-1} is equal to zero, using the proper critical values for this kind of models. On the contrary, in the equation for Δp_{t-1} , the lagged variable level of the price level p_{t-1} must be statistically different from zero with the hypothesis also evaluated with the proper distribution. Also, in a cointegration system for the exchange rate and currency, in the the unrestricted error correction model for Δe_{t-1} , the coefficient for e_{t-1} should be statistically not different from zero. But is there is cointegration, then in the unrestricted error correction model for Δm_{t-1} the coefficient for the variable m_{t-1} should be different from zero.

Table 1 shows the estimates for the four error correction models mentioned above. Only in two of them the key parameter is significant. The first two are for the pair exchange rate and the price level (e_t, p_t) and the other two are for the exchange rate and the circulant (e_t, m_t). From the appropriate estimated parameters, one can infer which the leading variable was for the nominal system during this period. The four models take the form of unconstrained error correction mechanisms, one for each pair of the variables here considered. The regression constants are not reported. The null hypothesis are described at the end of table.

For each model, the chief parameter corresponds to the lagged level of the variable which is the dependent one in the form of first difference. The highlighted cells in Table 1 contain the corresponding estimates of those parameters. The distribution of the t statistics for them is nonnormal. Tables for the critical values for this kind of models are provided by Ericsson and MacKinnon (2002). From their Table 3, one obtains the critical values. The first key parameter, that for e_{t-1} when the dependent variable is Δe_t , has a positive sign when it is required to be negative, as in any unrestricted error correction models. If one introduces as a regressor the contemporary value for the rate of inflation Δp_t , then one obtains the right sign for the estimate, which is -0.26. However, even in that case, the t statistics at -1.41 is far below the critical value of 10 percent of significance which is -3.37. With such value for the t statistic, one can infer that the price level is not a weakly exogenous variable for the exchange rate thus, the contemporary value of the rate of inflation cannot be used in a regression as an explanatory variable for the exchange rate. That is why the contemporary rate of inflation is

Table 1: Relationships of the Exchange Rate with the Price Level and the Circulant During the German Hyper-inflation

Regressors	Related variables			
	e, p		e, m	
	Dep. Var.		Dep. Var.	
	Δe_t	Δp_t	Δe_t	Δm_t
e_{t-1}	0.64 (1.95)	0.56 (5.76)	-0.42 (-2.47)	0.17 (5.09)
p_{t-1}	-0.56 (-1.64)	-0.56 (-5.76)	n.i. .	n.i. .
m_{t-1}	n.i. .	n.i. .	0.83 (3.74)	-0.19 (-4.81)
Δe_t	n.i. .	0.74 (15.17)	n.i. .	n.s. .
Δe_{t-1}	0.36 (2.69)	n.s. .	0.86 (1.89)	n.s. .
Δp_{t-1}	n.s. .	0.22 (5.84)	n.i. .	n.i. .
Δm_t	n.i. .	n.i. .	n.i. .	n.i. .
Δm_{t-1}	n.i. .	n.i. .	n.s. .	0.32 (1.93)
d_{Jul23}	n.i. .	-0.80 (-6.90)	n.i. .	n.i. .
d_{Oct23}	3.46 (7.43)	0.90 (4.41)	n.i. .	n.i. .
Period	21:03-23:11	21:03-23:11	21:04-23:11	21:03-23:05
T	33	33	33	27
Adjusted R^2	0.89	0.99	0.83	0.89
S.E.	0.42	0.10	0.51	0.06
Jarque-B	0.09	0.33	0.71	0.59
LM(2) autocor	0.80	0.21	0.66	0.07
LM(1) ARCH	0.86	0.63	0.28	0.58

t statistics are between parentheses.

The null hypothesis H_0 in each case is that the coefficient in the shaded area is nondifferent from zero.

n.s. means excluded for being nonsignificant.

For Jarque-B, LM(2) autocor LM(2) ARCH the p values are provided.

excluded in the model presented in Table 1. Thus, one concludes that the price level is not a weakly exogenous variable for the exchange rate. Moreover, as the lags of inflation are not significant either, one can also conclude that the price level does not help to predict the exchange rate. The price level does not cause the exchange rate in the Granger sense.

The next estimate to evaluate is that for p_{t-1} in the equation for Δp_t . The coefficient's estimate was -0.56, identical in absolute value to the coefficient for e_{t-1} in the same equation. This means that the cointegration vector was (1, -1), implying that the exchange rate and the price level changed in the same percentage amount during the German hyperinflation. Moreover, the t statistic for the coefficient of p_{t-1} was -5.76, which is more negative than the critical value at the 1 percent level of significance of -5.50 provided by Table 3 in Ericsson and MacKinnon (2002). This means that the exchange rate was weakly exogenous with respect to the parameters of the equation for the price level. The coefficient for the contemporary exchange rate depreciation was 0.74, meaning that almost 3/4 of the depreciation was transmitted in the same month to the price level and its test statistic was very high (15.17). Two dummy variables were included to ensure the normality of the regression errors.

Similarly, the third and four columns show the regressions for the pair (e_t, m_t) . The main coefficient for the third regression is -0.42 but it has a t statistic of -2.47, which is even far less negative than the critical value of -3.19 for a 10 percent significance. This implies that money is not a weakly exogenous variable for the exchange rate. On the contrary, the coefficient for lagged money in the fourth regression was -0.19, with a t statistic of -4.81. This is more negative than the critical value for the 1 percent level, which is -4.09. From this, one can conclude that the exchange rate is weakly exogenous with respect to money supply. For the last regression, the sample was cut short at May 1923 because of the wild behavior of money at the end of the hyperinflation that has been documented often (Cagan, 1956 and Georgoutsos and Kouretas 2004).

A complementary procedure shows that money is the statistically more endogenous variable in the system or, in other words, it is Granger-caused by both the exchange rate and the price level. To do this, consider that during the German hyperinflation all the relationships for the classical monetary model were holding, as suggested by Frenkel (1980). For this episode, both real income and the foreign price level are of little relevance given the large variability of the domestic nominal variables. Thus, equations (6) and (8) are reduced to the following:

$$p_t = m_t + v_t \quad (14)$$

$$p_t = e_t + rer_t \quad (15)$$

These equations for the log levels of the variables do not imply any particular Granger causality structure but they can be used to derive it. The first step consists in running a regression of the rate of change of each variable against the lagged levels of the other two and lags of the rate of change of the three variables. In Garces Diaz (2016, 2017) this was done only for the rate of inflation as the dependent variable. These regressions are:

$$\Delta e_t = \beta_p^e p_{t-1}^e + \beta_m^e m_{t-1} + \phi_p^e \Delta p_{t-1} + \phi_m^e \Delta m_{t-1} \quad (16)$$

$$\Delta p_t^p = \beta_e^p e_{t-1} + \beta_m^p m_{t-1} + \phi_e^p \Delta e_{t-1} + \phi_m^p \Delta m_{t-1} \quad (17)$$

$$\Delta m_t = \beta_e^m e_{t-1}^m + \beta_p^m p_{t-1} + \phi_e^m \Delta e_{t-1} + \phi_p^m \Delta p_{t-1} \quad (18)$$

Where the superscripts refer to the dependent variable and the subscripts to the regressors. These are unbalanced regressions due to the fact that the integration order of the dependent variable, which is in first difference, is lower than that of the regressors in levels. Because of this, the t statistics for the coefficients with the higher order of integration cannot be analyzed with standard inference. However, differently from what happens in the case when there is one nonstationary regressor, which can generate a spurious relationship, as shown by Granger and Newbold (1974), the presence of two nonstationary regressors helps to avoid this problem, provided the correct distributions are used in the inference. As discussed in Pagan and Wickens (1989), if the nonstationary regressors are unrelated to the dependent variable, then their coefficients must be zero. If they form a valid cointegration relationship they form a valid regression but with coefficients with a nonstandard distribution. This is done here transforming the unbalanced regressions into unrestricted error correction models and use the Ericsson and MacKinnon (2002) tables mentioned before.

The procedure is the following. First, when the monetary model does not work as prescribed by theory, the coefficients for the nonstationary variables should be zero in all three equations. So, this procedure is a simple way to test if the monetary model is working in a given sample or not. Second, if the monetary model is truly working, the nominal variables

e_t , p_t and m_t can be used as proxies of one another. In principle, the estimates of those parameters should be very similar in absolute value because of the restrictions on the parameters of the monetary model. The interesting part comes from the fact that if one regression is valid in the sense that the nonstationary variables are statistically significant, and at least one must be because otherwise there would be no cointegration, then the coefficients for the nonstationary variables will have the signs reversed. The variable with negative sign takes the place of the one for which the error mechanism is going to be formed. Table 2 shows the estimates for the unbalanced equations. Again, the table has shaded cells for the statistics emphasized.

Table 2: Unbalanced Regression Tests for Monetary Variables in German Hyper-inflation

Regressors	Dependent variables		
	Δe_t	Δp_t	Δm_t
e_{t-1}	n.i. ·	0.36 (2.40)	0.98 (4.26)
p_{t-1}	-0.10 (-0.25)	n.i. ·t	-0.95 (-4.26)
m_{t-1}	0.30 (1.25)	-0.35 (-1.78)	n.i. ·
Δe_t	n.i. ·	n.s. ·	0.45 (4.22)
Δe_{t-1}	0.36 (2.69)	1.05 (4.11)	n.s. ·
Δp_{t-1}	n.s. ·	1.68 (-4.65)	n.i. ·
Period	21:02-23:08	21:03-23:08	21:03-23:08
T	31	31	31
Adjusted R^2	0.33	0.63	0.80
S.E.	0.40	0.28	0.21
Jarque-B	0.12	0.68	0.26
LM(2) autocor	0.30	0.93	0.92
LM(1) ARCH	0.58	0.64	0.26

t statistics are between parentheses.

n.s. means excluded for being nonsignificant.

For Jarque-B, LM(2) autocor LM(2) ARCH the p values are provided.

For the first regression, that for Δe_t , the negative sign corresponds to the coefficient for Δe_{t-1} . Its t statistic is small, -0.25, very far from showing significance. For the next regression, corresponding to Δp_t , the t statistic in the shaded cell is -1.78, again very far from being significant. On the contrary, the t statistic in the shaded cell for regression corresponding to Δm_t is -4.26, which is more negative than the critical value of -4.09 in the Ericsson and

MacKinnon table 3. It is also important to notice that the absolute values for the coefficients of e_{t-1} and p_{t-1} in that regression, 0.98 and 0.95 respectively, are very similar. This is because as shown in the previous table the exchange rate and the price level are cointegrated with coefficients (1,-1). For these equations, the sample was shortened to the first half of 1923, as has been common practice in models on the German hyperinflation (as, for example, in Cagan's classical paper) because during the second part of 1923, money growth behaved wildly, in a way that has resisted a clear explanation within a model.

Thus, during the German hyperinflation, the pricing equation working was really (7) and not (1), as is often stated. In other words, there was an implicit price level determination regime with the exchange rate rather than money as the driving variable. Of course, this does not contradict that people were expecting increases in the money supply after rising prices. Money can be very well the final cause of inflation but there is at least a practical relevance in asking in general terms why is that Granger causality varies according to the monetary regime. This is best exemplified looking at two cases that share several similarities and are not pure cases of hyperinflation. Indeed, the approach applied to the German hyperinflation can be applied to any case regardless of the level of inflation provided there is enough information in the sample.

4.2 Mexico (1932-2013)

A cointegration analysis for Mexico and Brazil similar to that applied to the German hyperinflation episode is carried out in Garcés-Díaz (2016) but here an alternative approach is taken. This can also be applied to the cases when the inflation rate is determined instead of the price level, something where the concept of cointegration is appropriate. However, as the variables involved contain stochastic trends, regular Granger causality tests might not be applicable because the covariance matrix of the joint distribution of the VAR parameters is singular (Killian and Lütkepohl, 2017). However, Toda and Yamamoto (1995) showed that there is a simple solution to this problem. This consists in adding an additional lag to the VAR process fitted to the data for each degree of integration of the series after the optimal number of lags has been determined. This produces a nonsingular covariance matrix for the parameters related to the lags optimally determined. The Wald test is then applied to the optimal number of lags alone ignoring the additional ones arising from the integration of the series.

The Toda-Yamamoto test can account for both long-run and short-run Granger causality but it cannot distinguish between them. When there is long-run causality, then the series must be cointegrated, as discussed above, but the converse is not true. In the cases of Mexico and Brazil, for most of the sample there was price-level determination and cointegration, as shown in Garcés Díaz (2016). However, when both countries adopted inflation targeting, there is no long-run causality so the test indicates the presence or absence of short-run causality. In the case of the United States there is no evidence of price level determination. Only Sweden, among all developed countries, experienced with it during a brief period, as mentioned before. Thus one must consider that in the cases of Mexico and Brazil, the Toda-Yamamoto tests confirm the evidence of long-run Granger causality for the respective subsample. In the case of the United States, the result must be interpreted as evidence of short-run causality. These are the tests applied below. As the data periodicity is annual and it is in turn divided into subsamples (regimes), there is a constrain to the number of lags one can include in the corresponding VARs. This is critical because some of the criteria suggests more than one lag and there is an ambiguity on the degree of integration of the time series, as shown in Garcés Díaz (2016). Because of this, all of the VARs are run with two lags plus an additional one that covers the degree of integration of all the series that will be presumed to be one despite that ambiguity.

The Mexican case produces the clearest application of the hypothesis of simultaneous changes in pricing equations and Granger causality. The country is usually included in the high inflation category (for example, Jones 2011), but it has a more complex and interesting story than that. It actually had an episode of low inflation, virtually identical in average to that of the US (1956 to 1971). Low inflation was again the norm since 2001 onwards. Its highest level of annual inflation was 99 percent in 1987 although it had a month of 12-month inflation of 150 per cent in 1988. The country has applied several types of exchange rate systems. Most interesting, the theoretical monetary relationships, for example PPP and the QEM, appear in its data with stunning clarity for much of the sample.

In Mexico's case, it is easy to find the dates of change for the pricing equation and Granger causality because there were well-known public events that signaled them. The first episode goes from 1932, when the country like many others abandoned the Gold standard, to 1981. In this long span, the central bank had as an objective to maintain the stability of the nominal exchange rate, despite several devaluations. This required to implicitly determine the price

level, which was tied to the level of currency even though there was no explicit official statement of doing so. Thus, for this period, the pricing equation at work was (1). The implications of that were that the QEM held and that money Granger-caused the price level.

In 1982, Mexico looked for financial assistance from the IMF. In the agreement with that institution, there were two key provisions that propelled a change in the price equation. First, the central bank could not longer provide financing for the public deficit through primary emission of money. Second, the IMF recommended a policy of competitive devaluations. This shifted the attention of the price setters from money to the exchange rate as the central bank's instrument to drive the price level. As in the previous regime, this was never stated as an official objective but the behavior of the price level was better described with a price level determination regime. Thus, since 1983 to 2003, the price equation was (13). This changed the Granger causality structure among the variables. Interestingly enough, the year 1982 as period of transition does not fit well in any of the two regimes. A similar transition period happens also in the case of Brazil, described next.

At the end of the 1990s, the Mexican central bank announced its intention to apply an inflation targeting regime. This was a big change because for the first time since its foundation in 1925, it was attempting to abandon any kind of policy setting better that could be described with a price level determination pricing equation. This caused that the new price equation had the form (9). This again caused a change in the Granger causality structure. Since then, both the exchange rate and money aggregates ceased to Granger-cause inflation.

The changes in Granger causality can be appreciated in Table 3, which contains the Toda-Yamamoto (1995) tests for Granger Causality for nonstationary data to the log levels of the price level (p_t), money (m_t) and the exchange rate (e_t) organized by pairs and in different regimes. However, as happens in the United States and many other economies, simple correlations between the output gap, or other measures of economic activity, are not easy to get. A possible reason for this, besides perhaps not being closely related as some economists suggest, is provided in the discussion for the United States below: for the first time, Mexico and the United States had similar pricing equations.

There are several alternatives of impulse-response functions for these VARs. Figure 2 shows the impulse-response functions for pairs of the first differences in monetary variables for the three regimes determined by different pricing equations. These confirm the structure of Granger causality for each regime. Similar results are obtained with VARs in levels and

Table 3: Mexico: Bivariate Granger Causality Tests For Nonstationary Variables in Each Regime

Dependent Variable: Price Level p_t						
	Regime 1		Regime 2		Regime 3	
Excluded Variable	$\chi^2(2)$	p-value	$\chi^2(2)$	p-value	$\chi^2(2)$	p-value
Inflationary Money $(m - y)_t$	9 .73	0 .01	2 .27	0 .32	0 .26	0 .88
Exchange Rate e_t	2 .17	0 .34	7 .02	0 .03	3 .43	0 .18

Dependent Variable: Inflationary Money $(m - y)_t$						
	Regime 1		Regime 2		Regime 3	
Excluded Variable	$\chi^2(2)$	p-value	$\chi^2(2)$	p-value	$\chi^2(2)$	p-value
Price Level p_t	1 .73	0 .42	15 .89	0 .00	2 .14	0 .34
Exchange Rate e_t	1 .52	0 .22	11 .80	0 .00	0 .23	0 .63

Dependent Variable: Exchange Rate e_t						
	Regime 1		Regime 2		Regime 3	
Excluded Variable	$\chi^2(2)$	p-value	$\chi^2(2)$	p-value	$\chi^2(2)$	p-value
Price Level p_t	2 .23	0 .33	1 .56	0 .46	1 .23	0 .54
Inflationary money $(m - y)_t$	7 .57	0 .01	0 .26	0 .61	2 .46	0 .11

The test was modified following Toda and Yamamoto (1995).

The Wald test statistic is distributed as a $\chi^2(2)$.

also imposing cointegration restrictions. Actually, in this case is possible to construct inflation models for each regime based on the long-run relationships and their respective error-correction mechanisms. They provide good forecasting performance out of sample, as shown in Garcés Díaz (2016).

However, the property of switching Granger causality makes this a more complex task than usual because most cointegration tests methods are based on the assumption of invariant Granger causality or, equivalently, the constancy of the feedback parameters. In a companion paper (Garcés Díaz 2016), this procedure is described for Mexico and other five major Latin American Economies. The equations obtained for Mexico are an error-correction model based on money for the period 1932-1981; an error-correction model based on the exchange rate for the 1983-2000 sample and; a model of noise for the period 2001-2013.

The sample for the last regime is rather short (13 data points) so it is hard to find a correlation between inflation and a measure of economic activity slack (defined here as the output gap obtained from a Hodrick-Prescott filter), which tends to be weak in the United States as well, as discussed below. However, when the movements are sharp, these two variables can be more correlated. Figure 3 shows inflation and the output gap for the said period. Only during the sharp contraction following the world financial crisis of 2008 both variables seem to be closely related with the expected sign. A longer period going forward might reveal a closer relationship but that is not a certainty.

Figure 2: Impulse Responses of Bivariate VARs in Each Regime for Mexico.

Figure 3: Output Gap and Inflation in Mexico 2001-2013

4.3 Brazil (1950-2013)

The case of Brazil is very similar to that of Mexico in terms of the use of pricing equations. The dates of change of pricing equations for both countries are even very close to each other. For the period 1950 to 1979, the pricing equation that determined inflation dynamics was that for price level determination based on money (equation (1)). There were two years of transition, 1980 and 1981, that do not fit well in any regime. For the sample 1982-1998, the working pricing equation was based on price level determination based on the exchange rate (1) (equation (7)). For the period 1999 to 2013, also related to the adoption of inflation targeting, the pricing equation had the form (9) and inflation became basically noise around a constant. As happened with the Mexican case, the tests reported in Table 4 indicate that Granger causality changed according to the movements from one regime into another. In the first one, the driving variable was money. In the second regime, the forcing nominal variable was the exchange rate and during the third regime neither money nor the exchange were systematic drivers of inflation. In Garcés Díaz (2016), detailed inflation models for each regime are estimated and outperform other alternatives in out-of-sample exercises.

4.4 United States (1880-2007)

United States inflation is one of the most studied economic phenomena and the models designed for that purpose are often applied to other economies. It might be surprising then that,

Table 4: Brazil: Bivariate Granger Causality Tests For Nonstationary Variables in Each Regime

Dependent Variable: Price Level p_t						
	Regime 1		Regime 2		Regime 3	
Excluded Variable	$\chi^2(2)$	p-value	$\chi^2(2)$	p-value	$\chi^2(2)$	p-value
Inflationary Money $(m - y)_t$	3 .99	0 .05	0 .14	0 .71	4 .27	0 .04
Exchange Rate e_t	3 .49	0 .06	25 .76	0 .00	31 .12	0 .00

Dependent Variable: Inflationary Money $(m - y)_t$						
	Regime 1		Regime 2		Regime 3	
Excluded Variable	$\chi^2(2)$	p-value	$\chi^2(2)$	p-value	$\chi^2(2)$	p-value
Price Level p_t	0 .91	0 .33	1 .77	0 .18	0 .53	0 .46
Exchange Rate e_t	1 .42	0 .23	29 .72	0 .00	1 .94	0 .16

Dependent Variable: Exchange Rate e_t						
	Regime 1		Regime 2		Regime 3	
Excluded Variable	$\chi^2(2)$	p-value	$\chi^2(2)$	p-value	$\chi^2(2)$	p-value
Price Level p_t	17 .00	0 .00	2 .50	0 .11	2 .01	0 .15
Inflationary money $(m - y)_t$	1 .58	0 .20	0 .30	0 .58	0 .91	0 .34

The test was modified following Toda and Yamamoto (1995).

The Wald test statistic is distributed as a $\chi^2(2)$.

despite this, the causes of inflation in the United States are still subject to debate. There have been two main theoretical approaches although they have also been mixed. None of them has fared particularly well in the last three decades when it comes to its forecasting performance.

The oldest approach is based on money. Although it is often stated that in the long run the only cause of inflation is money (Romer, 2011), the evidence for that has weakened because money growth has been losing its tight connection with inflation, as found by Lucas and Nicolini (2013). These authors attempt to solve the problem by proposing a new definition of the monetary aggregate M1, which in its official version had become undependable because of, for example, the introduction of “sweeps” by commercial banks (Ball, 2012).

However, there is not a clear mechanism through which money determines long run inflation while in the short run it might have no visible effect. Furthermore, although there have

been many attempts to show that money demands are stable in both the long-run (Benati, et al. 2016) and the short run (Ball, 2012), the procedures involved in these studies have ignored the possibility of a regime changes that could be hidden by the dominance of some periods. For example, Teles and Uhlig (2013) found for a large sample of countries that the relationship between the price level and money has decreased considerably since the 1990s. As since that period inflation has tended to become lower in most countries, its properties might be dominated by previous episodes of higher and more variable inflation and money growth.

Because of that, it would be better to consider equation (6) as a more proper pricing equation for the United States for the time when money had an effect on inflation, possibly from 1880 to the 60s or seventies. For many years already, it has been stated that money is not useful to forecast inflation in the United States as in the form of the P^* model (Christiano, 1989).

A more recent and popular alternative is the Phillips curve, adopted since the sixties by Keynesian economists as their pricing equation. According to Stock and Watson (2007), that model was able to produce good inflation forecasts from 1970 to 1983. However, since the outset of the Great Moderation (1984 to 2004 in their sample), they found that it had a dismal performance. More recent work has confirmed this finding for later years. In particular, the failure of the Phillips curve to forecast inflation after the 2008 economic crisis (“the missing deflation” puzzle) and the posterior recovery (“the missing inflation” puzzle) has been widely commented. Some economists have expressed doubts that inflation in a stable economic environment can really be tied to specific causes. For example, Hall (2011) suggested to consider inflation as a “near-exogenous” process, echoing what many Keynesian models from the 1950s to the early 1970s assumed (Woodford, 2008). Nevertheless, Del Negro et al. (2015) made some modifications to the standard New Keynesian model and were able to produce better forecasts for that event and following years. However, they did not address the previous the problem of poor forecasting since the mid eighties.

Goodfriend and King (2009) provide an explanation for the lack of predictive power of economic activity on inflation. They suggest that if the central bank stabilizes economic activity in their “business as usual” practices, then inflation becomes determined only by its trend and output gap or unemployment measures cannot be used to forecast it. When the central bank actively attempts to control growing inflation by raising the interest rate to slow economic activity, as the Fed did during the seventies and early eighties, slackness

measures have predictive power in inflation models. This explanation is consistent with what was observed by Stock and Watson (2007).

Differently from other cases examined in this paper, the changes of regime in the United States are harder to see because its macroeconomic variables are relatively not very volatile, the information they contain is harder to extract. For example, Sims and Zha (2006) found necessary to include money (M2), along the federal funds rate, to be able to identify the monetary policy regimes that the Federal Reserve has followed . They argue that in the seventies, monetary aggregates were part of the discussions of monetary policy.

Because of this, the date of change of one regime into another was chosen as follows. There was a regime when money was tightly related to inflation from 1888, when the sample starts, to 1983 when the connection between them began to diminish and the Great Moderation was about to start. This separation of regimes reveals that not only the correlation between these variables weakened since the beginning of the eighties: Granger causality from money growth to inflation also disappeared since the Great Moderation, as can be seen in Table 5. As money grew faster since 2008 because of Federal Reserve's quantitative easing, it is clear that the causality has not been reestablished since then.

5 Conclusions

A promising approach to get a better understanding of inflation dynamics is to consider radical regime changes of the pricing equation, not only in the parameters of a specific type. In other words, one can ponder the possibility that the whole right-hand side of an inflation model changes, including the explanatory variables and the functional form. This approach has been somehow implicit in many studies. For example, for a period of hyperinflation there is one type of pricing equation (for example, a monetarist one) that is replaced by another when inflation is low (for example, a Keynesian one).

A first consideration is to explicitly decide if the monetary regime in place determines either the price level or the inflation rate as this settles the issue of how persistent different inflationary shocks are. Next, it is important to decide if the main objective of the monetary authority is to maintain price stability or something else (for example, budget deficit financing). Then it is necessary to identify the drivers of inflation that can be regarded as systematic

Table 5: Trivariate Granger Causality Tests For US Inflation, M2 and GDP

Dependent Variable: Inflation Δp_t				
	1888-1983		1984-2007	
Excluded Variable	$\chi^2(2)$	p-value	$\chi^2(2)$	p-value
Money Growth Δm_t	6 .86	0 .01	1 .37	0 .50
GDP Growth Δy_t	0 .17	0 .68	1 .01	0 .60

Dependent Variable: Money Growth Δm_t				
	1888-1983		1984-2007	
Excluded Variable	$\chi^2(2)$	p-value	$\chi^2(2)$	p-value
Inflation Δp_t	1 .80	0 .18	1 .45	0 .48
GDP Growth Δy_t	0 .57	0 .45	11 .37	0 .00

Dependent Variable: GDP Growth Δy_t				
	1888-1983		1984-2007	
Excluded Variable	$\chi^2(2)$	p-value	$\chi^2(2)$	p-value
Inflation Δp_t	2 .43	0 .12	3 .75	0 .15
Money Growth Δm_t	0 .06	0 .81	0 .92	0 .63

The test was modified following Toda and Yamamoto (1995).

The Wald test statistic is distributed as a $\chi^2(2)$.

so one does not have to deal with many factors that, although they might be important at some periods, it is not necessary they are always acting. Thus, one has to look carefully for the variables that are the acting systematic inflationary factors at each point of time. By applying that procedure, one has to give a central role to Granger causality and conditional forecasts of inflation. A variable that has no predicting value for inflation is discarded as a systematic cause of price changes. This approach solves several puzzles such as the declining contemporary and dynamic correlations of money growth, exchange rate movements and economic slack measures with inflation.

During the German hyperinflation, money is Granger-caused by the price level and not the other way around. This has been explained as a trick played by the rational expectations of economic agents. However, there are cases, as those presented here, where the direction of Granger causality is the opposite so currently there is at least an incomplete explanation for what happens to Granger causality properties in one case or another. For Mexico and Brazil,

the Toda-Yamamoto tests show that there was change in the direction of Granger causality at given dates, obtained through a review of historical events (for example, the announcement of the adoption of inflation targeting). In both cases, the money stock was the driving force behind price level movements. Then from around 1982 to the beginning of the 2000s, the exchange rate was the driving cause behind the price level determination. In none of these episodes the main concern of the central banks was price stability. However, since around the beginning of the 2000s both countries adopted inflation targeting and the pass-through of both money and exchange rate shocks on inflation were considerably diminished to tiny effects, if any. For the United States there was also a change in Granger causality. Before the eighties, money growth Granger caused inflation but since the mid eighties onwards, the causality changed direction and money became of little use to predict inflation. This is consistent with the dominance of cashless models of inflation in recent times.

Although the approach looks promising to solve some issues of the dynamics of inflation, there are others that are left unanswered. The results for the United States shows that identifying the direction of Granger causality is not enough to obtain a forecasting model that beats simple time series models. An additional weakness is that the dates of the regime changes have to be provided by the modeler. This seems to be a rather challenging problem with no easy solution. Nonetheless, the approach seems worth trying with different data sets. Among some projects, one can study general differences in the dynamics of inflation between advanced and emerging economies. Another project is the application of the approach to the data of some relevant papers on inflation models such as that of Hendry (2001 and 2015), who uses many explanatory variables for inflation instead of the very few that are suggested in this article.

References

- [1] Ball, L. (2012), “Short-run money demand,” *Journal of Monetary Economics*, Volume 59, Issue 7, November, Pages 622-633.
- [2] Barro, R.J. (1979) “Money and the Price Level Under the Gold Standard,” *The Economic Journal*, Vol. 89, No. 353 (March).

- [3] Benati, L., Robert E. Lucas, Jr., Juan Pablo Nicolini, and Warren Weber (2016), “International Evidence on Long Run Money Demand,” NBER Working Paper No. 22475, July.
- [4] Bobeica, E. and Marek Jarociński (2017), “Missing disinflation and missing inflation: the puzzles that aren’t,” European Central Bank Working Paper 2000, January.
- [5] Bresciani-Turroni, C. (1968), “The Economics of Inflation,” 3rd. reprint, John Dickens & Co. Ltd., Northampton. Available online in <http://mises.org/library/economics-inflation-study-currency-depreciation-post-war-germany>
- [6] Cagan, P. (1956), “The Monetary Dynamics of Hyperinflation,” in *Studies in the Quantity Theory of Money*, ed. by M. Friedman. University of Chicago Press, Chicago, IL.
- [7] Campbell, J. Y., and Robert J. Shiller. (1987) “Cointegration and tests of present value models.” *Journal of Political Economy* 95(5).
- [8] Campos, J. and Neil R. Ericsson (1999), “Constructive data mining: modeling consumers expenditure in Venezuela,” *Econometrics Journal*, volume 2, pp. 226-240.
- [9] Castle, J. and David F. Hendry (2007), “Forecasting UK Inflation: the Roles of Structural Breaks and Time Disaggregation,” Oxford University, Department of Economics, Discussion Paper Series, No. 309.
- [10] Christiano, L. J. (1989), “P*: Not the Inflation Forecaster’s Holy Grail,” *Quarterly Review*, Federal Reserve Bank of Minneapolis, Fall, pp. 3-18.
- [11] Citu, F.V. (2002) “Review of the literature on the comparison of price level targeting and inflation targeting,” Federal Reserve Bank of New Zealand, Publications.
- [12] Clements, M. P. and David F. Hendry (2008) “Forecasting Annual UK Inflation Using an Econometric Model over 1875-1991,” in *Forecasting in the Presence of Structural Breaks and Model Uncertainty*, Edited by David E. Rapach and Mark E. Wohar.
- [13] De Long, J. B. (1996) “America’s Only Peacetime Inflation: The 1970s,” NBER Historical Working Paper No. 84, May.

- [14] Del Negro, M., Marc P. Giannoni, and Frank Schorfheide (2015), "Inflation in the Great Recession and New Keynesian Models." *American Economic Journal: Macroeconomics*, 7 (1): 168-96.
- [15] Dotsey, M., Shigeru Fujita and Tom Stark (2017) "Do Phillips Curves Conditionally Help to Forecast Inflation", Federal Reserve Bank of Philadelphia, WP. No. 11-40.
- [16] Engels, C. and Kenneth D. West (2005), "Exchange Rates and Fundamentals," *Journal of Political Economy*, vol 113, no. 3.
- [17] Engle, R. F., David F. Hendry and Jean-Francois Richard (1983) "Exogeneity," *Econometrica*, Vol. 51, No. 2, pp. 277-304.
- [18] Ericsson, N. R. and James G. Mackinnon (2002), "Distributions of error correction tests for cointegration," *Econometrics Journal*, v.5, pp285-318.
- [19] Faust, J. and Jonathan H. Wright (2013) "Forecasting Inflation," *Handbook of Economic Forecasting*, Ch. 1, vol. 2, pp 2-56.
- [20] Frenkel, J. (1980) "Exchange Rates, Prices, and Money: Lessons from the 1920's," in *Interwar Macroeconomics From the Perspective of the 1970's*, May, American Economic Association, pp 235-242.
- [21] Friedman, M. (1987), "Quantity Theory of Money," *The New Palgrave Dictionary of Economics*, First Edition, Edited by John Eatwell, Murray Milgate and Peter Newman, MacMillan.
- [22] Garcés Díaz, D. G. (2016) "Changes in Inflation Predictability in Major Latin American Countries," Banco de México, Documento de Investigación No. 2016-20, December 2016.
- [23] Garcés Díaz, D. G. (2017) "Explaining Inflation with a Classical Dichotomy Model and Switching Monetary Regimes: Mexico 1932-2013," Banco de México, Documento de Investigación No. 2017-20, December 2017.
- [24] Georgoutsos D. A. and Georgios P. Kouretas (2004), "A Multivariate I(2) cointegration analysis of German hyperinflation," *Applied Financial Economics*, 14:1, 29-41.

- [25] Goodfriend, M. and Robert G. King (2009) “The Great Inflation Drift”, NBER Working Paper No. 14862, April.
- [26] Gordon, R.J. (2013) “The Phillips Curve is Alive and Well: Inflation and the NAIRU During the Slow Recovery,” NBER Working Paper No. 19390 August 2013.
- [27] Granger, C.W.J. and P. Newbold (1974) “Spurious Regressions in Econometrics,” *Journal of Econometrics* 2, 111-120.
- [28] Hall, R.E. (2002) “Controlling the Price Level”, *Contributions to Macroeconomics*, Volume 2, Issue 1, Article 5.
- [29] Hall, R.E. (2011) “The Long Slump”, *American Economic Review*, 101 (April 2011): 431-469.
- [30] Hall, T. E. and William R. Hart (2012) “The SamuelsonSolow Phillips Curve and the Great Inflation,” *History of Economics Review*, Volume 55, 2012 - Issue 1.
- [31] Hatanaka, M. (1996) “Time-Series-Based Econometrics: Unit Roots and Co-Integrations,” *Oxford University Press*.
- [32] Hendry, D. F. (2001), “Modelling UK inflation, 1875-1991”, *Journal of Applied Econometrics*, v. 16, Issue 3, pp. 255-275.
- [33] Hendry, D. F. (2015), “Introductory Macro-econometrics: A New Approach”, Timberlake Consultants Ltd.
- [34] Hendry, D. F. and Neil R. Ericsson (1991), “An Econometric Analysis of U.K. Money Demand in Monetary Trends in the United States and the United Kingdom by Milton Friedman and Anna J. Schwartz,” *The American Economic Review*, Vol. 81, No. 1, pp. 8-38.
- [35] Jones, C. I. (2011) “Macroeconomics,” Second Edition, W.W. Norton & Company.
- [36] Jonung, L. (1979) “Knut Wicksell’s Norm of Price Stabilization and Swedish Monetary Policy in the 1930s,” *Journal of Monetary Economics*, 5, 459-496.
- [37] Juselius, K. (2006), “The Cointegrated VAR Model: Methodology and Applications”, *Advanced Texts in Econometrics*, Oxford University Press.

- [38] Killian, L. and Helmut Lütkepohl (2017), "Structural Vector Autoregressive Analysis," Cambridge University Press, 2017.
- [39] Levy, D. (2002) "Cointegration in Frequency Domain," *Journal of Time Series Analysis* 23.
- [40] Lütkepohl, H. (2004) "Vector Autoregressive and Vector Error Correction Models," in *Applied Time Series Econometrics*, Lütkepohl and Markus Krätzig (ed), Cambridge University Press.
- [41] Lucas, R.E. and Juan Pablo Nicolini (2013) "On the Stability of Money Demand", Meeting Papers 353, Society for Economic Dynamics.
- [42] MacDonald, R. and Mark P. Taylor (1993) "The Monetary Approach to the Exchange Rate: Rational Expectations, Long-Run Equilibrium, and Forecasting," *Staff Papers*, International Monetary Fund, Vol. 40, No. 1.
- [43] Moosa, I. A. and George Towadros (1999), "The Monetary Model of Exchange Rates in the History of Economic Thought", *History of Economics Review*, Volume 29 - Issue 1.
- [44] Makinen, G.E. and Thomas Woodward (1988), "The Transition from Hyperinflation to Stability: Some Evidence," *Eastern Economic Journal*, Vol. 14, No. 1, pp. 19-26.
- [45] Pagan, A. R. and M. R. Wickens (1989), "A survey of some recent econometric methods," *Economic Journal* 99, pp. 962-1025.
- [46] Roberts, J. M. (1995), "New Keynesian Economics and the Phillips Curve", *Journal of Money, Credit and Banking*, Vol. 27, No. 4, Part 1, November, pp. 975-984.
- [47] Romer, D. (2011), "Advanced Macroeconomics", McGraw-Hill Series Economics.
- [48] Samuelson, P. and Robert M. Solow (1960) "Analytical Aspects of Anti-Inflation Policy," *The American Economic Review*, Vol. 50, No. 2, Papers and Proceedings of the Seventy-second Annual Meeting of the American Economic Association, May, pp.177-194.
- [49] Sargent, T. J. and Neil Wallace (1973), "Rational Expectations and the Dynamics of Hyperinflation," *International Economic Review*, vol. 14, no. 2.

- [50] Sargent, Thomas; Williams, Noah; Zha, Tao (2009), “The Conquest of South American Inflation.”, *Journal of Political Economy*, April 2009, v. 117, iss. 2, pp. 211-56.
- [51] Sims, C. and Tao Zha, (2006), “Were There Regime Switches in U.S. Monetary Policy?”, *The American Economic Review*, Vol. 96, No. 1, March, pp. 54-81.
- [52] Stock, James W. and Mark W. Watson (2007), “Why Has Inflation Become Harder to Forecast?” *Journal of Money, Credit, and Banking*, Supplement to Vol. 29, No.1 (February): 3-33.
- [53] Teles, P. and Harald Uhlig (2013) “Is Quantitative Theory Still Alive?,” European Central Bank, Working Paper Series, No. 1605, November.
- [54] Toda, H. and Taku Yamamoto (1995), “Statistical inference in vector autoregressions with possibly integrated processes,” *Journal of Econometrics*, Volume 66, Issues 1-2, March-April, Pages 225-250.
- [55] Woodford, M. (2008), “How Important is Money in the Conduct of Monetary Policy?” *The Journal of Money, Credit, and Banking*, Volume 40, Issue 8, December, pp 1561-1598.

A Data Sources

All the series analyzed in the paper were transformed to natural logarithms. As discussed in Garcés Díaz (2016), the integration properties of the data were evaluated for the whole sample and they are ambiguous in some cases as some could be taken as $I(2)$ instead of $I(1)$. There is no point in analyzing such property for each regime as the results would be undependable given the small size of the subsamples.

The data for this paper come from the following sources:

German Hyperinflation:

All data was copied directly from the tables in the book by Bresciani-Turroni (1968).

México:

The GDP (Y) for Mexico was constructed with the real series in local currency (base 2008) from the International Monetary Fund’ WEO Database October 2013 and extrapolated

backwards from 1979 to 1900 with the rates of change from the historical series of Instituto Nacional de Estadística, Geografía e Informática (INEGI).

The price level P , was constructed with the series of National Consumer Price Index from 2001 to 2013 and it was extrapolated backwards from 2000 to 1932 with the implied variation rates of the Wholesale Price Index for Mexico City.

The nominal exchange rate E , currency M , and P were obtained 1980-2002 Instituto Nacional de Estadística, Geografía e Informática (INEGI) at: <http://www.inegi.gob.mx>; or from Banco de México at: <http://www.banxico.org.mx>.

For the period 1940-1979, it was taken from the book “Estadísticas Históricas del INEGI.”

Brazil:

The price level data was obtained from the website Ipeadata. The rest of the data was taken from the International Finance Statistics (IFS).

United States:

CPI (P^{US}): Department of Labor Bureau of Labor Statistics, at:
<ftp://ftp.bls.gov/pub/special.requests/cpi/cpiait.txt>

M2 was obtained for the period 1960-2007 from the information system FRED by the Federal Reserve Bank of St. Louis. This series was extrapolated backwards to 1880 using the rates of growth of the M2 series taken from the data of Rapach and Wohart (2002) “Testing the monetary model of exchange rate determination: new evidence from a century of data,” *Journal of International Economics* 58 (2002) 359385.

GDP was obtained from FRED for the period 1947-2007 and it was extrapolated backwards to 1880 using the rates of growth of the M2 series taken from the data of Rapach and Wohart (2002).