CHARACTERISATION OF VERY STRONGLY PERFECT GRAPHS

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ABSTRACT

A graph G is said to be strongly perfect if every induced sub graph H has an independent set, meeting all maximal complete sub graphs of H . A strongly perfect graph is said to be very strongly perfect if it contains good independent set. It follows that every very strongly perfect graph is perfect not conversely. For example, the compliment of even cycle of length more than 4 is not very strongly perfect though it is perfect. In this paper, we characterise the classes of very strongly perfect graphs like cyclic graph, tadpole graph, barbell graph and friendship graph along with its independence number.

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1. INTRODUCTION

In this paper, graphs are finite and simple. A path in a graph $G(V,E)$ is an alternating sequence of vertices and edges. Every path contains maximal clique of size two i.e. K_2 such that all alternating set of vertices form an independent set which meets all maximal cliques. Therefore every path is very strongly perfect. A simple graph G is said to be complete if every distinct pair of vertices are adjacent. The complete graph on n -vertices is denoted by K_n . Since in K_n , every vertex is an independent set which meets its maximal clique (complete graph itself), every complete graph is very strongly perfect. A clique in a graph G is a maximal complete sub graph. An independent set is a set of vertices such that no two of which are adjacent. An independent set is said to be good independent if each of its vertex meets

maximal clique. The number of vertices in a largest independent set of graph G is called independence number denoted by $\psi(G)$.

The theory of perfect graph deals with the theoretical concepts of a clique and independent set. An application of perfect graph is in an urban science problem involving optimal routing of garbage trucks. Also perfect graphs can be used to solve polynomial time algorithm, based on the ellipsoid method, for finding a maximum stable set and minimal colouring. The major role of graph theory in computer applications is to develop the graph algorithms like data mining, image segmentation, networking, clustering, image capturing etc.

Due to the applications of perfect graphs, it is necessary to investigate its subclasses too. As very strongly perfect graph is the subclass of perfect graphs, here it is investigated the same graph.

2. VERY STRONGLY PERFECT GRAPH (VSP)

A graph G is said to be very strongly perfect if in every induced sub graph H , every vertex of H belongs to an independent set of H meeting all maximal cliques of H . Fig. 1 and 2 illustrate both very strongly and non very strongly perfect graphs.

Example 1

Figure 1: Very Strongly Perfect Graph

Here, $\{1\}$ is a independent set which meets all the maximal cliques of graph G .

Example 2

Figure 2: Non-Very Strongly Perfect Graph

Here, $\{v_4, v_5, v_6\}$ is an independent set which does not meet all maximal cliques.

3. CYCLE

A Cycle graph or Circular graph is a graph that consists of a single cycle, or in other words, some number of vertices connected in a closed chain (A path in which terminal vertices are coinciding is called cycle). The cycle graph with n vertices is denoted by C_n . The even cycle is a cycle with even number of vertices and the odd cycle is a cycle with odd number of vertices. The number of vertices in a C_n equals the number of edges, and every vertex has degree 2; that is, every vertex has exactly two edges incident with it. The following figure illustrates the cycle graph C_6 .

Example 3

Figure 3: C_6

3.1. Theorem

Every Even Cycle is very strongly perfect.

Proof:

Consider the cycle C_n of even length.

Therefore, $C_n = \{v_1, v_2, v_3, \dots, v_n, v_1\}$, where n is even.

Let I = independent set (set of nonadjacent vertices).

Without loss of generality, let $v_1 \in I, v_2 \notin I$ and so on.

Hence for all $j \in (1, 2, 3, \dots, n)$

We get, $v_j \in I$ if j is odd and $v_j \notin I$ if j is even.

As n is even, $v_n \notin I$.

Therefore $I = \{v_j / j \text{ is odd}\}$ which meets all the maximal cliques of length two (i.e.) K_2 .

Hence, G is very strongly perfect.

3.2. Theorem

Every Odd Cycle is non-very strongly perfect.

Proof:

Consider the cycle C_n of odd length.

Therefore, $C_n = \{v_1, v_2, v_3, \dots, v_n, v_1\}$, where n is odd.

Let I be the independent set such that $v_1 \in I, v_2 \notin I$ and so on.

Hence for all $k \in (1, 2, 3, \dots, n)$, we get $v_k \in I$ if k is odd and $v_k \notin I$ if k is even.

As n is odd, $v_n \in I$.

Therefore there exists an edge $e = v_n v_1$ in G belongs to independent set I .

That proves I contains two adjacent vertices.

Which gives contradiction to the fact that I is an independent set.

Hence, G is not very strongly perfect.

3.3. Proposition

Every even cycle graph C_n has the cardinality of independence number $\frac{n}{2}$.

Proof:

Let C_n be the cycle containing n number of vertices, where n is even.

Let I be the independent set containing alternating vertices of C_n which meets all cliques.

Since G has n vertices, I contains $\frac{n}{2}$ vertices which is maximum.

Hence the proof.

4. TADPOLE GRAPH

The (m, n) - Tadpole graph, also called a dragon graph, is the graph obtained by joining a cycle graph C_m to a path graph P_n with a bridge and it is denoted by $T_{m,n}$. The $(m, 1)$ -tadpole graph is sometimes known as the m -pan graph. The particular cases of the $(3,1)$ - and $(4,1)$ -tadpole graphs are also known as the paw graph and banner graph, respectively. The tadpole graph $T_{4,4}$ is illustrated in fig.4.

Example 4

Figure 4: $T_{4,4}$

Independent set = $\{v_1, v_4, v_6, v_8\}$ meets all maximal cliques.

4.1. Theorem

Every tadpole graph $T_{2m, n}$ is very strongly perfect.

Proof:

Let G be a tadpole graph.

Since G is constructed by joining a cycle of length $2m$ i.e. C_{2m} with a path of length n (i.e.,) P_n .

Also, we see in theorem 3.1 that every even cycle is very strongly perfect.

As P_n is a path of n vertices, therefore every alternating set of vertices from the path forms the independent set I which meets maximal clique k_2 .

Since G is the union of cycle C_{2m} and path P_n , both are very strongly perfect.

Hence, G is a very strongly perfect.

4.2. Theorem

Every tadpole graph $T_{2m+1, n}$ is non-very strongly perfect.

Proof:

Let G be a tadpole graph.

G is constructed by joining a cycle of length $2m+1$ i.e. C_{2m+1} with a path of length n i.e. P_n . C_{2m+1} is a odd cycle which is a non- very strongly perfect graph and every path is very strongly perfect graph .The union of C_{2m+1} and P_n results a non-very strongly perfect graph.

Hence, G is non- very strongly perfect graph.

4.3. Proposition

Independence number of tadpole graph $T_{2m, n}$ is $m + \frac{n}{2}$.

Proof:

Let $T_{2m, n}$ be a tadpole graph.

$\Rightarrow$ there exists a cycle of length $2m$ which gives m number of vertices and there is a path of length n which gives $\frac{n}{2}$ number of vertices belonging to an independent set. Therefore maximum number of vertices belonging to an independent set is $m + \frac{n}{2}$.

Hence the proof.

5. BARBELL GRAPH

The n -barbell graph is the simple graph obtained by connecting two copies of a complete graph K_n by a bridge. Bridge is an edge whose deletion disconnects the graph. Fig. 5 illustrates the 4-barball graph.

Example 5

Figure 5: 4 - Barbell Graph

5.1. Theorem

Every n -barbell graph is very strongly perfect.

Proof:

Let G be an n -barbell graph.

Every complete graph is very strongly perfect.

If two very strongly perfect graphs are joined by a bridge (i.e.) K_2 , the resultant graph becomes very strongly perfect graph.

Hence, G is very strongly perfect.

5.2. Proposition

Every barbell graph has independence number two.

Proof:

Every barbell graph is obtained by connecting two copies of complete graph K_n by a bridge.

As each complete graph gives only one vertex which belongs to independent set, therefore there exists a largest independent set of size two.

Hence the proof.

6. FRIENDSHIP GRAPH

A Friendship graph F_k of $2k+1$ vertices is the graph obtained by taking k -copies of the cycle graph C_3 , with a vertex in common, (i.e.) the graph of k -triangles intersecting in a single vertex. Hence every friendship graph contains a single vertex which is adjacent to all the remaining vertices with unit distance. It is a graph with girth 3 and diameter 2. Friendship graph F_6 is illustrated in fig. 6.

Example 6

Figure 6: F_6

6.1. Theorem

Every Friendship Graph is very strongly perfect.

Proof:

Let G be a friendship graph

$\Rightarrow$ there exists a vertex $v \in V$, which is adjacent to all the other vertices of G .

$\Rightarrow \{v\}$ is an independent set of cardinality one which meets all the maximal cliques of size three.

$\Rightarrow G$ is very strongly perfect.

6.2. Proposition

Every friendship graph F_k has independence number k .

Proof:

Let G be a friendship graph F_k .

$\Rightarrow G$ has k copies of maximal cliques of size three (i.e K_3).

$\Rightarrow$ in each K_3 , there is only one vertex which lies in independent set.

Therefore independent set consists of at most k number of vertices.

Hence the proof.

7. CONCLUSION

Here, it is discussed the structural properties of very strongly perfect graphs on cycle, tadpole, barbell and friendship graphs. In future, this investigation will be more applicable for the remaining graph classes also.

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