Programming as a didactic technique in the teaching-learning process of the complex variable subject

La programación como técnica didáctica en el proceso enseñanza – aprendizaje de la asignatura variable compleja

Universidad Politécnica del Estado de Guerrero

ALANÍS, Reynaldo †, MOTA, Juan and URUETA, Daniel Edahi

ID 1st Author: Reynaldo, Alanís / ORC ID: 0000-0001-5397-7016

ID 1st Coauthor: Juan, Mota / ORC ID: 0000-0001-6099-9408

ID 2nd Coauthor: Daniel, Urueta / ORC ID: 0000-0002-8741-6978

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Abstract

The use of computer programming in some programming language as a teaching tool in the teaching - learning process of the Complex Variable subject is proposed and the results of the major in Telematics Engineering at the Universidad Politécnica del Estado de Guerrero are presented. It will be indicated in which subjects the proposed technique was used, the programming language used in its implementation, a schematic model of the process followed by its implementation and the obtained results. The methodology used that will be detailed in the corresponding section is to locate the subjects of the subject, being: arithmetic of complex numbers and transcendental functions with argument of complex numbers for those that present on the one hand greater difficulty in understanding and, on the other, are those that can be programmed in a simple but meaningful way. The subject of arithmetic of complex numbers will be explained in detail. The programming language used is the one that the students master, since this subject is taught in the fifth semester of the major in Telematics Engineering.

Teaching techniques, Teaching Mathematics, Education

Resumen

Se propone el uso de la programación de computadoras en algún lenguaje de programación como herramienta didáctica en el proceso enseñanza – aprendizaje de la asignatura Variable Compleja y presentan los resultados de la experiencia en la carrera de Ingeniería en Telemática en la Universidad Politécnica del Estado de Guerrero. Se indicará en que temas de la asignatura se utilizó la técnica propuesta, el lenguaje de programación utilizado en su implementación, un modelo esquemático del proceso seguido de su implementación y los resultados obtenidos. La metodología utilizada que se detallará en la sección correspondiente es ubicar los temas de la asignatura, siendo: aritmética de los números complejos y funciones trascendentales con argumento de números complejos por ser los que presentan por un lado mayor dificultad en el entendimiento por parte del alumno y por otro son los que se pueden programar de manera sencilla pero significativa. Se explicará en detalle el tema de la aritmética de números complejos. El lenguaje de programación utilizado es el que los estudiantes dominan, dado que esta asignatura se cursa en el quinto cuatrimestre de la carrera de Ingeniería en Telemática.

Técnicas didácticas, Enseñanza de las Matemáticas, Educación

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† Researcher contributing as first author.

Introduction

Information technologies have been used in recent years as tools for support in the learning-teaching process in academic environments from elementary school to higher levels and this is known, encouraged and accepted (Salinas 2004). Teachers and students use different computer technologies in both academic and research activities, from large or personal computers, to gadgets, among others. End-user applications are frequently used, such as: PC applications and/or gadgets, calculators, spreadsheets, browsers, and many others. The coding of applications is a formal process and the errors that can be made (logical errors) or even before, in the compilation (syntactic errors) are evidenced. On the other hand, those specific applications are evidenced during the execution of the final application to an area of knowledge, how difficult but structured to learn some subjects in the area of basic sciences such as Mathematics. The use of programming is explored as a didactic technique both for teaching-learning the subject of, in this case, Complex Variable, as well as for reviewing and reinforcing programming skills. The use of programming languages and the coding of programs in the corresponding compilers can be a helpful tool in learning formal subjects (Fernández 2013).

In polytechnic universities, the basic educational model is that of Competency-Based Learning (Tobón 2008). The core competencies are: know-how-to-do, know-how-to-be and know-how-to-know (Tobón 2008). The know-how-to-do is to obtain knowledge in the development of related products in the area of knowledge. The know-how-to-be is that the student can develop solutions to problems of their socio-economic environment. Finally, know-how-to-know is that the student, in addition to the knowledge presented by the teacher, can be able to obtain, by searching in different sources, information to structure and obtain additional and complementary knowledge on specific topics related to their area. The Network and Telecommunications Engineering major consists of a series of subjects that allow the development of products that can be academic but also a solution to the needs identified in the student's socio-economic environment.

The subject of Complex Variable was used as an academic context in the development of the competence know-how-to-know, know-how-to-do and with less emphasis know-how-to-be, relying on previous knowledge and experience in coding, with basic programming concepts and implementing them in some programming language. If we focus mainly on the fundamental concepts of programming, that is, the bases or structures of program construction: sequence, decision and repetition as well as analysis of the different programming languages, being able to be procedural and/or object oriented, mainly in C, C++, Java or even C#. Developing a program necessarily implies including, in addition to coding, specific tasks aimed at building useful, efficient and reusable code (Pressman 2010), such as:

1. Investigate and diagnose the problem to be solved;
2. Perform an analysis to create a model of the system to be developed and to solve the identified problem and finally
3. Design the elements to be programmed (using analysis methodologies and the design of programs such as flow charts and concepts of the problem area, the knowledge acquired or necessary to acquire in the area of Complex Variable).

For the diagnostic activity, a recommended strategy for its usefulness is to be guided using the steps of the Diagnosis of the Current Situation (Pressman 2010). It helps the developer to identify the need or problem to solve; the environment where the need or problem is detected; the environment where the solution will be implemented; and the feasibility of the solution. The students in this semester have not completed the subject of Software Engineering, however basic aspects or principles of this were included, particularly, the initial phases of the development of computer-based systems: analysis and designing of programs. The generic process model was explained and for the analysis and designing phase, flowcharts were used.

The Complex Variable subject begins with the classification of numbers and identifying the creation of complex numbers and the justification of their need. Focusing on Complex Variable, it begins with the definition of complex numbers and Arithmetics, as well as their properties.

The different ways of representing a complex number are explained. It is in this part of the course that the complications and difficulties of understanding complex numbers begin. Given, for example, a complex number in its binomial representation, identify its module and argument, that is, locate the complex number on the Argand plane (Churchill 1986). Subsequently, the arithmetic of complex numbers continues: addition, subtraction, multiplication, division, powers and roots. The foregoing further complicates the process of understanding, especially the root part, since students are accustomed to only obtaining a root even though there may be two or more depending on the order of the root. For this, it was necessary to understand the representation of Moiver and Euler, especially for potency and obtaining roots. It is followed later with functions of Complex Variable. Of these, trigonometric, logarithmic and exponential functions present a great difficulty for students to understand and manipulate them. Since programming is a discipline that requires a clear understanding of what is going to be codified, as well as the handling of exceptions, it can support the understanding of the issues mentioned.

If it is not understood how to determine, for example, the quadrant in which the complex number is to be plotted based on its argument, it cannot be programmed and the results will be wrong (Distéfano, ML, Aznar, MA, & Pochulu, M. 2012). If the student wants to experiment by changing values of complex numbers, it can be tedious and prone to error, since the procedure to perform it involves arithmetic operations, as well as trigonometric functions. To program the above, it is necessary to develop the algorithm and, for this, it is necessary to clearly understand the process of transforming the complex number into a rectangular representation to a graph.

The knowledge and mastery of the algebraic skills of complex numbers, as well as the Complex Variable are of paramount importance for students, professionals and researchers in engineering and science. The difficulties involved in teaching and learning the above motivate teachers to search and try different strategies, so it is justified to explore the use of programming as a didactic technique.

Objectives

The objective of this paper is to present the use of programming as a strategy and didactic tool in the teaching-learning process of the subject of Complex Variable, as well as to present the workflow and the results of its application.

Methodology

A didactic technique is a didactic process that helps in part in the realization of learning (Carrasco 2004). There are various techniques and each of them has advantages and areas of opportunity. For the Complex Variable course, the exhibition teaching technique was applied, which consists of presenting the information in an orderly manner using various means of support. The Complex Variable course begins with the presentation of the agenda. The explanation of the content of the subject begins with the construction of the set of numbers: natural (N), integer (Z), rational (Q), irrational (I), real (R), and finally, imaginary and complex are presented. Figure 1 shows said construction.

Figura 1 Set of numbers

The above is addressed through the solution of algebraic expressions of the form $x^n + b = c$; $a, b, c \in R$ and $n \in N$ in such a way that when trying to solve the equation in particular:

$$x^2 + 1 = 0 \quad (1)$$

The need to include the imaginary numbers ai , (with $a \in R$) that represent the even root of negative numbers is imperative, in the form:

$$i = \pm \sqrt[2]{-1} \quad (2)$$

(In this case, the square root of -1). Complex numbers are constructed when it comes to solving a quadratic equation of the form:

$$ax^2 + bx + c = 0 \quad (3)$$

Using the discriminant that results when applying the general formula:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (4)$$

$$b^2 - 4ac < 0 \quad (5)$$

The solution of which is

$$x_{1,2} = a \pm bi \quad (6)$$

With a and $b \in \mathbb{R}$.

This is the binomial representation of a complex number. Subsequently, the Argand plane is exposed and the graphic form of the complex number is presented, as shown in Figure 2.

Figure 2 Graphical representation of a complex number

Subsequently, the polar representation of the complex number $z = (r, \phi)$ in Figure 3 is shown.

Figure 3 Polar graphical representation of a complex number z

Where r is the module of z and ϕ the angle or argument. Depending on the values of a and b , the complex number is located in one of the four quadrants of the Argand plane. The following table shows the details.

a (x axis)	b (y axis)	Quadrant
+	+	I
-	+	II
-	-	III
+	-	IV

Table 1 Location of the complex number in the Argand plane

The r module is calculated as follows:

$$r = \sqrt{a^2 + b^2} \quad (7)$$

And ϕ , the argument of the complex number z , is generally calculated as follows:

$$\phi = \tan^{-1}\left(\frac{b}{a}\right) \quad (8)$$

Based on table 1, ϕ is calculated to locate z in the Argand plane according to the following table:

a	b	ϕ	Quadrant
+	+	$\tan^{-1}\left(\frac{b}{a}\right)$	I
-	+	$\tan^{-1}\left(\frac{b}{a}\right) + \pi$	II
-	-	$\tan^{-1}\left(\frac{b}{a}\right) + \pi$	III
+	-	$\tan^{-1}\left(\frac{b}{a}\right) + 2\pi$	IV

Table 2 Calculation of the value of the complex number argument for the Argand plane

It is in this moving from binomial algebraic representation to the graphic part where students have more difficulties and where the greatest number of mistakes are made. A set of exercises was applied where the following errors were identified:

- 1) When calculating the module of z , not only the coefficient b of the imaginary part was squared but also included i ;
- 2) The same happened when obtaining the argument of z without knowing how to perform the division of bi by a ;
- 3) Errors when placing z in the corresponding quadrant in the argand plane.

Subsequently, the properties of complex numbers and the arithmetic operations of addition, subtraction, multiplication, division, powers and roots were exposed.

Again, there were difficulties in learning the above. For the addition, subtraction, multiplication and division the binomial representation was used and later, when the power and roots were explained, the representation of Moivre (Churchill 1986) was explained and the multiplication and division operations were resumed and again the bidirectional conversion of algebraic z -graphic. Again, a questionnaire was applied to evaluate the arithmetic operations as well as the Moivre representation of $z = rCIS(\theta) = r\cos(\theta) + risen(\theta)$. The most frequent errors, when they were able to solve them, were the same as those mentioned, in addition to the difficulty of obtaining the multiple roots of $z^{1/n}$ in particular, where

$$z^{1/n} = r^{1/n} CIS\left(\frac{\theta + 2k\pi}{n}\right) \quad k = 1, 2, \dots, n-1 \quad (9)$$

The explanation was made using the didactic exposition technique and supported by exercises and problems. Subsequently, Euler's representation of the complex numbers $z = re^{i\theta}$ was explained, which will illustrate the trigonometric, logarithmic and exponential functions. The trigonometric functions are evaluated and explained:

$$sen(z) = \frac{e^{iz} - e^{-iz}}{2i} \quad (10)$$

$$cos(z) = \frac{e^{iz} + e^{-iz}}{2} \quad (11)$$

$$tan(z) = \frac{(e^{iz} - e^{-iz})}{i(e^{iz} + e^{-iz})} \quad (12)$$

$$ln(z) = ln(r) + i(\theta + 2k\pi) \quad k = 0, \pm 1, \pm 2 \dots \quad (13)$$

Again, a questionnaire was applied to diagnose the progress on these last issues and the mistakes mentioned above were made, as well as the difficulty of calculating trigonometric and logarithmic functions.

It was proposed as product evidence to develop a program in the programming language of their liking that would automate all the operations mentioned. For this, the traditional software development process was explained: Analysis, Design, Construction, Testing, and Installation (Pressman 2011). The sequence of the mentioned phases is presented below.

Figure 4 Traditional process for program development

The work model shown in the following figure was developed and implemented.

Figure 5 Workflow of the Complex Variable subject

As it can be seen, the process is systematized and the activity where programming is used as a didactic technique is included. For the last activity, development process applies the sequence that was shown in Figure 4. The following is broken down.

Figure 6 Development process of product evidence

Once the product evidence is proposed, it begins with the revision of the notes and appropriate literature in Mathematics.

Subsequently, the writing of the research report begins, the contents of which are: introduction, objectives, theoretical framework, development, results, conclusions and references.

In the theoretical framework, the student writes the corresponding theory and develops the steps to follow for the programming of the application.

The external characteristics of the program, exception handling, user conditions, program installation and validation test plan are indicated. It is in this section where the analysis and design of the program is carried out.

In the development, what is explained in the theoretical framework is coded, as well as the semantic and logical correctness of the program is evaluated.

In the results, the program's correctness is finally evaluated, comparing the results of the program output with the expected results. If there is a discrepancy, both the theoretical framework and the analysis, design and coding of the program are reviewed until the program is correct.

The calculation of r is not complicated. For example, the student understands that only the coefficients of z are used to obtain the result. Since in order to program something it is important to determine the type of variables to be used, their declaration would be real or decimal.

During the exposition of the theory, only whole numbers are used. Since the arithmetic operations are requested to be programmed, it must be validated that when dividing two complex numbers z_1 and z_2 , the module of z_2 must be different from zero. For the other operations it is not necessary.

It was requested that the numbers z be transformed to their polar and Euler's form, obtaining not only the module but also the argument, it is in the calculation of the argument where the students should pay attention and "explain" to the computer how to obtain r and the argument. The corresponding flowchart is presented in the following figure.

Figure 7 Flowchart reading of z 's

Since it was requested to work with two complex numbers, it is validated that the number of data to be read is two, otherwise, error. This process of "explaining" to the machine helps the student pay attention to what they are going to do. Subsequently, r_1 and r_2 would be calculated, as well as the corresponding arguments. The corresponding flowchart is shown in the following figure. Emphasis is placed on the identification of the quadrant where z_1 and z_2 are to be located. Again, the fact that the student "explains" to the machine how to identify the quadrant leads the student to attend to it.

Figure 8 Flowchart to obtain module, argument and quadrant of z_1

The calculation of the argument of z_2 is not detailed but is the same as that of z_1 .

Once having the module and argument of z_1 and z_2 , the required operations can be performed, as well as represent them in their binomial, polar, Euler and even graphic form. The student can program these operations sequentially or can do it by selection. If z_1 is to be divided by z_2 , it must validate that z_2 is different from zero.

To program the calculation of trigonometric functions, the student must develop formulas 10 and 11 in their binomial form first and calculate their polar form.

The following figure shows the binomial form of $\sin(z)$ using only exponential and trigonometric functions with real arguments. The deduction of this formula helps the student to exercise not only what is seen in the subject of Complex Variable, but also the algebraic part.

$$\sin(z) = \frac{1}{2}e^{-y}\sin(x) + \frac{1}{2}e^y\sin(x) + i\left(\frac{1}{2}e^y\cos(x) - \frac{1}{2}e^{-y}\cos(x)\right)$$

Figure 9 Binomial representation of $\sin(z)$

The $\cos(z)$ function is developed in the same way and the $\tan(z)$ function can be evaluated by first calculating $\sin(z)$, then $\cos(z)$ to later perform the division by calling the routine that calculates the arithmetic operation in the program or developing it as equation 12.

The function $\ln(z)$ is easier to calculate and it is interesting how students discover that logarithms of negative real numbers and the famous Euler equation can be calculated:

$$\ln(-1) = i\pi \quad (14)$$

Results

As mentioned in the previous section, the presentation and explanation of the content of the subject of Complex Variable was initially supported using the didactic technique of exposure.

Exercise questionnaires were used on the topics of arithmetic of complex numbers and trigonometric functions. The students made, when they were able to solve the exercises, several errors and deficiencies.

When proposing to carry out a program to implement the obtention of the module and argument of z , the arithmetic operations, as well as to evaluate the trigonometric functions and the natural logarithm function, the students had to “teach and explain” to the computer how to perform these operations, they were forced to review the basic theory about complex numbers, the management of attributions, consult the teacher with specific questions on the subject, review algebra and discover the application of programming in somewhat arid subjects, such as arithmetic and trigonometry of complex numbers.

After the completion, execution and evaluation of the program, questionnaires on the topics were again applied and showed a substantial improvement in the results. The questionnaires not only focused on the operational part, but also on the reasoning. They were asked to solve non-numerical exercises and approached them satisfactorily. In the process of research, analysis and design of the program, the students asked for advice and their questions were formulated more concrete and timely. The undersigned, in advising them, identified the progress in the knowledge and mastery of the subject. At times, the students showed that they already understood what the operations meant but were looking for a way to “exploit” the computer on how to perform these operations. Some students took advantage of the programming language libraries (Phyton) and they were asked to explain the algorithms of the bookstores or how they thought these functions had been programmed and included them in detail in the research report.

Conclusions

An improvement is perceived when comparing the results obtained when diagnosing learning and progress through problems using only the didactic exposure technique with the results after the completion of the program and preparation of the research report. The advice given to the students during the development of this work helped the subscriber to evaluate the progress in the management and knowledge of the topics addressed. The work was developed in teams of two students and this not only supported the know-how-to-do and know-how-to-know skills but also the know-how-to-be, as it favored and strengthened collaborative work.

The know-how-to-know competence was addressed and strengthened by having students investigate on their own to clarify some points in the development of, for example, trigonometric functions. The know-how-to-do skill was developed by having students create an application that would help them solve some exercises related to complex numbers and arithmetic operations. With some simple modifications, the student could take this application to an exploration and research instrument. For example, if complex numbers are very large in their real and imaginary part, they may encounter the problem of calculations that in turn require evaluating trigonometric, exponential and/or logarithmic functions and would lead them to review their knowledge in programming. Some improvements that can be made include that the application can be executed via the Internet or developed for mobile devices.

In general, the objective that the students, through the formalism of programming, learn and master in a better way the topics addressed at the beginning of the Complex Variable course was achieved.

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