Fuzzy Reliability Based on Hesitant and Dual Hesitant Fuzzy Set Evaluation

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Abstract

In this study, we calculate hesitant and dual hesitant fuzzy reliability of linear and circular consecutive 2-out-of-4:G system from reliability function and Weibull distribution. After that, we introduced some basic methods for investigating the fuzzy reliability in the form of upper and lower membership and non-membership function having an aggregation operator with equal weights. At last, two numerical examples are also illustrated with the considered technique.

Keywords- Fuzzy reliability, Hesitant fuzzy, Dual hesitant fuzzy, Triangular fuzzy number, Linear and circular consecutive k -out-of- n :G system.

1. Introduction

In recent years, the theory of fuzzy set is widely used in the field of reliability which is based on real-life engineering systems and management. Various researchers used different types of fuzzy set having triangular and trapezoidal fuzzy numbers such as intuitionistic fuzzy, hesitant fuzzy, bifuzzy and picture, etc. for the analysis of fuzzy reliability. Zadeh (1965) explained the relationship between set theory and fuzzy set theory having membership grade zero and one. The author discussed the various operations in the perspective of fuzzy set theory like union, intersection, multiplication, and division, etc. and many more applications in the real world. Atanassov (1983) discussed the intuitionistic fuzzy theory based on the crisp set and define membership and non-membership functions. The author described various properties based on crisp theory and also defined operations as a complement, union, intersection, multiplication, and division. Torra and Narukawa (2009) described the extension of fuzzy set theory based on

intuitionistic fuzzy called hesitant fuzzy having membership function value between zero and one. The authors also defined various types of operations and implementation of the crisp set for interval membership. Zhang (2013) first introduced the various power aggregations operators defined on extended hesitant fuzzy set and explained the relationship between properties and operations of hesitant fuzzy which utilized decision-making problems. Rodríguez et al. (2014) introduced hesitant fuzzy set deals with some uncertainty of various kinds of problems and also discussed membership grade. Hao et al. (2017) determined the dual hesitant set of membership and non-membership grade which depend on intuitionistic fuzzy set theory and defined some basic laws of operations on aggregation operators to improve the results.

Liu et al. (2007) calculated the fuzzy reliability and mean time to failure (MTTF) of series-parallel, parallel-series, and cold standby repairable systems. The authors also described various results, how to calculate the fuzzy reliability of the un-repairable system of lifetime random variables. Kumar et al. (2013) discussed the fundamentals of the fuzzy and intuitionistic fuzzy set having membership and non-membership failure rate of series and parallel systems. The authors extended the idea of the fuzzy set by the idea of intuitionistic fuzzy set and also discussed a new idea to generate the membership and non-membership functions of the fuzzy reliability. Kumar and Ram (2018) also calculated the hesitant fuzzy of the proposed system from aggregation and Weibull distribution in the form of interval fuzzy. Kumar and Ram (2019) evaluated the fuzzy reliability of some basic environmental system as series, parallel and combinations of these systems based on dual hesitant fuzzy set and Weibull distribution. The authors used the concept of the Markov chain technique for evaluating the system reliability in the form of membership and non-membership function. Kumar et al. (2019 and 2020) determined the fuzzy reliability of the considered consecutive- u -out-of- v : F system having non-identical components from the Markov chain process and determined interval fuzzy from intuitionistic and hesitant fuzzy set both membership and non-membership function. Negi and Singh (2019) computed fuzzy reliability and fuzzy MTTF of linear m -consecutive weighted- u -out-of- r -from- v : F system based on universal generating function and exponentially distributed.

From the above discussion, it is clear that many researchers analyzed the reliability of various systems series, parallel and complex systems using fuzzy intervals like fuzzy theory, intuitionistic, bifuzzy, interval-valued fuzzy, hesitant fuzzy and dual hesitant fuzzy, etc. Keeping the above facts in the view, in the present work we have evaluated the hesitant and dual hesitant fuzzy using aggregation operator of linear and circular consecutive-2-out-of-4 using reliability function and Weibull distribution.

2. Definitions

2.1 Fuzzy Set

The relation between the fuzzy set theory and the crisp set theory was first discussed by Zadeh (1965) based on its membership function of α -cut set. A fuzzy set Z is to describe the membership grade $\mu_Z : A \rightarrow [0, 1]$, where $\mu_Z(A)$ belongs to membership component z in a fuzzy A for all $a \in A$ and $A = (a_1, a_2, \dots, a_n)$ is the non-empty set of class.

2.2 Intuitionistic Fuzzy Set

The fuzzy set theory defined from the membership and non-membership grade having their α and β -cut set is called intuitionistic fuzzy set theory is given by Atanassov (1983). It is defined as a set C to be a subset of b as:

$$C = \{ \langle b, \mu_C(b), \nu_C(b) \rangle : b \in B \}$$

where, $\mu_C : b \rightarrow [0, 1]$ and $\nu_C : b \rightarrow [0, 1]$ with $\mu_C(b) + \nu_C(b) \leq 1, \forall b \in B$.

2.3 Hesitant Fuzzy Set

The hesitant fuzzy set is the extended form of the fuzzy set theory discussed by Torra and Naukara (2009) of a crisp set. The hesitant fuzzy set B based on membership function such as

$$B = \{ (y, h_B(a)) / a \in A \}$$

where, $h_B(a)$ is the set of the value $[0, 1]$ and component $a \in A$ shows the membership grade of the set B and h is a hesitant fuzzy component of the set.

Xia and Xu (2011) discussed the functional of the hesitant fuzzy component are as:

- (i) $h^\alpha = \bigcup_{\beta \in h} \{ \delta^\alpha \}, \alpha > 0,$
- (ii) $\alpha h = \bigcup_{\beta \in h} \{ 1 - (1 - \beta)^\alpha \}, \alpha > 0,$
- (iii) $h_1 \oplus h_2 = \bigcup_{\beta_1 \in h_1, \beta_2 \in h_2} \text{Max} \{ \beta_1 + \beta_2 - \beta_1 \beta_2 \},$
- (iv) $h_1 \otimes h_2 = \bigcup_{\beta_1 \in h_1, \beta_2 \in h_2} \text{Min} \{ \beta_1 \beta_2 \}.$

Basic functional of hesitant fuzzy components, (Torra, 2010) as

- (i) $h^c = \bigcup_{\beta \in h} \{ 1 - \beta \},$
- (ii) $h_1 \cup h_2 = \bigcup_{\beta_1 \in h_1, \beta_2 \in h_2} \text{Max} \{ \beta_1 \beta_2 \},$
- (iii) $h_1 \cap h_2 = \bigcup_{\beta_1 \in h_1, \beta_2 \in h_2} \text{Min} \{ \beta_1 \beta_2 \}.$

2.4 Dual Hesitant Fuzzy Set

The concept of dual hesitant fuzzy set (DHFS) was discussed by Zhu et al. (2012), Chen and Huang (2017) presented the membership and non-membership grade. Let us consider A be a set of DHFS, then A be defined as:

$$D_h = \{ \langle a, C(a), E(a) \rangle : a \in A \},$$

where, $C(a)$ and $E(a)$ are sets which values lie in $[0, 1]$ and shows the possible membership and non-membership function of the component $a \in A$ to the set D_h under valid operations.

$$0 \leq \alpha, \beta \leq 1, 0 \leq \alpha + \beta \leq 1,$$

where, $\alpha \in C(a)$, $\beta \in E(a)$, $\alpha \in C(a) = \bigcup_{\alpha \in C(a)} \max \{\alpha\}$ and $\beta \in E(a) = \bigcup_{\beta \in E(a)}$ for all $a \in A$.

2.5 Operations on DHFS

If D_{h_1} and D_{h_2} two set DHFS then

$$(i) D_{h_1} \cup D_{h_2} = \{C \in (C_1 \cup C_2) \mid C \geq \max(C_1, C_2), E \in (E_1, E_2) \mid E \leq \min(E_1, E_2)\},$$

$$(ii) D_{h_1} \cap D_{h_2} = \{C \in (C_1 \cap C_2) \mid C \leq \min(C_1, C_2), E \in (E_1, E_2) \mid E \geq \max(E_1, E_2)\},$$

$$(iii) D^c = \begin{cases} \bigcup_{\alpha \in C, \beta \in E} \{\{\alpha\}, \{\beta\}\}, & \text{if } E \neq \gamma, C \neq \gamma \\ \bigcup_{\alpha \in h} \{\{1-\alpha\}, \{\gamma\}\}, & \text{if } E = \gamma, C \neq \gamma \\ \bigcup_{\beta \in E} \{\{\gamma\}, \{1-\alpha\}\}, & \text{if } C = \gamma, E \neq \gamma. \end{cases}$$

3. Triangular Fuzzy Number

(a) Triangular fuzzy number χ of lower, upper and modal value is a real value function R basis on the membership grade function $\mu_\chi : R \rightarrow [0,1]$, (Chang, 1996) as

$$\mu_\chi(z) = \begin{cases} \frac{x - \chi^L}{\chi^M - \chi^L}, & z \in [\chi^L, \chi^M] \\ \frac{x - \chi^U}{\chi^M - \chi^U}, & z \in [\chi^M, \chi^U] \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where, $\chi_L \leq \chi_M \leq \chi_U$.

If we have two triangular fuzzy numbers $\chi_1 = (\chi_{L_1}, \chi_{M_1}, \chi_{U_1})$ and $\chi_2 = (\chi_{L_2}, \chi_{M_2}, \chi_{U_2})$, and these are some operators which are expressed as:

$$(i) \chi_1 \oplus \chi_2 = (\chi_{L_1}, \chi_{M_1}, \chi_{U_1}) \oplus (\chi_{L_2}, \chi_{M_2}, \chi_{U_2}) = (\chi_{L_1} + \chi_{L_2}, \chi_{M_1} + \chi_{M_2}, \chi_{U_1} + \chi_{U_2}),$$

$$(ii) \chi_1 \otimes \chi_2 = (\chi_{L_1}, \chi_{M_1}, \chi_{U_1}) \otimes (\chi_{L_2}, \chi_{M_2}, \chi_{U_2}) = (\chi_{L_1} \chi_{L_2}, \chi_{M_1} \chi_{M_2}, \chi_{U_1} \chi_{U_2}),$$

$$(iii) \chi_1 = \chi_2 \Leftrightarrow \chi_{L_1} = \chi_{L_2}, \chi_{M_1} = \chi_{M_2}, \chi_{U_1} = \chi_{U_2},$$

$$(iv) \chi_1 \leq \chi_2 \Leftrightarrow \chi_{L_1} \leq \chi_{L_2}, \chi_{M_1} \leq \chi_{M_2}, \chi_{U_1} \leq \chi_{U_2},$$

$$(v) \beta \cdot \chi_1 = (\beta, \beta, \beta)(\chi_{L_1}, \chi_{M_1}, \chi_{U_1}) = (\beta \chi_{L_1}, \beta \chi_{M_1}, \beta \chi_{U_1}), \beta > 0, \beta \in R,$$

$$(vi) \chi_1^{-1} = (\chi_{L_1}, \chi_{M_1}, \chi_{U_1})^{-1} = (1/\chi_{U_1}, 1/\chi_{M_1}, 1/\chi_{L_1}).$$

(b) Let us consider $g_i (i=1,2,...,n)$ be a class of hesitant fuzzy component and $W = (w^1, w^2, ..., w^n)^t$ be define weight vector of $g_i (i=1,2,...,n)$ belongs $W^i \in [0,1]$ and $\sum_{i=1}^n W^i = 1$, then weighted averaging operator also called score function of a triangular fuzzy set is

$$G^n \rightarrow G = (g_1, g_2, ..., g_n) = \bigotimes_i^n (W^i g_i) \\ = \left\{ \left(1 - \prod_{i=1}^n (1 - \chi_{L_i})^{W^i}, 1 - \prod_{i=1}^n (1 - \chi_{M_i})^{W^i}, 1 - \prod_{i=1}^n (1 - \chi_{U_i})^{W^i} \right) \right\} \quad (2)$$

4. Fuzzy Weibull Distribution

In the case of reliability evaluation, Weibull distribution is generally used with hazard function and lifetime components are as follows:

$$f(t) = \begin{cases} \beta \theta^{-1} (t \theta^{-1})^{\beta-1} e^{-(t \theta^{-1})^\beta}, & x > 0, \theta > 0, \beta > 0 \\ 0, & \text{otherwise} \end{cases}, \quad \text{namely, } \theta = \text{scale and } \beta = \text{shape parameter.}$$

Hence, the reliability function of the system having Weibull distribution is

$$= e^{-\left\{ \left(\frac{x-\eta}{\theta} \right)^\beta \right\}} \quad (3)$$

Here, $\eta = 0$ the failure rate $= e^{-\left\{ (x \theta^{-1})^\beta \right\}}$.

5. Linear and Circular Consecutive u -out-of- v System

Linear and circular consecutive system are two types such as linear and circular consecutive u -out-of- v :F and G systems, if at least consecutive u component failed then it is called failed system and if at least consecutive u components work it called working system respectively. The system components have two conditions either working or failed. These are some applications of the proposed system as Vacuum systems in accelerators and field of integrated circuits. If all components of the system having equal reliability *i.e.* $R_1=R_2=R_3=...R_v=R$ then, reliability and unreliability function of the linear and circular consecutive u -out-of- v :G system can be determined as:

$$p(v; u) = p(v-1; u) + kr^k (1 - p(v-u-1; u)) \text{ and}$$

$$q(v, u) = \sum_{t=v-u+1}^v \sum_{m=1}^{(t+u)v} k^2 r^{v-t+m-1} . q(t-m-1, u)$$

also calculate the reliability of circular consecutive- u -out-of- v :G system
 $q(i; j) = 1 - q((i, j); u)$ (Zhang, 1988).

5.1 Reliability Function of Linear Consecutive 2-out-of-4:G System

Consider a linear consecutive 2-out-of-4: G system having the same reliability components and evaluate the reliability function of the system as:

First, we assess the reliability function using the formula of linear consecutive 2-out-of-4:G system see (Zhang, 1988) as:

$$p(n; k) = p(n-1; k) + ur^k(1 - r(n-k-1; k)) \quad (4)$$

Taking $n=4$ and $k=2$, put in Eq. (4), then we can express as:

$$\begin{aligned} p(0, 2) &= p(-1, 2) + q(-3, 2) = 0 \\ p(1, 2) &= 0 \\ p(2, 2) &= r^2 \\ p(3, 2) &= 2r^2 - r^3 \\ p(4, 2) &= 3r^2 - 2r^3 \end{aligned} \quad (5)$$

5.2 Reliability Function of Circular Consecutive 2-out-of-4:G System

Using Eq. (4), to estimate the reliability of circular consecutive 2-out-of-4:G system same as a previous linear system, now we will start from Eq. (5) to get unreliability and reliability function such as:

$$p(4, 2) = 3r^2 - 2r^3.$$

Using the formula (Zhang, 1988) for $v=4$ and $u=2$ we have,

$$q(v, u) = \sum_{t=v-u+1}^v \sum_{m=1}^{(t+u)v} u^2 r^{v-t+m-1} \otimes q(t-m-1, u),$$

then,

$$\begin{aligned} q(4, 2) &= \sum_{t=3}^4 \sum_{m=1}^{(t+2)} u^2 r^{3-t+m} \otimes q(t-m-1, 2) \\ &= \sum_{m=1}^1 u^2 r^m q(2-m, 2) + \sum_{m=1}^2 u^2 r^{m-1} q(3-m, 2) \\ &= u^2 [2rq(1, 2) + q(2, 2)] \\ &= u^2 [2r(1 - p(1, 2) + 1 - p(2, 2))] \\ q(4, 2) &= 1 + 4r^3 - 4r^2 - r^4, \text{ hence} \end{aligned}$$

$$p(4, 2) = 1 - q(4, 2)$$

$$p(4, 2) = 4r^2 - 4r^3 + r^4 \quad (6)$$

6. Numerical Example

We considered the hesitant membership function ϕ from a triangular fuzzy number of lifetime distribution are obtained, if membership function defined as $\varphi = \varphi_1; (1.5, 2, 2.5)$, $\varphi_2; (1.75, 1.85, 1.95)$, $\varphi_3; (2.25, 2.5, 2.75)$ having hesitant fuzzy number within the same weightage given $W_1 = W_2 = W_3 = 1/3$, hence α -cut is described as: $\phi = \phi_1; (1.5 + 0.5\alpha, 2.5 - 0.5\alpha)$, $\phi_2; (1.75 + 0.10\alpha, 1.95 - 0.10\alpha)$, $\phi_3; (2.25 + 0.25\alpha, 2.75 - 0.25\alpha)$.

Now with the help of Eqs. (3), (5) and (6), we obtain fuzzy reliability of linear and circular consecutive 2-out-of-4: G system for all α , taking $\beta = 0.5$ and $t = 5$ is

$$R(t)(\alpha) = 3e^{\left(-2\left(\frac{t}{1.5+0.5\alpha}\right)^\beta\right)} - 2e^{\left(-3\left(\frac{t}{1.5+0.5\alpha}\right)^\beta\right)},$$

$$3e^{\left(-2\left(\frac{t}{2.5-0.5\alpha}\right)^\beta\right)} - 2e^{\left(-3\left(\frac{t}{2.5-0.5\alpha}\right)^\beta\right)};$$

$$3e^{\left(-2\left(\frac{t}{1.75+0.10\alpha}\right)^\beta\right)} - 2e^{\left(-3\left(\frac{t}{1.75+0.10\alpha}\right)^\beta\right)},$$

$$3e^{\left(-2\left(\frac{t}{1.95-0.10\alpha}\right)^\beta\right)} - 2e^{\left(-3\left(\frac{t}{1.95-0.10\alpha}\right)^\beta\right)}$$

$$3e^{\left(-2\left(\frac{t}{2.25+0.25\alpha}\right)^\beta\right)} - 2e^{\left(-3\left(\frac{t}{2.25+0.25\alpha}\right)^\beta\right)},$$

$$3e^{\left(-2\left(\frac{t}{2.75-0.25\alpha}\right)^\beta\right)} - 2e^{\left(-3\left(\frac{t}{2.75-0.25\alpha}\right)^\beta\right)};$$

$$R(t)(\alpha) = 4e^{\left(-2\left(\frac{t}{1.5+0.5\alpha}\right)^\beta\right)} - 4e^{\left(-3\left(\frac{t}{1.5+0.5\alpha}\right)^\beta\right)} + e^{\left(-4\left(\frac{t}{1.5+0.5\alpha}\right)^\beta\right)},$$

$$4e^{\left(-2\left(\frac{t}{2.5-0.5\alpha}\right)^\beta\right)} - 4e^{\left(-3\left(\frac{t}{2.5-0.5\alpha}\right)^\beta\right)} + e^{\left(-4\left(\frac{t}{2.5-0.5\alpha}\right)^\beta\right)};$$

$$4e^{\left(-2\left(\frac{t}{1.75+0.10\alpha}\right)^\beta\right)} - 4e^{\left(-3\left(\frac{t}{1.75+0.10\alpha}\right)^\beta\right)} + e^{\left(-4\left(\frac{t}{1.75+0.10\alpha}\right)^\beta\right)},$$

$$4e^{\left(-2\left(\frac{t}{1.95-0.10\alpha}\right)^\beta\right)} - 4e^{\left(-3\left(\frac{t}{1.95-0.10\alpha}\right)^\beta\right)} + e^{\left(-4\left(\frac{t}{1.95-0.10\alpha}\right)^\beta\right)};$$

$$4e^{\left(-2\left(\frac{t}{2.25+0.25\alpha}\right)^\beta\right)} - 4e^{\left(-3\left(\frac{t}{2.25+0.25\alpha}\right)^\beta\right)} + e^{\left(-4\left(\frac{t}{2.25+0.25\alpha}\right)^\beta\right)},$$

$$4e^{\left(-2\left(\frac{t}{2.75-0.25\alpha}\right)^\beta\right)} - 4e^{\left(-3\left(\frac{t}{2.75-0.25\alpha}\right)^\beta\right)} + e^{\left(-4\left(\frac{t}{2.75-0.25\alpha}\right)^\beta\right)}.$$

Hence, Table 1, Table 2 and Figures 1-6 discussed the fuzzy reliability of considered system based on reliability function and hesitant fuzzy membership grade using the above equations are as:

Table 1. Reliability of the linear system.

α -cut	$R_1[\alpha]$	$R_2[\alpha]$	$R_3[\alpha]$
0	(0.0778, 0.1772)	(0.1021, 0.1219)	(0.1521, 0.2020)
0.1	(0.0826, 0.1772)	(0.1030, 0.1209)	(0.1546, 0.1996)
0.2	(0.0874, 0.1671)	(0.1040, 0.1199)	(0.1571, 0.1971)
0.3	(0.0923, 0.1621)	(0.1050, 0.1189)	(0.1596, 0.1946)
0.4	(0.0971, 0.1571)	(0.1060, 0.1179)	(0.1621, 0.1921)
0.5	(0.1021, 0.1521)	(0.1070, 0.1169)	(0.1646, 0.1896)
0.6	(0.1070, 0.1470)	(0.1080, 0.1159)	(0.1671, 0.1871)
0.7	(0.1120, 0.1420)	(0.1090, 0.1149)	(0.1697, 0.1847)
0.8	(0.1169, 0.1370)	(0.1100, 0.1139)	(0.1722, 0.1822)
0.9	(0.1219, 0.1320)	(0.1110, 0.1129)	(0.1747, 0.1797)
1	(0.1269, 0.1269)	(0.1120, 0.1120)	(0.1772, 0.1772)

Figure 1. Membership function of $R_1(\alpha)$ for linear system.

Figure 2. Membership function of $R_2(\alpha)$ for linear system.

Figure 3. Membership function of $R_3(\alpha)$ for linear system.

Table 2. Reliability of the consecutive system.

α -cut	$R_1[\alpha]$	$R_2[\alpha]$	$R_3[\alpha]$
0	(0.0878, 0.1824)	(0.1122, 0.1315)	(0.1598, 0.2042)
0.1	(0.0926, 0.1780)	(0.1131, 0.1305)	(0.1621, 0.2021)
0.2	(0.0975, 0.1735)	(0.1141, 0.1296)	(0.1644, 0.1999)
0.3	(0.1024, 0.1689)	(0.1151, 0.1286)	(0.1667, 0.1978)
0.4	(0.1073, 0.1644)	(0.1160, 0.1276)	(0.1689, 0.1956)
0.5	(0.1122, 0.1598)	(0.1170, 0.1267)	(0.1712, 0.1934)
0.6	(0.1170, 0.1551)	(0.1180, 0.1257)	(0.1735, 0.1913)
0.7	(0.1219, 0.1505)	(0.1190, 0.1248)	(0.1757, 0.1891)
0.8	(0.1267, 0.1458)	(0.1199, 0.1238)	(0.1780, 0.1869)
0.9	(0.1315, 0.1410)	(0.1209, 0.1228)	(0.1802, 0.1847)
1	(0.1363, 0.1363)	(0.1219, 0.1219)	(0.1824, 0.1824)

Figure 4. Membership function of $R_1(\alpha)$ for consecutive system.

Figure 5. Membership function of $R_2(\alpha)$ for consecutive system.

Figure 6. Membership function of $R_3(\alpha)$ for consecutive system.

Now, we have calculated hesitant fuzzy by using of aggregation operator from Eq. (2), and obtained the results for the considered system as shown in Table 3 and Table 4, correspondingly Figure 7 and Figure 8, show the triangular membership function.

Table 3. Reliability of the linear system after aggregation operator.

α -cut	$R_1[\alpha]$
0	(0.1112, 0.1677)
0.1	(0.1139, 0.1665)
0.2	(0.1146, 0.1620)
0.3	(0.1195, 0.1597)
0.4	(0.1222, 0.1562)
0.5	(0.1250, 0.1534)
0.6	(0.1278, 0.1488)
0.7	(0.1307, 0.1477)
0.8	(0.1335, 0.1448)
0.9	(0.1363, 0.1420)
1	(0.1392, 0.1392)

Figure 7. Membership function of linear consecutive 2-out-4: G system.

Table 4. Reliability of the circular consecutive system after aggregation operator.

α -cut	$R_1[\alpha]$
0	(0.1204, 0.1733)
0.1	(0.1231, 0.1707)
0.2	(0.1258, 0.1673)
0.3	(0.1285, 0.1656)
0.4	(0.1312, 0.1630)
0.5	(0.1339, 0.1604)
0.6	(0.1366, 0.1578)
0.7	(0.1393, 0.1552)
0.8	(0.1419, 0.1526)
0.9	(0.1446, 0.1499)
1	(0.1473, 0.1473)

Figure 8. Membership function of circular consecutive 2-out-4: G system.

6.1 Numerical Illustration

The fuzzy reliability of linear and circular consecutive 2-out-4: G system is obtained from the dual hesitant fuzzy set and triangular fuzzy number with Weibull distribution. If triangular fuzzy is defined as $s = (1, 1.25, 1.50, 1.75, 2)$ and α -cut of dual hesitant fuzzy number is (Chen and Huang, 2017) as

$$R(t)[\alpha] = \max(1 + 0.5\alpha, 1.25 + 0.5(1 - \alpha)); \min(2 + 0.5\alpha, 1.75 + 0.25(1 - \alpha)).$$

For all α , taking $\beta = 0.5$ and $t = 5$

$$R(t)(\alpha) = \max \left[3e^{\left(-2\left(\frac{t}{1+0.5\alpha}\right)^\beta\right)} - 2e^{\left(-3\left(\frac{t}{1+0.5\alpha}\right)^\beta\right)}, \right. \\ \left. 3e^{\left(-2\left(\frac{t}{1.25+0.5(1-\alpha)}\right)^\beta\right)} - 2e^{\left(-3\left(\frac{t}{1.25+0.5(1-\alpha)}\right)^\beta\right)} \right] \\ \min \left[3e^{\left(-2\left(\frac{t}{2+0.5\alpha}\right)^\beta\right)} - 2e^{\left(-3\left(\frac{t}{2+0.5\alpha}\right)^\beta\right)}, \right. \\ \left. 3e^{\left(-2\left(\frac{t}{1.75+0.25(1-\alpha)}\right)^\beta\right)} - 2e^{\left(-3\left(\frac{t}{1.75+0.25(1-\alpha)}\right)^\beta\right)} \right].$$

$$R(t)(\alpha) = \max \left[4e^{\left(-2\left(\frac{t}{1+0.5\alpha}\right)^\beta\right)} - 4e^{\left(-3\left(\frac{t}{1+0.5\alpha}\right)^\beta\right)} + e^{\left(-4\left(\frac{t}{1+0.5\alpha}\right)^\beta\right)}, \right. \\ \left. 4e^{\left(-2\left(\frac{t}{1.25+0.5(1-\alpha)}\right)^\beta\right)} - 4e^{\left(-3\left(\frac{t}{1.25+0.5(1-\alpha)}\right)^\beta\right)} + e^{\left(-4\left(\frac{t}{1.25+0.5(1-\alpha)}\right)^\beta\right)} \right]; \\ \min \left[4e^{\left(-2\left(\frac{t}{2+0.5\alpha}\right)^\beta\right)} - 4e^{\left(-3\left(\frac{t}{2+0.5\alpha}\right)^\beta\right)} + e^{\left(-4\left(\frac{t}{2+0.5\alpha}\right)^\beta\right)}, \right. \\ \left. 4e^{\left(-2\left(\frac{t}{1.75+0.25(1-\alpha)}\right)^\beta\right)} - 4e^{\left(-3\left(\frac{t}{1.75+0.25(1-\alpha)}\right)^\beta\right)} + e^{\left(-4\left(\frac{t}{1.75+0.25(1-\alpha)}\right)^\beta\right)} \right].$$

Hence, Table 5 and Table 6 show the results of fuzzy reliability based on dual hesitant fuzzy set from Eqs. (1)-(6) of considered system are as:

Table 5. Reliability of the linear system.

α -cut	Max (α , β)	Min (α , β)
0	(.0052, .0236)	(.0318, .0318)
0.1	(.0061, .0221)	(.0336, .0310)
0.2	(.0070, .0206)	(.0353, .0301)
0.3	(.0079, .0191)	(.0371, .0293)
0.4	(.0090, .0177)	(.0389, .0285)
0.5	(.0101, .0163)	(.0407, .0276)
0.6	(.0112, .0149)	(.0425, .0268)
0.7	(.0124, .0137)	(.0444, .0260)
0.8	(.0137, .0124)	(.0462, .0252)
0.9	(.0149, .0112)	(.0481, .0244)
1	(.0163, .0101)	(.0500, .0236)

Table 6. Reliability of the circular system.

α -cut	Max (α , β)	Min (α , β)
0	(.0069, .0306)	(.0409, .0409)
0.1	(.0080, .0286)	(.0431, .0399)
0.2	(.0092, .0262)	(.0453, .0388)
0.3	(.0104, .0248)	(.0475, .0377)
0.4	(.0118, .0230)	(.0498, .0367)
0.5	(.0132, .0212)	(.0521, .0356)
0.6	(.0146, .0195)	(.0543, .0346)
0.7	(.0162, .0178)	(.0567, .0336)
0.8	(.0178, .0162)	(.0590, .0326)
0.9	(.0195, .0146)	(.0613, .0315)
1	(.0212, .0132)	(.0637, .0306)

7. Conclusion

Fuzzy reliability is an essential part of engineering and real-life systems based on uncertainty, there is a different kind of uncertainty of fuzziness for problem formulation. The hesitant and dual hesitant fuzzy approaches to determine the fuzzy reliability of linear and circular consecutive 2-out-of-4: G system using aggregation operators, Weibull distribution and reliability function have been discussed. Table 1 and Table 2 show the hesitant fuzzy membership function in figure 1 to figure 6. Table 3 and Table 4 illustrate the hesitant fuzzy after using aggregation operator with the same weight is given $W_1=W_2=W_3=1/3$ in figure 7 and figure 8. In the case of dual hesitant fuzzy, Table 5 and Table 6 demonstrate the membership and non-membership function in the form of maximum and minimum α and β cut sets.

Conflict of Interest

The authors confirm that there is no conflict of interest to declare for this publication.

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