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The Radial Radio Number and the Clique Number of a Graph

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Abstract: ***Let*** $G(V(G),E(G))$ ***be a graph. A radial radio labeling, f, of a connected graph G is an assignment of positive integers to the vertices satisfying the following condition:*** $d(u,v)+|f(u)-f(v)|\geq 1+r(G)$***, for any two distinct vertices*** $u,v\in V(G)$***, where*** $d(u,v)$ ***and*** $r(G)$ ***denote the distance between the vertices u and v and the radius of the graph G, respectively. The span of a radial radio labeling f is the largest integer in the range of f and is denoted by span(f). The radial radio number of G,*** $r(G)$***, is the minimum span taken over all radial radio labelingsof G. In this paper, we construct a graph a graph for which the difference between the radial radio number and the clique number is the given non negative integer.***

Keywords: diameter, frequency assignment problem, radius, radio labeling, radio number, radial radio number, radial radio number.AMS Subject Classification Code(2010):05C78

I. INTRODUCTION

In this paper, by a graph, we mean only finite, simple, undirected and connected graph. For basic notations and terminology, we follow [4]. Let $G=(V(G),E(G))$ be a graph. The *distance* $d(u,v)$ between any two vertices u and v, is the length of a shortest $(u,v)-$ path in G. The *eccentricity*, $e(u)$, of a vertex u in $V(G)$ is the distance of a vertex farthest from u. The *radius* of a graph G is the minimum eccentricity among all the vertices and is denoted by $r(G)$ or r. The *diameter* of G is the maximum eccentricity among all the vertices and is denoted by $diam(G)$ or d. The relation between $r(G)$ and $diam(G)$ is given by the inequality $r(G)\leq diam(G)\leq 2r(G)$ [8]. For further details on distance in graphs, one can refer [5].

For a subset S of $V(G)$, let $<S>$ denote the induced subgraph of G induced by S. A *clique* C is a subset of $V(G)$ with maximum number of vertices such that $<C>$ is complete. The *clique number* of a graph G, denoted by $\omega(G)$ or ω, is the number of vertices in a clique of G.

In 1960's Rosa[12] introduced the concept of graph labeling. A *graph labeling* is an assignment of numbers to the vertices or edges or both, satisfying some constraints. Rosa named the labeling introduced by him as $\beta-valuation$ and later on it becomes a very famous interesting graph labeling called *graceful labeling*, which is the origin for any graph labeling problem. Motivated by the real life problems, many mathematicians introduced various labeling concepts[9]. Here, we see one of the familiar graph labelings in graph theory.

The problem of assigning frequencies to the channels for the FM radio stations is known as *Frequency Assignment Problem* (FAP). This problem was studied by W. K. Hale[10].

In a telecommunication system, the assignment of channels to FM radio stations play a vital role. Motivated by the FAP, Chartrand et al.[6] introduced the concept of radio labeling. For a given k, $1\leq k\leq diam(G)$,a *radio k-coloring*, f, is an assignment of positive integers to the vertices satisfying the following condition:

$$d(u,v)+|f(u)-f(v)|\geq 1+k \qquad (1)$$

for any two distinct vertices $u,v\in V(G)$. Whenever, $diam(G)=k$, the radio k- coloring is called a *radio labeling*[7] of G. The *span* of a radio labeling f is the largest integer in the range of f and is denoted by $span(f)$. The *radio number* of G is the minimum span taken over all radio labelings of G and is denoted by $rn(G)$. Motivated by the work of Chartrand et al., on radio labeling, KM. Kathiresan and S. Vimalajenifer[11] introduced the concept of radial radio labeling. A *radial radio labeling* f of G is a function $f:V\rightarrow\{1,2,...\}$ satisfying the condition,

$$d(u,v)+|f(u)-f(v)|\geq 1+r(G) \qquad (2)$$

for any two distinct vertices $u,v\in V(G)$. This condition is obtained by taking $k=r(G)$ in (1). The above condition is known as *radial radio condition*. The *span* of a radial radio labeling f is the largest integer in the range of f. The *radial radio number* is the minimum span taken over all radial radio labelings of G and is denoted by $rr(G)$.

That is, $rr(G)=\min_{f}\max_{v\in V(G)} f(v)$, where the minimum runs over all radial radio labelings of G.

Let f be a radial radio labeling of a graph G and let C be a clique in G.

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Then the minimum label in C under f is denoted by $m_f(C)$. That is, $m_f(C) = \min_{v\in C} f(v)$.

Also, the maximum of all such $m_f(C)$, where the maximum runs over all cliques in G is denoted by $Cmm_f(G)$ and is called the *clique min max of G* under f.

In this paper, we construct some families of graphs with radial radio number $\omega + m$, for any given $m \geq 0$ and $\omega \geq 3$.

Now, we present some basic results, which are helpful for further investigation.

The following three theorems have been proved in [2].

Theorem 1.1 Let G be a simple connected graph with a full vertex. Then $rr(G) \geq \omega(G)$.

Theorem 1.2 Let G be any simple connected graph. Then $rr(G) \geq \Delta(r-1)+2$, where Δ and r are the maximum degree and the radius of G, respectively.

Theorem 1.3 Let f be a radial radio labeling of a graph G. Then $span(f) \geq Cmm_f(G) + (\omega-1)r$, where ω is the clique number of G and r is the radius of G.

The next theorem has been established in [3].

Theorem 1.4 For any G, $rr(G) = 2$ if and only if $G \cong K_{1,n}$, $n \geq 1$.

And we stated the following theorem which has been proved in [11].

Theorem 1.5 Let W_n, $n \geq 3$ be the wheel graph. Then

$$rr(W_n) = \begin{cases} 3, & \text{if n is even} \\ 4, & \text{if n is odd} \end{cases}$$

Throughout this paper, let $\oplus$ denote the addition modulo $n-1$.

II. RADIAL RADIO NUMBER AND CLIQUE NUMBER

In this section, we construct graphs, for which the radial radio number is $\omega + m$, where $\omega \geq 3$ is the clique number and $m \geq 0$ is any given integer.

When $m = 0$, it is trivial that, K_ω is the required graph with radial radio number ω. Therefore, we assume that, $m \geq 1$.

Theorem 2.1 *For any given* $m \geq 1$, *there is a graph* G *with* $\omega = 3$ *and* $rr(G) = \omega + m$.

Proof. Given that $\omega = 3$.

When $m = 1$, W_n, n is odd is the required graph. We have, by Theorem 1.5 $rr(W_n) = 4$.

When $m = 2$, consider the graph G' with vertex set $V(G') = \{x, y, z, w, v\}$ andedgeset $E(G') = \{xy, yz, xz, xw, vw\}$.

Define $g : V(G') \to \{1,2,3,...\}$ such that

$g(x) = 1$

$g(y) = 3$

$g(z) = 5$

$g(w) = 4$

$g(v) = 2$

It is obvious that, g is a radial radio labeling for G' and so $span\ g = 5$ and $rr(G) \leq 5$. Also $\Delta(G') = 3$ and $r = 2$, by Theorem 1.2, we have $rr(G') \geq 5$. Thus $rr(G') = 5$. Therefore, assume that $m \geq 3$. Consider a graph G with vertex set $V(G) = \{x, y, z, w, v_1, v_2, ..., v_m\}$ and edge set $E(G) = \{xy, xz, yz, xw\} \cup \{wv_i : 1 \leq i \leq m\}$.We have $r = 2$ .Define $f : V(G) \to \{1,2,3,...\}$ such that

$f(x) = 1$

$f(y) = 3$

$f(z) = 5$

$f(v_i) = i+1$, $1 \leq i \leq m$

$f(w) = f(v_m) + 2 = m+3$

Now, we have to show that f is a radial radio labeling of G. That is, to prove that f satisfies the following condition:

$$d(u,v) + |f(u) - f(v)| \geq 3 \qquad (3)$$

for any two distinct vertices $u, v \in V(G)$.

Since $d(x, y) = 1$ and $d(x, y) + |f(x) - f(y)| = 1 + |1-3| \geq 3$. Hence the pair (x, y) satisfies (3). Similarly, the pairs (x,z), (y,z) and (x,w) satisfy (3).

For the pairs (v_i, v_j), $1 \leq i \neq j \leq m$, we have $d(v_i, v_j) + |f(v_i) - f(v_j)| = 2 + |i+1-(j+1)| \geq 3$.

Thus the pairs (v_i, v_j), $1 \leq i \neq j \leq m$, satisfies (3).Since $d(w, v_i) + |f(w) - f(v_i)| = 1 + |m+3-(i+3)| > 3$, the pairs (w, v_i), $1 \leq i \leq m$ satisfy (3).

From the above discussion, we conclude that f is a radial radio labeling of G. This implies that, $span\ f = m+3$ and hence

$rr(G) \leq m+3$ (4)

Also, we have $\Delta(G) = m+1$ and $r = 2$, by Theorem 1.2,

$rr(G) \geq m+3$ (5)Combining the inequalities (4) and (5), we get $rr(G) = 3+m$. ■

Theorem 2.2 For any given $m \geq \omega - 1 \geq 3$, there exists a graph G with $rr(G) = \omega + m$.

Proof. For $m = 3$ and $\omega = 4$, consider the graph G' with vertex set $V(G') = \{v_1, v_2, v_3, v_4, u_1, u_2\}$ and the edge set $E(G') = \{v_i v_j : 1 \leq i \neq j \leq 4\} \cup \{v_4 u_i : i = 1,2\}$. It is easy to verify that, $rr(G') = \omega + m = 7$.

Assume that, $\omega \geq 5$ and $m \geq 4$.

Consider the graph G with vertex set $V(G) = \{v_1, v_2, ..., v_\omega, u_1, u_2, ..., u_{m-1}\}$ and the edge set $E(G) = \{v_i v_j : 1 \leq i \neq j \leq \omega\} \cup$

$\{v_i u_j : 1 \le i \le \lfloor \omega/2 \rfloor + 1 \; and \; \lfloor \omega/2 \rfloor + 1 \le j \le m-1\} \cup$
$\{v_i u_j : \lfloor \omega/2 \rfloor + 2 \le i \le \omega \; and \; 1 \le j \le \lfloor \omega/2 \rfloor\}$.
Here, $r(G) = 2$ .Define

$f : V(G) \to \{1,2,3,...\}$ such that

$f(v_i) = 2i-1$, $1 \le i \le \omega$

$f(u_i) = f(v_i)+1$, $1 \le i \le \lfloor \omega/2 \rfloor$

$f(u_j) = f(v_i)+1, \lfloor \omega/2 \rfloor + 2 \le i \le \omega$ and $\lfloor \omega/2 \rfloor + 1 \le j \le \omega - 1$

$f(u_j) = f(u_{j-1})+1$, $\omega \le j \le m$

Now,we have to show that f satisfies the following condition:

$$d(u,v) + |f(u) - f(v)| \ge 3 \qquad (6)$$

for any two distinct vertices u and v of G.

Case 1 Consider the pair (v_i, v_j), $1 \le i \ne j \le \omega$.

Now, $d(v_i,v_j) + |f(v_i) - f(v_j)| = 1 + |(2i-1)-(2j-1)| \ge 3$.

Thus the pair (v_i, v_j), $1 \le i \ne j \le \omega$ satisfies (6).

Case 2 Consider the pair (u_i, u_j), $1 \le i \ne j \le m-1$.

We have $d(u_i,u_j) \ge 2$, $1 \le i \ne j \le m-1$.

Subcase 2a If $1 \le i \ne j \le \omega - 1$, then $d(u_i,u_j) = 2$.

Now, $d(u_i,u_j) + |f(u_i) - f(u_j)| =$

$2 + |f(v_i) + 1 - (f(v_j)+1)| \ge 3$

Subcase 2b If $\lfloor \omega/2 \rfloor + 1 \le i \ne j \le \omega - 1$, then $d(u_i,u_j) = 2$.

Also, $d(u_i,u_j) + |f(u_i) - f(u_j)| =$

$2 + |f(v_s) + 1 - (f(v_t)+1)| \ge 3$,where $\lfloor \omega/2 \rfloor + 2 \le s \ne t \le \omega$.

Subcase 2c If $\omega \le i \ne j \le m-1$, then $d(u_i,u_j) = 2$.

We have, $d(u_i,u_j) + |f(u_i) - f(u_j)| =$

$2 + |f(u_{i-1}) + 1 - (f(u_{j-1})+1)| \ge 3$.

Subcase 2d If $1 \le i \le \lfloor \omega/2 \rfloor$ and $\lfloor \omega/2 \rfloor + 1 \le j \le \omega - 1$, then $d(u_i,u_j) = 3$.

Since $d(u_i,u_j) = 3$, it is obvious that the pair (u_i,u_j), $1 \le i \le \lfloor \omega/2 \rfloor$ and $\lfloor \omega/2 \rfloor + 1 \le j \le \omega - 1$, satisfies (6).

Thus the pair (u_i,u_j), $1 \le i \ne j \le m-1$ satisfies (6).

Case 3 Consider the pair (v_i,u_j), $1 \le i \le \omega$ and $1 \le j \le m-1$.It is easy to verify that the pair (v_i,u_j), $1 \le i \le \omega$ and $1 \le j \le m-1$ satisfies (6).

Thus every pair of vertices of G satisfies (6) and so f is a radial radio labeling of G. Also, $span\ f = \omega + m$ and hence $rr(G) \le \omega + m$.

Also, by Theorem 1.3, we have $rr(G) \ge m + \omega$. Thus $rr(G) = \omega + m$. ■

Theorem 2.3 For any given $\omega \ge 4$, there exists a graph G with $rr(G) = \omega + 1$.

Proof.

Take $V(G) = \{x, v_1^{(1)}, v_2^{(1)}, ..., v_{\omega-1}^{(1)}, v_1^{(2)}, v_2^{(2)}, ..., v_{\omega-1}^{(2)}, ..., v_1^{(\omega)}, v_2^{(\omega)}, ..., v_{\omega-1}^{(\omega)}\}$ and

$E(G) = \{x v_i^{(j)} : 1 \le i \le \omega - 1, 1 \le j \le \omega\} \cup$
$\{v_i^{(j)} v_k^{(j)}, 1 \le i \ne k \le \omega - 1, 1 \le j \le \omega\} \cup$
$\{v_1^{(1)} v_1^{(j)} : 2 \le j \le \omega\} \cup$
$\{v_i^{(2)} v_k^{(i+1)} : 2 \le i \le \omega - 1, 2 \le k \le \omega - 1\}$.

Here, $rad(G) = 1$ .Define $f : V(G) \to \{1,2,3,...\}$ such that

$f(x) = \omega$

$f(v_i^{(2)}) = i$, $1 \le i \le \omega - 1$

For $1 \le j \le \omega - 1$ and $2 \le i \le \omega - 1$,

$$f(v_j^{(i+1)}) = \begin{cases} f(v_j^{(i)}) + 1, & if \; f(v_j^{(i)}) + 1 \le \omega - 1 \\ f(v_j^{(i)}) \oplus 1, & \text{otherwise} \end{cases}$$

$f(v_1^{(1)}) = \omega + 1$

$f(v_j^{(1)}) = j - 1$, $2 \le j \le \omega - 1$

Now,we have to show that f is a radial radio labeling for G. The radial radio condition for G is

$$d(u,v) + |f(u) - f(v)| \ge 2 \quad (7)$$

for any two distinct vertices u and v of G.

Case 1 Consider the pair $(x, v_1^{(1)})$. Since $d(x, v_1^{(1)}) = 1$, we have $d(x, v_1^{(1)}) + |f(x) - f(v_1^{(1)})| = 1 + |\omega - (\omega+1)| \ge 2$.

Therefore, the pair $(x, v_1^{(1)})$ satisfies (7).

Case 2 Consider the pair $(x, v_i^{(1)})$, $2 \le i \le \omega - 1$.

Since $d(x, v_i^{(1)}) = 1$, we have $d(x, v_i^{(1)}) + |f(x) - f(v_i^{(1)})| = 1 + |\omega - (\omega+1)| \ge 2$. and hence the pair $(x, v_i^{(1)})$, $2 \le i \le \omega - 1$ satisfies (7).

Case 3 Consider the pair $(x, v_i^{(j)})$, $2 \le i \le \omega - 1$ and $2 \le j \le \omega$. Here, $d(x, v_i^{(j)}) = 1$, we have $d(x, v_i^{(j)}) + |f(x) - f(v_i^{(j)})| \ge 2$, since $f(v_i^{(j)}) \le \omega - 1$, for all $2 \le i \le \omega - 1$ and $2 \le j \le \omega$.Thus the pair $(x, v_i^{(j)})$, $2 \le i \le \omega - 1$ and $2 \le j \le \omega$ satisfies (7).

Case 4 Consider the pair $(v_i^{(j)}, v_s^{(t)})$, $2 \le j,t \le \omega$, $1 \le i,s \le \omega - 1$.

Subcase 4a If $i = s$ and $j \ne t$, then $d(v_i^{(j)}, v_i^{(t)}) = 2$ and so it is easy to verify that the pair $(v_i^{(j)}, v_i^{(t)})$ satisfies (7).

Subcase 4b If $i \ne s$ and $j = t$, then $d(v_i^{(j)}, v_s^{(j)}) = 1$. Now, $d(v_i^{(j+1)}, v_s^{(j+1)}) + |f(v_i^{(j+1)}) - f(v_s^{(j+1)})| =$

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$$\begin{cases} 1+|\{f(v_i^{(j)})+1\}-\{f(v_s^{(j)})+1\}|, \text{if } f(v_i^{(j)})+1, \\ \qquad f(v_s^{(j)})+1 \le \omega-1 \\ 1+|\{f(v_i^{(j)})\oplus 1\}-\{f(v_s^{(j)})+1\}|, \text{if } f(v_i^{(j)})+1 > \omega-1 \\ \qquad f(v_s^{(j)})+1 \le \omega-1 \\ 1+|\{f(v_i^{(j)})+1\}-\{f(v_s^{(j)})\oplus 1\}|, \text{if } f(v_i^{(j)})+1 \le \omega-1, \\ \qquad f(v_s^{(j)})+1 > \omega-1 \\ 1+|\{f(v_i^{(j)})\oplus 1\}-\{f(v_s^{(j)})\oplus 1\}|, \text{if } f(v_i^{(j)})+1 > \omega-1, \\ \qquad f(v_s^{(j)})+1 > \omega-1 \end{cases}$$

This implies that, $d(v_i^{(j+1)},v_s^{(j+1)})+|f(v_i^{(j+1)})-f(v_s^{(j+1)})|\ge 2$ and hence the pair $(v_i^{(j)},v_s^{(j)})$ satisfies (7). Hence the pair $(v_i^{(j)},v_s^{(t)})$, $2\le j,t\le\omega$, $1\le i,s\le\omega-1$ satisfies (7).

Case 5 Consider the pair $(v_i^{(1)},v_k^{(j)})$, $2\le j\le\omega$, $1\le i,k\le\omega-1$. Since $d(v_i^{(1)},v_k^{(j)})=2$, for all $2\le j\le\omega$ and $1\le i,k\le\omega-1$, it is easy to verify that the pair $(v_i^{(1)},v_k^{(j)})$, $2\le j\le\omega$, $1\le i,k\le\omega-1$ satisfies (7).

From the above discussion, we conclude that f is a radial radio labeling of G and hence $span\ f=\omega+1$. This forces that, $rr(G)\le\omega+1$.

We can see that G contains ω copies of K_ω.

Let $V(K_\omega^{(i)})=\{x,v_1^{(i)},v_2^{(i)},\ldots v_{\omega-1}^{(i)}\}:1\le i\le\omega$. Then from the definition of G, we have $d(x,v)=1$, for all $v\in V(G)-\{x\}$. By Theorem 1.1, we have $rr(G)\ge\omega$. Suppose $rr(G)=\omega$. Then there exists a radial radio labeling c such that $span\ c=\omega=rr(G)$.

Consider the copies $K_\omega^{(i)}$, $2\le i\le\omega$. Since $d(v_j^{(i)},v_k^{(i)})=1$, $1\le j\ne k\le\omega-1$ and $r=1$, the integers 1,2,3,..., ω are enough to label the vertices of $K_v^{(i)}$, for some i. First, if we label the vertices of $K_\omega^{(3)}$ with integers 1,2,3,..., ω. Since $d(v_i^{(2)},v_k^{(i+1)})=1$, $c(v_i^{(2)})\ne c(v_k^{(i+1)})$, $2\le i\le\omega-1$, $1\le k\le\omega-1$. Also, $d(v_i^{(2)},v_1^{(j)})=1$, $1\le i\le\omega-1$, $3\le j\le\omega$, so that $c(v_i^{(2)})=c(v_1^{(j)})$. This happens only when $c(v_i^{(2)})\ne c(v_1^{(k)})$, $3\le j\ne k\le\omega$.

Suppose $c(v_i^{(2)})=c(v_1^{(k)})$, for some j and k, then we need a new label other than 1,2,3,... ω, which is a contradiction to our assumption.

Consider $K_\omega^{(2)}$. Since $d(v_i^{(2)},v_j^{(i+1)})=1$, $2\le i\le\omega-1$, $2\le j\le n$ and $d(v_i^2,v_j^{(k)})=2$, $2\le i,k\le\omega-1$ and $k\ne i+1$, $c(v_i^{(2)})=c(v_j^{(k)})$, $2\le i,k\le\omega-1$, $k\ne i+1$. Also, $d(v_i^{(2)},v_1^{(i+1)})=2$, $c(v_i^{(2)})=c(v_1^{(i+1)})$, $2\le i\le\omega-1$. Finally, we have to label $K_\omega^{(1)}$. Since $d(x,v_1^{(1)})=1$ and $d(v_1^{(1)},v_i^{(1)})=1$, $2\le i\le\omega-1$ and label of each $v_1^{(i)}$ is distinct, we can not label $v_1^{(1)}$ with integers 1,2,3,..., ω, which is a contradiction. From the above discussion, we conclude that $rr(G)>\omega$, which implies $rr(G)\ge\omega$. This completes the proof. ■

The following corollary is the generalization of the above theorem.

Corollary 2.4 For any given $\omega\ge 4$, and $2\le m<\omega$, there exists a simple connected graph G with $rr(G)=\omega+m$.

Proof. Consider the graph G with $V(G)$ and $E(G)$ are defined as follows:

$$V(G)=\{x,v_i^{(j)},\ 1\le i\le\omega-1,\ 1\le j\le\omega\}$$

$$E(G)=\{xv_i^{(j)}:1\le i\le\omega-1,1\le j\le\omega\}\cup \{v_i^{(j)}v_k^{(j)},1\le i\ne k\le\omega-1,1\le j\le\omega\}\cup \{v_i^{(1)}v_1^{(j)}:1\le i\le m,2\le j\le\omega\}\cup \{v_i^{(2)}v_k^{(i+1)}:2\le i\le\omega-1,2\le k\le\omega-1\}.$$

Here, $rad(G)=1$. Define $f:V(G)\to\{1,2,3,\ldots\}$ such that

$$f(x)=\omega$$
$$f(v_i^{(1)})=\omega+i,\ 1\le i\le m$$
$$f(v_{m+i}^{(1)})=i,\ 2\le i\le\omega-m$$
$$f(v_i^{(2)})=i,\ 1\le i\le\omega-1$$

For $1\le j\le\omega-1$ and $2\le i\le\omega-1$,

$$f(v_j^{(i+1)})=\begin{cases} f(v_j^{(i)})+1, & \text{if } f(v_j^{(i)})+1\le\omega-1 \\ f(v_j^{(i)})\oplus 1, & \text{otherwise}\end{cases}$$

By the above theorem, we can easily show that $rr(G)=\omega+m$, $\omega\ge 4$ and $m<\omega$. ■

By combining all the above results, we conclude the following:

Theorem 2.5 For any given $m\ge 0$, there exists a graph G for which $rr(G)=\omega+m$, where $\omega\ge 3$. ■

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