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ANALYSIS OF COMPLEX COMPOSITE BEAM BY USING TIMOSHENKO BEAM THEORY & FINITE ELEMENT METHOD

Prabhat Kumar Sinha¹, Rohit¹

¹Mechanical Engineering Department

Sam Higginbottom Institute of Agriculture, Technology & Sciences

(Deemed-to-be-University) Allahabad, 211007, U.P., India

(Formerly Allahabad Agriculture Institute)

ABSTRACT

Fiber-reinforced composites, due to their high specific strength, and stiffness, which can be tailored depending on the design requirement, are fast replacing the traditional metallic structures in the weight sensitive aerospace and aircraft industries. An analysis Timoshenko beam theory for complex composite beams is presented. Composite materials have considerable potential for wide use in aircraft structures in the future, especially because of their advantages of improved toughness, reduction in structural weight, reduction in fatigue and corrosion problems. The theory consists of a combination of three key components: average displacement and rotation variables that provide the kinematic description of the beam, stress and strain moments used to represent the average stress and strain state in the beam, and the use of exact axially-invariant plane stress solutions to calibrate the relationships between all these quantities. The Euler-Bernoulli beam theory neglects Shear deformations by assuming that plane sections remain plane and perpendicular to the neutral axis during bending. As a result, shear strains and stresses are removed from the theory. Two essential aspects of Timoshenko's beam theory are the treatment of shear deformation by the introduction of a mid-plane rotation variable, and the use of a shear correction factor [36].

Keywords: Timoshenko Beam Theory, Finite Element Analysis.

INTRODUCTION

Composite materials in this era have considerable potential for wide use in aircraft structures in the future, especially because of their advantages of improved toughness, reduction in structural weight, reduction in fatigue and corrosion problems [2]. Most of the structures experiences severe dynamic environment during their service life; thus the excited motions are likely to have large amplitudes. The analysis of complex composite structures is far more complex due to anisotropy, material couplings, and transverse shear flexibility effects compared to their isotropic counterparts [5]. The use of composite materials requires complex analytical methods in order to predict accurately their response to external loading, especially in severe environments, which may induce geometrically non-linear behavior and material nonlinearity. This requires appropriate design criteria and accurate estimation of the fatigue life. In addition to the usual difficulties encountered generally in the non-linear analysis of structures, related to the fact that the theorem of superposition does not hold, existence and uniqueness of the solutions are generally not guaranteed [1].

The main objective of this work is to analyze the complex composite beams. The geometrically non-linear analysis of composite beam exhibits specific difficulties due to the anisotropic material behavior, and to the higher non-linearity induced by a higher stiffness, inducing tensile mid-plane forces in beam higher, than that observed with conventional homogeneous materials [1]. These structures with complex boundary conditions, loadings and shapes are not easily amenable to analytical solutions and hence one has to resort to numerical methods such as finite elements [11]. A considerable amount of effort has gone into the development of simple beam bending elements based on the Timoshenko Beam Theory for homogeneous isotropic beam. The advantages of this approach are (i) it accounts for transverse shear deformation, (ii) it requires only C0 continuity of the field variables, (iii) it requires refined equivalent single-layer theory, and (iv) it is possible to develop finite elements based on 6 engineering degrees of freedom viz. 3 translations and 3 rotations [14]. However, the low-order elements, i.e. the 3-node triangular, 4-node and 8-node quadrilateral elements, locked and exhibited violent stress oscillations [10]. Unfortunately, this element which is having the shear strain becomes very stiff when used to model thin structures, resulting inexact solutions. This effect is termed as shear-locking which makes this otherwise successful element unsuitable. Many techniques have been tried to overcome this, with varying degrees of success. The most prevalent technique to avoid shear locking for such elements is a reduced or selective integration scheme [8]. In all these studies shear stresses at nodes are inaccurate and need to be sampled at certain optimal points derived from considerations based on the employed integration order. The use of the same interpolation functions for transverse displacement and section rotations in these elements results in a mismatch of the order of polynomial for the transverse shear strain field. This mismatch in the order of polynomials is responsible for shear locking [7].

The Euler-Bernoulli beam theory neglects shear deformations by assuming that plane sections remain plane and perpendicular to the neutral axis during bending. As a result, shear strains and stresses are removed from the theory. Shear forces are only recovered later by equilibrium: $V = dM/dx$ [5]. In reality, the beam cross-section deforms somewhat like what is shown in Figure 1a. This is particularly the case for deep beam, i.e., those with relatively high cross-sections compared with the beam length, when they are subjected to significant shear forces. Usually the shear stresses are highest around the neutral axis, which is where; consequently, the largest shear deformation takes place. Hence, the actual cross-section curves.

Instead of modeling this curved shape of the cross-section, the Timoshenko beam theory retains the assumption that the cross-section remains plane during bending [7]. However, the assumption that it must remain perpendicular to the neutral axis is relaxed. In other words, the Timoshenko beam theory is based on the shear deformation mode in Figure 1b. Various boundary conditions have been considered. The effect of variations in some material and/or geometric properties of the beam have also been studied.

1. (a)

Actual shear deformation

(b)

Average shear deformation

LITERATURE REVIEW

A rigorous mathematical model widely used for describing the vibrations of beams is based on the Thick beam theory (Timoshenko, 1974) developed by Timoshenko in the 1920s. This partial differential equation based model is chosen because it is more accurate in predicting the beam's response than the Euler-Bernoulli beam theory [14].

Historically, the first important beam model was the one based on the Euler-Bernoulli Theory or classical beam theory as a result of the works of the Bernoulli's and Euler. This model, established in 1744, includes the strain energy due to the bending and the kinetic energy due to the lateral displacement of the beam [16]. In 1877, Lord Rayleigh improved it by including the effect of rotary inertia in the equations describing the flexural and longitudinal vibrations of beams by showing the importance of this correction especially at high frequency frequencies [15]. In 1921 and 1922, Timoshenko proposed another improvement by adding the effect of shear deformation. He showed, through the example of a simply-supported beam, that the correction due to shear is four times more important than that due to rotary inertia and that the Euler-Bernoulli and Rayleigh beam equations are special cases of his new result [18]. As a summary, four beam models can be pointed out in Table 1, the Euler-Bernoulli beam and Timoshenko Beam models being the most widely used.

Table 1.

Effect Beam Model	Lateral displacement	Bending moment	Rotary inertia	Shear deformation
Euler – Bernoulli	+	+	-	-
Rayleigh	+	+	+	-
Shear	+	+	-	+
Timoshenko	+	+	+	+

METHODOLOGY USED

As we have seen, the Timoshenko Beam Theory accounts for both the effect of rotary inertia and shear deformation, which are neglected when applied to Euler-Bernoulli Beam Theory [22]. The external and internal loading of the beam depends on its geometrical and material properties as well as the external applied torque. The geometrical properties refer mainly to its length, size and shape of its cross-section such as its area A , moment of inertia I with respect to the central axis of bending, and Timoshenko's shear coefficient k which is a modifying factor ($k < 1$) to account for the distribution of shearing stress such that effective shear area is equal to kA . The material properties refer to its density in mass per unit volume ρ , Young's modulus or modulus of elasticity E and shear modulus or modulus of rigidity G [23].

1. Mathematical Formulation

A Timoshenko beam takes into account shear deformation and rotational inertia effects, making it suitable for describing the behavior of short beams, sandwich composite beams or beams subject to high-frequency excitation when the wavelength approaches the thickness of the beam.

The resulting equation is of 4th order, but unlike ordinary beam theory - i.e. Bernoulli-Euler theory - there is also a second order spatial derivative present [16].

(Fig. 1: Deformation in Timoshenko Beam element)

In static Timoshenko beam theory without axial effects, the displacements of the beam are assumed to be given by

$$u_x(x, y, z) = -z\phi(x); u_y = 0; u_z = w(x)$$

Where (x, y, z) are the coordinates of a point in the beam, u_x, u_y, u_z are the components of the displacement vector in the three coordinate directions, ϕ is the angle of rotation of the normal to the mid-surface of the beam, and w is the displacement of the mid-surface in z -direction [25]. The governing equations are the following uncoupled system of ordinary differential equations is:

$$\frac{dw}{dx} = \phi - \frac{1}{kAG} \frac{d}{dx} \left(EI \frac{d\phi}{dx} \right)$$

Stiffness Matrix

In the finite element method and in analysis of spring systems, a stiffness matrix, K , is a symmetric positive semi index matrix that generalizes the stiffness of Hook's law to a matrix, describing the stiffness of between all of the degrees of freedom so that

$$F = - kx$$

Where F and x are the force and the displacement vectors, and

$$U = \frac{1}{2} * x^T kx$$

Is the system's total potential energy [30].

Mass Matrix

A mass matrix is a generalization of the concept of mass to generalized bodies. For static condition mass matrix does not exist, but in case of dynamic case mass matrix is used to study the behavior of the beam element. When load is suddenly applied or loads are variable nature, mass & acceleration comes into the picture [29].

2. FINITE ELEMENT FORMULATION

FEM is a numerical method of finding approximate solutions of partial differential equation as well as integral equation. The method essentially consists of assuming the piecewise continuous function for the solution and obtaining the parameters of the functions in a manner that reduces the error in the solution .By this method we divide a beam in to number of small elements and calculate the response for each small elements and finally added all the response to get global value. Stiffness matrix and mass matrix is calculate for each of the discretized element and at last all have to combine to get the global stiffness matrix and mass matrix [30]. The shape function gives the shape of the beam element at any point along longitudinal direction. This shape function also calculated by finite element method. Both potential and kinetic energy of beam depends upon the shape function. To obtain stiffness matrix potential energy due to deflection and to obtain mass matrix kinetic energy due to application of sudden load are use. So it can be say that potential and kinetic energy of the beam depends upon shape function of beam obtain by FEM method [32].

Formulation of Hermite Shape Function

Beam is divided in to element. Each node has two degrees of Freedom.

Degrees of freedom of node j are Q_{2j-1} and Q_{2j} Q_{2j-1} is transverse displacement and Q_{2j} is slope or rotation.

$Q = [Q_1 Q_2 Q_3 \dots Q_{10}]^T$ Q is the global displacement vector.

Local coordinates

$$q = [q_1, q_2, q_3, q_4]^T$$

$$= [v_1, v_2, v_3, v_4]$$

Hermite shape function for an element of length l_e [32].

RESULT AND DISCUSSION

In the present analysis the mathematical formulation and finite element formulation for loaded complex composite beam have been done. The beam is modeled by Timoshenko beam theory. This essentially consists of assuming the piecewise continuous function for the solution and obtaining the parameters of the functions in a manner that reduces the error in the solution. By this method we divide a beam in to number of small elements and calculate the response for each small elements and finally added all the response to get global value. By taking Timoshenko beam theory we have taken shear deformation into consideration which other theories neglect to make the beam analysis simplified. Due to this we can be able to formulate a composite beam that would be much more reliable for fabrication of structures that are under continuous loading.

CONCLUSION

As a result, shear strains and stresses are removed from the theory. Two essential aspects of Timoshenko's beam theory are the treatment of shear deformation by the introduction of a mid-plane rotation variable, and the use of a shear correction factor. Timoshenko beam theory we have taken shear deformation into consideration which other theories neglect to make the beam analysis simplified

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