A STUDY ON REGULARIZATION FUNCTIONS AND REGULATION PARAMETERS IN IMAGE RESTORATION

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ABSTRACT

The aim of this paper is to apply the regularization functions namely TV norm, l_1 norm and l_0 norm and regularization parameters with these norms in image restoration. This class of problems results from combining a linear observation model with a non-quadratic regularizer. Improved Iterative Shrinkage Thresholding algorithm (IISTA) and Iterative Shrinkage Thresholding algorithm (ISTA) are employed for comparison. These algorithms are performed through a recursive application of two simple procedures such as linear filtering and soft thresholding.

Key words: ISTA, IISTA, image restoration, inverse problems, l_0 norm, l_1 norm, l_2 data fidelity term, regularization function, total variation.

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1. INTRODUCTION

Inverse problems abound in many application areas of image processing, remote sensing, radar imaging, tomographic imaging, microscopic imaging, astronomic imaging, digital photography [1][2][3][12]. Image restoration is one of the earliest and most classical linear inverse problems in imaging dating back to the 1960s [1]. In an inverse problem, the goal is to estimate an unknown original image $\mathbf{x}$ from a noisy observation $\mathbf{y}$, produced by an operator $\mathbf{K}$ applied to $\mathbf{x}$, when $\mathbf{K}$ is a linear. Many applications to linear inverse problems LIPs define a solution $\mathbf{x}$ a restored image as a minimizer of a convex objective function $f: \mathbf{x} \rightarrow \mathbb{R} = [-\infty, +\infty]$ given by

$$f(\mathbf{x}) = 1/2 \|\mathbf{y} - \mathbf{K}\mathbf{x}\|^2 + \lambda\phi(\mathbf{x}) \quad (1)$$

where $\mathbf{K}: \mathbf{X} \rightarrow \mathbf{Y}$ is the linear direct operator, $\mathbf{X}$ and $\mathbf{Y}$ are real Hilbert spaces (both with norm denoted as $\|\cdot\|$), $\phi: \mathbf{X} \rightarrow \mathbb{R}$ is a function, $\lambda \in [0, +\infty]$ is a parameter.

In a regularization framework minimizing f is seen as a way of overcoming the ill-conditioned or singular nature of $\mathbf{K}$, which precludes inverting it. In this context ϕ is called regularizer and λ is called regularization parameter [5].

In a finite dimensional Bayesian setting, the reasoning behind (1) is as follows: Assume that $y=kx+w$, where w is a sample of a white zero-mean Gaussian random vector/field of variance σ^2 , let $p(x)$ be the adopted prior, thus the logarithm of a posteriori density is $\log p(x|y) = -f(x)$ upto a constant with $\lambda = \sigma^2$ and $\phi(x) = -\log p(x)$: maximum posteriori (MAP) estimate are thus minimizer of f . Despite the possible interpretation of (1)

ϕ simply referred as the regularize in equation (1). The intuitive meaning of f is simple: minimizing it corresponds to looking for a compromise between the lack of fitness of a candidate estimate X to the observed data, measured by $\|y - Kx\|^2$ and its degree of undesirability given by $\phi(x)$. The regularization parameter λ controls the relative weight of the two terms. Examples of total variation (TV) regularization [8][13] and wavelet based regularization [14][15]. The non-differentiable nature of f together with the huge dimension of its argument for example 512×512 image $X = R^{262144}$, place its minimization beyond the reach of standard off-the-shelf optimization methods.

This paper strictly concerned with algorithms for minimization [1] and discusses different choices of λ and ϕ . The rest of the paper is organized as follows. Section 2 discusses preliminaries. Section 3 explains proposed work. Section 4 analyses the experimental results. Section 5 concludes the paper.

2. PRELIMINARIES

2.1. General Restoration Model

In general, the image restoration problems have the form

$$\min_{x \in R^n} \varphi(x) = f(x) + \tau c(x) \quad (2)$$

$f: R^n \rightarrow R$ is smooth and convex data fidelity term, usually

$$f(x) = \frac{1}{2} \|Ax - y\|_2^2 \quad (3)$$

$C: R^n \rightarrow R$ is a regularization or penalty function, typically convex often non-differentiable.

If $A=I$, it has a denoising problem.

If c is a proper and convex φ is strictly convex, there is a unique minimizer. Thus the shrinkage thresholding function is

$$\Psi_\lambda = \arg \min_z \frac{1}{2} \|Z - u\|_2^2 + \lambda c(z) \quad (4)$$

is a well defined Moreau-proximal mapping.

3. PROPOSED WORK

3.1. Regularization Functions

Three types of regularization functions l_0 norm, l_1 norm and TV norm are studied with respect to restoration. Iterative shrinkage thresholding algorithm (ISTA) [9] and Improved Iterative shrinkage thresholding (IISTA) [10] is used to measure the performance.

If $c(z) = \|z\|_0$, l_0 norm then $\Psi_\lambda = \text{hard}(z, \lambda)$ where $\text{hard}(z, \lambda) = x^* (\text{abs}(x) > \sqrt{2\lambda})$.

If $c(z) = \|z\|_1$, l_1 norm then $\Psi_\lambda(z) = \text{soft}(z, \lambda)$ where

$\text{soft}(z, \lambda) = \text{sign}^*(|z| - \lambda)_+$ and

$$(a)_+ = \begin{cases} 0 & \text{if } a < 0 \\ a & \text{if } a \geq 0 \end{cases}$$

Note that both functions are component wise application

If $c(z) = \text{TV}(z)$, total variation function then equation becomes

$$\Psi_\lambda(z) = \arg \min_z \frac{1}{2} \|Az - y\|_2^2 + \tau \text{TV}(z) \quad (5)$$

Where $\text{TV}(z) = \sqrt{(\Delta_i^h z)^2 + (\Delta_i^v z)^2}$ where $\Delta_i^h(z)$ and $\Delta_i^v(z)$ are linear operators corresponding to horizontal and vertical first order differences at point i respectively i.e, $\Delta_i^h z = z_i - z_j$ where j is the first order neighbour to the left of i , and $\Delta_i^v z = z_i - z_k$ where k is the first order neighbour above i . This equation is isotropic and not differentiable.

τ is a regularization parameter in equation (5) are analyzed for three types regularization functions.

4. EXPERIMENTAL RESULTS

The Experiments are carried with MATLAB R2010a. The role of regularization functions and regularization parameters are studied using ISTA and IISTA algorithms under blur image of size 4x4 and 9x9. The increase of blur size shows that increase of iterations for restoration

Tables 1-4 gives result of regularization function role with number of iterations. The study of regularization parameter τ with ISNR, objective function and regularizations is consolidated in Tables 5-8.

Table 1 ITERATIONS vs. ISNR using ISTA Blur size 9x9

Iterations	ISNR		
	TV norm	l_1 norm	l_0 norm
500	6.275	6.285	6.285
1000	6.690	6.688	6.688
1500	6.985	6.955	6.955
2000	7.210	7.174	7.210
2500	7.398	7.125	7.367
3000	7.557	7.149	7.530
3500	7.655	7.141	7.648
4000	7.786	7.137	7.781
4500	7.865	7.157	7.879
5000	7.993	7.163	7.974

Table 2 ITERATIONS vs. Objective function using ISTA Blur size 9x9

Iterations	Objective function		
	TV norm (10^3)	l_1 norm(10^4)	l_0 norm(10^3)
500	9.54921	2.81700	5.42570
1000	9.23162	2.80714	5.18368
1500	9.09400	2.79831	6.95500
2000	9.05012	2.80361	4.98930
2500	8.91964	2.79718	4.88313
3000	8.92602	2.80016	4.89099
3500	8.92025	2.80091	4.85595
4000	8.92264	2.79913	4.83579
4500	8.80493	2.80014	4.81461
5000	8.84212	2.80633	4.84594

Table 3 ITERATIONS Vs ISNR and Objective function using IISTA Blur size 9x9

Iterations	ISNR		Objective function	
	TV norm	l_0 norm	TV norm (10^3)	l_0 norm(10^3)
50	6.613	6.160	9.70883	5.53750
100	6.951	6.972	9.14136	5.02918
150	7.612	7.622	9.01817	4.81195
200	8.077	8.127	8.81832	4.78740
250	8.452	8.410	8.81019	4.79088
300	8.660	8.612	8.81834	4.80032
350	8.452	8.682	8.79188	4.72642
400	8.750	8.801	8.78827	4.76248
450	8.719	8.788	8.80850	4.79617
500	8.736	8.718	8.79433	4.78165

Table 4 ITERATIONS vs. ISNR using IISTA Blur size 4x4

Iterations	ISNR		
	TV norm	l_1 norm	l_0 norm
50	6.649	6.431	6.610
60	6.841	6.523	6.710
70	7.037	6.525	6.753
80	7.130	6.523	6.852
90	7.322	6.544	6.880
100	7.482	6.549	6.903
110	7.560	6.578	6.891
120	7.651	6.554	6.842
130	7.764	6.535	6.891
140	7.823	6.521	6.909
150	7.918	6.606	6.827

Table 5 Regularization parameter τ vs. ISNR using ISTA Number of Iterations=50

τ	ISNR		
	TV norm	l_1 norm	l_0 norm
10^{-1}	6.427	5.916	5.920
10^{-2}	6.002	6.000	6.002
10^{-3}	5.941	5.879	5.930
10^{-4}	5.902	5.923	5.936
10^{-5}	5.894	5.898	5.895
10^{-6}	5.901	5.895	5.932
10^{-7}	5.921	5.894	5.882
10^{-8}	5.914	5.902	5.914
10^{-9}	5.899	5.921	5.916
10^{-10}	5.905	5.914	5.924
10^{-20}	5.850	5.899	5.898
10^{-30}	5.898	5.905	5.891
10^{-38}	5.89	5.899	5.912

Table 6 Regularization parameter τ vs. Objective function using ISTA Number of Iterations=50

τ	Objective function		
	TV norm (10^4)	l_1 norm (10^4)	l_0 norm (10^4)
10^{-1}	6.9238	28.1120	1.1537
10^{-2}	1.3130	3.2429	0.5330
10^{-3}	0.5531	0.7370	0.4730
10^{-4}	0.4734	0.4958	0.4620
10^{-5}	0.4570	0.4635	0.4606
10^{-6}	0.4600	0.4560	0.4597
10^{-7}	0.4576	0.4561	0.4589
10^{-8}	0.4600	0.4623	0.4644
10^{-9}	0.4614	0.4576	0.4579
10^{-10}	0.4630	0.4600	0.4629
10^{-20}	0.4584	0.4614	0.4641
10^{-30}	0.4584	0.4629	0.4617
10^{-38}	0.4594	0.4649	0.4556

Table 7 Regularization parameter τ vs. ISNR using IISTA Number of iterations=50

τ	ISNR		
	TV norm	l_1 norm	l_0 norm
10^{-1}	6.780	5.959	5.974
10^{-2}	6.544	6.543	6.554
10^{-3}	5.928	5.913	5.942
10^{-4}	5.890	5.935	5.910
10^{-5}	5.934	5.909	5.898
10^{-6}	5.931	5.917	5.994
10^{-7}	5.906	5.945	5.915
10^{-8}	5.924	5.892	5.917
10^{-9}	5.860	5.926	5.933
10^{-10}	5.907	5.928	5.908
10^{-20}	5.920	5.938	5.910
10^{-30}	5.916	5.890	5.958
10^{-38}	5.934	5.906	5.930

Table 8 Regularization parameter τ vs. Objective function using IISTA Number of iteration =50

τ	Objective function		
	TV norm (10^4)	l_1 norm (10^4)	L_0 norm (10^4)
10^{-1}	5.8518	28.1210	1.1564
10^{-2}	1.2533	3.2316	0.5159
10^{-3}	0.5021	0.6792	0.4112
10^{-4}	0.4031	0.4319	0.3946
10^{-5}	0.3986	0.3999	0.4002
10^{-6}	0.3994	0.3969	0.3963
10^{-7}	0.3995	0.3956	0.3981
10^{-8}	0.3969	0.3952	0.4023
10^{-9}	0.3952	0.4002	0.3961
10^{-10}	0.3925	0.3938	0.3965
10^{-20}	0.3957	0.3992	0.3959
10^{-30}	0.3991	0.3967	0.3924
10^{-38}	0.3976	0.3995	0.4001

Figure 1 and Figure 2 shows restoration using different norms of ISTA and IISTA algorithms.

Figure 1 A) Original image B) noisy and blur image Blur size 9x9 Restored using ISTA C) TV norm D) l_1 norm and E) l_0 norm Number of iterations is 5000.

Figure 2 A) Original image B) noisy and blur image blur size 4x4 Restored using IISTA C) TV norm D) l_1 norm and E) l_0 Number of Iterations is 150

5. CONCLUSION

The observations shows that under blur image of size 9x9 and 4x4, TV norm shows better ISNR than l_0 norm and l_1 norm for both ISTA and IISTA. In the case of l_1 norm using 9x9 blur image size shows recovery is not possible after a certain number of iterations. The study of regularization parameter shows better performance for l_0 norm than TV norm and l_1 norm. It is observed that when the value of τ decreases the performance of algorithms is also decreases. The future study may be continued for the role of different types of noise.

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