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BUCKLING ANALYSIS OF SHELLS SUBJECTED TO COMBINTED LOADS

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ABSTRACT

A semi-analytical isoparametric finite element with three nodes per element and five degrees of freedom per node has been used for the solution. Moderately thick shell theory has been used for the analysis. Second order strains with the in plane and transverse non-linear terms are used for the derivation of geometric matrix. Full Fourier expansion is used in the circumferential direction to overcome the coupling that arises due to material anisotropy and torque pre-stress. Comparison of the results obtained due to finite element is made with simplified solutions using two thin shell theories with and without shear deformation. The effects of combined load (axial compression and external pressure) on pre-buckling characteristics of composite circular cylindrical and conical shells of various geometric properties have been presented.

Keywords: Combined Loads, Composite, Finite Element Method, Moderately Thick Shell, Pre-Buckling,

INTRODUCTION

The load carrying capacity of shell structures leads to wide application and this is evident from the review of Noor and Burton [1]. This is ever on the increase with composite materials due to their desirable material characteristics. Stability for the buckling load has been analysed by many researchers. Simitses [2] made a survey of buckling analysis of imperfect cylindrical shells.

Some of the research works reported in literature are by Vogiatidis and Shi [3] and Lou and Yaniv [4]. They are analysed the shells using finite element method and Galerkin's method respectively. Lou and Yaniv [4] analysed composite circular cylindrical shell subjected to axial compression" bending and combined loads for their buckling characteristics. Chang and Lin [5] analysed the cross-ply laminated circular cylindrical shell for their free vibration characteristics and buckling loads. In their study circular cylindrical shells subjected to axial force and lateral pressure is analysed. Foster [6] conducted experimental study on the buckling characteristics of composite conical shells due to axial compression with imposed defects.

Most of the research works reported for buckling characteristics are for isotropic or orthotropic circular cylindrical shells and using thin shell theories. In the case of anisotropic shells (angle wound and with torque pre-stress the normal Fourier expansion (either symmetric. or anti-symmetric modes alone) cannot represent the buckling behavior properly. This is because of the coupling between symmetric and anti-symmetric modes. Sivadas [7] and many others have explained this point. Sivadas [7] used full Fourier expansion in circumferential direction (both symmetric and anti-symmetric modes). In the present study an attempt has made to study the buckling characteristics of pre-stressed circular conical shells. The buckling due to axial compression, external pressure and combined loads are considered. A semi-analytical isoparametric finite element with 3 nodes per element and 5 degrees of freedom per nodes has been used for the solution. Full Fourier expansion is used in the circumferential direction to overcome the coupling arise due to material anisotropy and torque pre-Stress. Second order strains with the in-plane and transverse deformation in non-linear

terms are used for the derivation of geometric matrix. An moderately thick shell theory with shear deformation and rotatory inertia has been used to analyze the buckling behavior. The convergence study and comparison with simplified solutions are also conducted.

THEORETICAL FORMULATION

In the present analysis, shell theory with second degree non-linear strains for pre-stressing are used. In this case three of the assumptions of the first approximation shell theory have been retained and are,

- i) Normal strain in thickness direction is zero, ($\epsilon_z = 0$)
- ii) Thickness normal stress is neglected, *i.e.*, ($\sigma_z \cong 0$)
- iii) Deflections are small.

The strains in terms of linear and non-linear components can be represented by,

$$\boldsymbol{\epsilon} = \boldsymbol{\epsilon}_L + \boldsymbol{\epsilon}_{NL} \quad (1)$$

Where $\boldsymbol{\epsilon}_L$ and $\boldsymbol{\epsilon}_{NL}$ are linear and non-linear strain tensors. The linear and nonlinear displacement relations are given by Sivadas [7]. Substituting the non-linear strains in total strains (eqn (1)), the eqn (1) can be written as,

$$\boldsymbol{\epsilon} = \boldsymbol{\epsilon}_L + \frac{1}{2} \mathbf{G} \boldsymbol{\beta} \quad (2)$$

Where $\mathbf{G}$ and $\boldsymbol{\beta}$ and details of the strain components are given by Sivadas [7] The five components of linear stresses in the shell are represented by,

$$\boldsymbol{\sigma} = \mathbf{D} \boldsymbol{\epsilon}_L \quad (3)$$

Where

$$\boldsymbol{\sigma}_L = \begin{Bmatrix} \sigma_L^1 \\ \sigma_L^2 \end{Bmatrix}, \quad \boldsymbol{\epsilon}_L = \begin{Bmatrix} \epsilon_L^1 \\ \epsilon_L^2 \end{Bmatrix}$$

$$\boldsymbol{\sigma}^i = \{\sigma_s \quad \sigma_\theta \quad \tau_{s\theta} \quad \tau_{\theta z} \quad \tau_{sz}\}^T$$

$$\boldsymbol{\epsilon}_L^i = \{\epsilon_s \quad \epsilon_\theta \quad \gamma_{s\theta} \quad \gamma_{\theta z} \quad \gamma_{sz}\}^T$$

and $\mathbf{D}$ is the matrix of elastic constants. 1 and 2 represent the quantities corresponding to symmetric and anti-symmetric modes.

FINITE ELEMENT FORMULATION

The displacement in conical coordinate (s, θ, z) are taken in the form,

$$\mathbf{Y}(s, \theta, z, t) = \begin{Bmatrix} u(s, \theta, z, t) \\ v(s, \theta, z, t) \\ w(s, \theta, z, t) \end{Bmatrix} \quad (4)$$

$$= \mathbf{Y}_0(s, z) + \mathbf{Y}_p(s, \theta, z, t) \quad (5)$$

where $\mathbf{Y}_0$ is the displacement vector due to initial stress. P is an index varying from 0 to ∞

In anisotropic shells the terms D_{13} , D_{23} , and D_{45} lead to the coupling between in-plane strains and twist and between two shear strains. This leads to a non-canonical form for the governing equations that prevents the normal (either symmetric or anti-symmetric modes alone) application of semi-analytical finite element to such shells. It is found that this difficulty also occurs in the case of torque pre-stress. This difficulty can be overcome by using full Fourier series

expansion in the circumferential direction. In the present analysis a semi-analytical isoparametric axisymmetric element is used. The shape functions and the derivation of this element have also been discussed by Sivadas [7]

Substituting the displacement relation in strain displacement relation and manipulating the various equations, the strain energy of an element becomes,

$$\Pi = \int_V \sum_{n=0}^{\infty} \delta \mathbf{q}_e^T [\mathbf{B}_{ne}^T \mathbf{D} \mathbf{B}_{ne} + \mathbf{G}_{ne}^T \hat{\sigma}_0 \mathbf{G}_{ne}] \mathbf{q}_e dV \quad (6)$$

Performing the appropriate variational manipulations in eqn.(6), one can obtain the stiffness matrix, $\mathbf{K}_{sn}$ and the geometric matrix, $\mathbf{K}_{gn}$ due to initial stresses of an element for a particular circumferential mode number 'n' and finally the equation of motion of the shell becomes, for a particular circumferential mode number n and is given by,

$$\mathbf{K}_{sn} q_n + \lambda \mathbf{K}_{gn} q_n = F \quad (7)$$

where F is the forcing function. The buckling loads can be calculated from eqn. (7) which is a first order eigenvalue problem.

SIMPLIFIED SOLUTION

To compare the results obtained using finite element analysis, the stability equations corresponding to thin shell theory with and without shear deformation has been derived. A circular cylindrical shell is considered for this purpose. Some of the assumptions made are,

- The effects of bending and shear stress are neglected in pre-stress, and only membrane effects are retained.
- The variation of in-plane stresses in axial direction also neglected.
- The stress resultants are assumed constant along the axial direction and are given by,

$$1. \quad \text{due to external pressure, } N_x^0 = 0 \text{ and } N_{\theta}^0 = -PR \quad (8)$$

$$2. \quad \text{due to axial compression } N_x^0 = -\frac{F_x}{2\pi R_0} \text{ and } N_{\theta}^0 = 0 \quad (9)$$

where P and F_x are external pressure and axial compressive loads and R is mean surface radius. These assumptions are valid for long shells. For short shells it is an oversimplification. The pre-buckling linear analysis for short shells is meaningless because the stress induced due to buckling load is far greater than the breaking stress of the material. The equations of motion of shells with this assumption are derived (not given). The shape functions assumed for the solution are,

$$\begin{aligned} u_0(s, \theta) &= u_0(s) + \sum_{n=0} U_n \cos \lambda s \cos n\theta \\ v_0(s, \theta) &= v_0(s) + \sum_{n=1} V_n \sin \lambda s \sin n\theta \\ w_0(s, \theta) &= w_0(s) + \sum_{n=0} W_n \sin \lambda s \cos n\theta \\ \phi_1(s, \theta) &= \Phi_1(s) + \sum_{n=0} \Phi_{1n} \cos \lambda s \cos n\theta \\ \phi_2(s, \theta) &= \Phi_2(s) + \sum_{n=1} \Phi_{2n} \sin \lambda s \sin n\theta \end{aligned}$$

The superscript '0' represent the displacements and rotations due to static initial load. This shape functions will satisfy the governing equations and boundary conditions. The corresponding boundary conditions (SS2) are, $v_0 = w_0 = 0$ and $N_x = M_x = 0$ for shell theory without shear deformation and with shear deformation in addition to above boundary condition the rotation, $\theta_2 = 0$ is also satisfied at both ends ($x=0$ and l). This shape functions can handle the multilayered shells with inplane and bending coupling but not anisotropic shells. Manipulating the matrices one can get a matrices of size 5X5 and 3X3 respectively. These matrices can be solved for the critical load.

RESULTS AND DISCUSSION

The convergence study of the element for buckling load due to external pressure and combined loads have been conducted for two boundary conditions (not presented) and it found that the results are converged to a practically acceptable limit with a few number of elements.

The length parameter, μ is the ratio of slant length to minimum middle surface radius (l/R_0). The axial load parameter, λ_x and the parameter due to external pressure λ_p are defined as,

$$\lambda_x = \frac{F_x(1 - \nu_{12}^2)}{E_2 R_0 h} \text{ and } \lambda_p = 12(1 - \nu_{12}^2) \beta^3 \frac{P}{E_2},$$

where β is the ratio of minimum middle surface radius to total thickness (R_0/h), F_x and P are axial force and external/internal pressure respectively.

E_2 , and ν_{12} , are the Young's modulus in transverse to fibre direction and major Poisson's ratio. In the present analysis the minimum reference surface radius and the value of B used are 0.25 and 10 respectively. The material properties of carbon/epoxy are used for the analysis are given in Table 1.

Table 1 Material properties of Carbon Composite used

E_1 (in GPa)	72.7
E_2 (in GPa)	7.2
G_{12} . (in GPa)	3.76
G_{13} (in GPa)	3.76
G_{23} . (in GPa)	3.76
ρ (mass density) Kg/m ³	1550
Major Poisson's ratio	0.26

Comparison of critical loads due to external pressure and axial compression is conducted and given in Table 2. The comparison is made using two thin shell theories, with and without shear deformation. The simplified equations of motion have been solved exactly. It can be seen that the values corresponding to FEM are slightly smaller than other two sets of values. In the case of shell subjected to axial compression, the FEM values are always less than other two simplified assumptions.

Figure 1 shows the variation of buckling load due external pressure on shells subjected to axial compression. The SS1 (only radial constraint) boundary condition has been considered for this study. For axially wound cylindrical shell the λ_p is minimum when the circumferential mode number, $n=2$. The effect of axial compression on λ_p corresponding to other values of n is very small in the range of λ_x , considered except when $n=1$. When $\phi=30$, and $\alpha=0$ (Figure 1(b)) the minimum buckling load due to external pressure and axial compression occurs when $n=5$ and 3 respectively. As expected the n at which the minimum buckling load occurs changes as λ_x changes. When $\lambda_x=0$, minimum occurs at $n=5$ and as λ_x increases, it change to $n=4$ and, when λ_x approaches to axial buckling load, minimum occurs when $n=3$. The change of buckling load when $n=3$ is greater than the other values of n presented as λ_x increases.

Figure 1 Effect of combined load on critical buckling loads of shells, $\mu=10$, $\beta=10$ (a) $\alpha=0, \varphi=0$, (b) $\alpha=0, \varphi=30$, (c) $\alpha=30, \varphi=30$, (d) $\alpha=0/90/0, \varphi=30$

Similar types of variations are observed in the case of other conical shells considered. The corresponding variations are shown in Figures 1(c) and Figure 1(d). It is observed that the buckling loads corresponding to the adjacent values of the n , lowest critical load due to axial compression decrease very drastically as λ_x increases. This can be seen from Figure 1(c) ($\alpha=30, \varphi=30$ and $n=1$ and 2) and in Figure 2(a) ($\alpha=0, \varphi=0$ and $n=1$) the buckling load decreases drastically. In this case, the minimum buckling load due axial compression occurs when $n=2$. Figure 2(b) shows the effect of λ_x on λ_p of single layer and three layer shells. Here also one can observe the decrease in λ_p as λ_x increases.

Table 2 Comparison of Buckling load parameters of orthotropic (carbon/epoxy) cylindrical shells

(a) Due to External pressure			
μ	Without shear	With shear	FEM
2	36.1507(4)	33.8368(4)	31.9192(4)
5	12.2486(3)	12.1613(3)	10.9541(3)
10	7.2277(2)	7.4996(2)	6.8423(2)
20	3.8632(2)	4.1455(2)	3.8011(2)
30	3.5415(2)	3.8245(2)	3.5120(2)
40	3.4730(2)	3.7564(2)	3.4514(2)
50	3.4508(2)	3.7343(2)	3.4328(2)
(b) Due to axial compression			
1	1.4329(3)	1.0602(3)	1.0443(3)
2	0.7354(3)	0.6956(3)	0.6898(3)
5	0.6453(2)	0.6502(2)	0.6490(2)
10	0.6931(2)	0.7196(2)	0.6504(2)
20	0.8164(1)	0.8164(1)	0.6514(2)
30	0.5150(1)	0.5151(1)	0.5149(1)
40	0.3398(1)	0.3397(1)	0.3466(1)
50	0.2364(1)	0.2365(1)	0.2358(1)

Figure 2 Effect of combined load on critical buckling loads of shells, $\mu=10$, $\beta=10$
(— $n=1$, - - - - $n=2$) (a) $\alpha=0, \phi=0$, (b) $\square \alpha=0, \phi=30$, $\blacksquare \alpha=0/90/0, \phi=30$

Table 3 shows the critical loads of some of the typical conical/cylindrical shells subjected to external pressure/axial compression in slant length direction. The number in bracket shows the corresponding circumferential mode number. It is observed that the n at which the lowest critical load due to external pressure occurs shift to lower circumferential modes for cylindrical shells and to higher values of n for conical shells.

Table 3 Lowest Buckling load parameters of shells for different geometric parameters and corresponding “n” where it occurs (bracket)

(a) Due to External pressure						
ϕ	α	$\mu=1$	2	5	10	20
0	0	176.2236(7)	30.8158(4)	10.6713(3)	6.5486(2)	3.7964(2)
30	0	388.9662(1)	29.9145(6)	2.4848(5)	0.6793(5)	0.1518(7)
60	0	287.024(1)	42.9836(7)	10.4491(5)	0.1742(6)	0.0348(7)
30	30	207.9252(4)	28.8889(5)	4.6449(5)	1.3153(5)	0.2991(6)
30	60	134.3122(5)	40.2952(5)	11.1033(4)	3.1950(4)	0.7313(5)
30	0/90/0	269.1528(1)	19.6581(5)	3.8601(4)	1.0564(6)	0.2407(7)
(b) Due to Axial compression						
0	0	0.4463(1)	0.5400(2)	0.5277(2)	0.5806(2)	0.6334(2)
30	0	0.3856(1)	0.5752(2)	0.5101(3)	0.5837(3)	0.6135(6)
60	0	0.3825(1)	0.5261(1)	0.3125(2)	0.3367(3)	0.3585(5)
30	30	0.4001(1)	0.4538(1)	0.5611(1)	0.6154(1)	0.6160(1)
30	60	0.3035(1)	0.3535(1)	0.4511(1)	0.5491(1)	0.8715(2)
30	0/90/0	1.1796(3)	0.7443(3)	0.6865(3)	0.7349(3)	0.8562(3)

CONCLUSIONS

A study on critical buckling load of composite shells of different geometric properties is conducted using a semi-analytical finite element method. Moderately thick shell theory with shear deformation is made use of for the analysis. The values are compared with simplified equations of motion. Some of the findings in the study are listed below

1. The circumferential mode at which the critical buckling load occurs due to the compression in slant length and external pressure are different.
2. As compression load in slant length direction increases, circumferential mode at which the lowest buckling load occurs changes. This depends on the material and geometric properties of the shell used.

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