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Reliability Equivalence Factors for Coherent System using Survival Signature

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Abstract
This study presents a methodology aimed at enhancing the performance of coherent systems through the application of survival signature analysis, focusing on the calculation of reliability equivalence factors (REFs). In the context of system improvement, the selection of reliability improvement strategies, such as reduction and duplication, depends on various factors like space limitations, costs, and other constraints. The importance of REF lies in their ability to quantify the extent of reliability improvement, providing a clear metric for decision-makers to assess the cost-effectiveness of various enhancement strategies. The analysis focuses on two distinct types of REFs, namely, mean reliability equivalence factors (MREFs) and survival reliability equivalence factors (SREFs), targeted at reliability enhancement via strategies including component failure rate reduction and the implementation of warm standby duplication. Both perfect and imperfect switching scenarios in warm duplication are examined, with survival signature analysis applied to determine the system's survival function and mean time to failure (MTTF). The methodology's effectiveness is illustrated through a case study of a six-unit bridge system, where the components are modeled using exponential and Weibull distributions. REFs are evaluated for sequential upgrades in either individual components or entire component types. The study also conducts a comparative analysis between the reliability and MTTF of the original and improved systems across different improvement techniques.

Keywords- Reliability analysis, Coherent systems, Survival signature, Reliability equivalence factor, Reduction method, Warm standby duplication method.

1. Introduction

In coherent systems, it is crucial to ensure that each component performs optimally, as failures can have a significant impact on the entire system. Evaluating these systems is vital to determine their potential for reliable operation over time. In the analysis of coherent systems, various configurations, and redundancy strategies are considered to enhance reliability. Such strategies often involve duplicating components to minimize the impact of failures. There can be three kinds of such strategies within a system. These are hot standby, warm standby and cold standby. In the first type of the strategy, a backup unit is kept ready close-by and it starts working instantly in case of a failure in the primary unit. However, in cold standby, the backup unit is kept offline and is activated if the primary unit fails. The warm standby serves as a compromise between the two, with the backup unit running at a lower capacity. In critical operations, redundancy in the systems provides more reliability and fault tolerance and ensures uninterrupted operation but it also introduces some problems like increase in costs and complexity and underutilization of potential resources. The efficiency of conventional duplication methods is lowered due to limitations pertaining to the cost and the space in designing systems such as high-tech medical equipment and mobile phone batteries. Undoubtedly, these strategies are capable to upgrade the reliability of the systems but may not be financially justified if the components of the system are very expensive and space is limited. The Redundancy Allocation Problem is an important issue in system reliability and requires the best allocation of the redundancy within a system to boost its performance along with certain constraints. Researchers are exploring different ways to improve the reliability of systems. The reduction approach is one such strategy that focuses on lowering the failure rates of specific components by the application of scaling factor ρ, which ranges between 0 and 1. For high-tech equipment, the reduction method may be economically justified. This approach minimizes the failure rates and offers a cost-efficient and space-saving means to improve the system reliability.

There are different types of reliability enhancement strategies and among them, the REFs have emerged as a critical parameter. This parameter quantifies the improvement in reliability achieved through specific design modifications or strategies, bringing one design's reliability level equivalent to another. Initially introduced by Råde (1993a), the concept of REFs has since been extensively researched. Råde (1993b) evaluated REFs for a single component and systems with two components in series and parallel, using the survival function as a performance metric. Sarhan (2000) utilized the survival function to calculate REFs for systems consisting of *n* independent and non-identical components. Sarhan (2002) extended the concept of REFs to radar systems in aircraft, deriving the MTTF of the systems after improvement. Furthermore, the author formulated a theorem for the systems under consideration, in the context of various methods used to enhance system reliability. Sarhan (2009) examined the equivalence of different designs of general series-parallel systems by considering reliability and MTTF characteristics. In all the above-mentioned studies, failure rates are assumed to follow an exponential distribution. Xia and Zhang (2007) extended the concept of REFs to include the gamma distribution, focusing on parallel systems and thereby broadening its applicability. Pogany et al. (2013) and Pogany et al. (2014) expanded the scope of these studies to composite systems with components following a gamma-Weibull distribution, also integrating both hot and cold duplication strategies. El-Damcese (2009) studied REFs for series-parallel systems, where the failure rates of components are time-dependent and follow the Weibull distribution. The author also evaluated the fractiles of both the original and the improved system. Further, Migdadi and Al-Batah (2014) examined the same system configuration, where elements follow the Burr-type X distribution, and investigated the shape parameter. Alghamdi and Percy (2017) also expanded the REF concept to these systems, considering exponentiated Weibull distributed components. Migdadi et al. (2019) studied the reliability and MTTF of the general series-parallel system by considering reduction strategies, as well as hot, cold, and hybrid duplication strategies with general exponential distributed components. Recently, Mustafa et al. (2023) improved the performance of the series-parallel system by assuming independently and identically distributed (*i.i.d.*) Lindley components with three parameters. In their study, they calculated two REFs and determined fractiles, finding that, in a specific numerical example, the cold duplication technique provided the best results. El-Faheem et al. (2022) obtained REFs for a radar system consisting of three units, having Rayleigh distribution with two parameters. A novel approach was used by Etminan et al. (2023) to compute a closed form of ρ in the system where the reduction method is applied to a single component. They considered REFs as the measures of component importance. The incessant development pertaining to the research work on REFs with particular reference to accommodate several component distributions and system configurations, emphasizes their importance in increasing the reliability of the engineering systems.

Some instrumental approaches that provide detailed perception of the reliability of modern systems are stochastic modeling, use of the universal generating function, signature analysis, and survival signature methods. Stochastic modeling (Dhillon and Yang, 1992; El-Sherbeny, 2013; Chopra and Ram, 2017). employs probabilistic analysis for assessing the reliability of systems emphasizing randomness and uncertainty. Contrary to this, Ushakov (1986), Levitin (2005), Triantafyllou (2021b) and Ram et al. (2023) used the universal generating functions which provide algebraic representations of performance distributions. Several researchers (Samaniego, 1985; Boland, 2001; Navarro et al., 2010; Kumar and Singh, 2019; Triantafyllou, 2021a) used the signature analysis. Coolen and Coolen-Maturi (2012) and Coolen-Maturi et al. (2021) used the survival signature methods which mainly focus on how component configurations affect system survivability, emphasizing the structural dependencies. Coolen and Coolen-Maturi (2012) introduced the survival signature approach as a pivatol analytical tool to determine the reliability characteristics of the system and it is relevant to this study. Aslett (2012) developed the ReliabilityTheory R package to facilitate the calculation of survival signature. The evidences of a wide range of applications of the survival signature reflect its versatility and practical feasibility. Samaniego and Navarro (2016) and Chopra and Kumar (2022) demonstrated its successful application for stochastic order comparisons. Moreover, it is indispensable to examine the significance of different components in the coherent system, thus assisting in specifying the pivotal components which influence the system's reliability, significantly. It is also applicable in the sensitivity analysis as marked by the research workers (Feng et al., 2016; Miro et al., 2019). Chatwattanasiri et al. (2016) considered the uncertain stress-based reliability of components while optimizing the system redundancy. The survival signature approach was used by Wang et al. (2023) to assess the reliability of standby redundant systems. To determine the reliability and crucial components of a piston making system for maintenance planning, Ge and Zhang (2019) employed the survival signature approaches with a multi-level strategy. A load-sharing-based reliability model embodied with the survival signature was proposed by Li et al. (2020) to determine the system reliability and explore the reliability sensitivity of redundant components. Ling et al. (2023) analyzed system reliability depending upon the survival signature to assist decision-making and constructed a solution to the Redundancy Allocation Problem. Alghamdi (2022) recently employed a novel and unique approach to determine the REFs for complex systems. This research work utilized hot as well as cold duplication strategies and studied REFs by applying the survival signature method and thus demonstrated its potential for enhancing the evaluation of REFs and system reliability.

The aforementioned studies have demonstrated systems improvement and the calculation of REFs in various systems, including their application in real-world scenarios such as radar systems. However, there are relatively few studies that utilize the Survival Signature method. The survival signature concept, that relies solely on the system structure and is independent of the lifetime distribution of components, may offer distinct advantages in evaluating REFs for complex systems comprising multiple types of components. To our knowledge, no study has yet explored REFs using the warm duplication method in conjunction with the survival signature. Addressing this research gap, our study employs the survival signature approach to compute MREFs and SREFs. This method considers warm standby redundancy, incorporating both perfect and imperfect switching, to enhance system reliability. We have applied the theoretical results to a bridge system comprising six components, where some components follow an exponential distribution while others adhere to a Weibull distribution. This system, mirroring critical structures in communication networks, electrical grids, transportation networks, and control systems, exemplifies the practical significance of our study in real-world engineered systems. Following the introduction, section 2 discusses survival signature-based survival function and MTTF concepts. Section 3 outlines the methodology, describing the survival functions of the enhanced system utilizing the reduction and warm duplication approach. In section 4, we define REFs based on system reliability and MTTF. Section 5 provides a numerical example for empirical validation. Lastly, section 6 discusses the conclusions derived from the findings and suggests potential directions for future research.

2. Definitions and Notations

In a system composed of m *i.i.d.* components, the survival signature, represented by $\Phi(l)$, quantifies the probability of the system remaining operational, under the condition that exactly l components are functioning properly. Consider x_i as the operational status of the i^{th} (for $i = 1, 2 \dots, m$) component at time t, where $x_i = 0$ denotes the non-functioning state and $x_i = 1$ indicates the component is operational. There exists $\binom{m}{l}$ distinct state vectors $x = (x_1, x_2, \dots, x_i, \dots, x_m)$, where exactly l components are operational, satisfying the condition $\sum_{i=1}^{m} x_i = l$. The collection of these state vectors is represented by s_l. The survival signature for a system with *i.i.d.* elements, as defined by Coolen and Coolen-Maturi (2012), is mathematically represented as follows:

$$\Phi(l) = \frac{\sum_{x \epsilon s_l} \phi(x)}{|s_l|} = \binom{m}{l}^{-1} \sum_{x \epsilon s_l} \phi(x) \tag{1}$$

where, $\phi(x)$ is the structure function, which assumes the value 1 when the system is working and 0 otherwise. The system survival function $R(t)$, based on the survival signature, can be written as,

$$R(t) = \sum_{l=0}^{m} \Phi(l) \binom{m}{l} \left[1 - \overline{F(t)}\right]^{m-l} \left[\overline{F(t)}\right]^{l} \tag{2}$$

where, $\overline{F(t)}$ represents the survival function of the *i.i.d.* components.

MTTF, the average time a coherent system can function before failing, is a critical indicator of system reliability. This information is useful for estimating the lifetime cost of a system and for making informed decisions concerning maintenance and repair. For the coherent system, the expression for MTTF is given by,

$$MTTF = \int_0^{\infty} R(t)\ dt \tag{3}$$

Consider a coherent system comprising m independent components categorized into n distinct types. For each type i ($1 \le i \le n$), there are m_i components, resulting in a total of m components across all types. Each type's components are assumed to have an identical distribution of failure times. In this context, system's state can be represented by a vector x, defined as $x = (x^1, x^2, \dots, x^n)$. The vector $x^i = \left(x_1^i, x_2^i, \dots, x_{m_i}^i\right)$ specifically describes the states of the m_i components pertaining to type i. For such systems, the survival signature is denoted as $\Phi(l_1, l_2, \dots, l_n)$, which quantifies the probability of the system being operational under the condition that exactly l_i components of type i are functional, where $l_i \in \{0, 1, \dots, m_i\}$ for each i. Furthermore, for every type i, there exist $\binom{m_i}{l_i}$ distinct state vectors x^i, in which precisely l_i components are operational ($x_s^i = 1$), satisfying the condition $\sum_{s=1}^{m_i} x_s^i = l_i$. The collection of such state vectors for all types is represented by the set $s_{l_1, l_2, \dots, l_n}$. The survival signature $\Phi(l_1, l_2, \dots, l_n)$ is stated as,

$$\Phi(l_1, l_2, \dots, l_n) = \left[\prod_{i=1}^{n} \binom{m_i}{l_i}^{-1}\right] \sum_{x \epsilon s_{l_1, l_2, \dots, l_n}} \phi(x) \tag{4}$$

In this framework, the system survival function $R(t)$, in terms of the survival signature, is expressed as,

$$R(t) = \sum_{l_1=0}^{m_1} \dots \sum_{l_n=0}^{m_n} \left[\Phi(l_1, l_2, \dots, l_n) \prod_{i=1}^{n} \binom{m_i}{l_i} \left[1 - \overline{F_i(t)}\right]^{m_i - l_i} \left[\overline{F_i(t)}\right]^{l_i}\right] \tag{5}$$

where, $\overline{F_i(t)}$ represents the survival function of the components of the type i.

3. Structure of Improved System

Enhancing the reliability of a coherent system is possible through lowering the malfunction probabilities of its components or by incorporating parallel components. The framework employs a reduction and duplication method, with hot, cold, and warm duplication variants along with perfect and imperfect switches.

3.1 Reduction Method

The reduction method focuses on adjusting the hazard rate of selected components using a scaling factor ρ within the range $(0,1)$. Employing this reduction approach can enhance system performance in a manner similar to a duplication strategy, but without increasing the system's size, thereby rendering it more efficient and cost-effective. Moreover, this method simplifies system maintenance and makes it easier to manage. Assume a coherent system consisting of m units, categorized into n distinct types, each with m_i components. In an effort to enhance the performance of some k_i components within the system, where $k_i \in \{1, \dots, m_i - 1\}$, through the reduction strategy, the count of component types in the system increases to $(n+1)$. This modification retains the intrinsic characteristics while introducing a new category with k_i components, effectively replacing an equivalent number of type i components. Assuming the lifetime distribution $F_{n+1}(t)$ for these k_i components, and employing Equation (5), the survival function for the improved system, denoted as $R_{k_i}^R(t)$, is formulated as follows:

$$R_{k_i}^R(t) = \sum_{l_1=0}^{m_1} \sum_{l_2=0}^{m_2} \dots \sum_{l_i=0}^{m_i-k_i} \sum_{l_{i+1}=0}^{m_{i+1}} \dots \sum_{l_n=0}^{m_n} \sum_{l_{n+1}=0}^{k_i} \Bigg[\Phi(l_1, l_2, \dots, l_n, l_{n+1}) \left(\prod_{s=1,\ s\neq i}^{n} \binom{m_s}{l_s} \left[1 - \overline{F_s(t)}\right]^{m_s-l_s} \left[\overline{F_s(t)}\right]^{l_s} \right) \binom{m_i - k_i}{l_i} \left[1 - \overline{F_i(t)}\right]^{m_i-k_i-l_i} \left[\overline{F_i(t)}\right]^{l_i} \binom{k_i}{l_{n+1}} \left[1 - \overline{F_{n+1}(t)}\right]^{k_i-l_{n+1}} \left[\overline{F_{n+1}(t)}\right]^{l_{n+1}} \Bigg] \tag{6}$$

The survival function for the enhanced k_i components can be calculated by multiplying the hazard rate of the targeted components with ρ. This results in an updated system survival function, achieved by substituting the component's survival function $\overline{F_{n+1}(t)}$ with the survival function $\left(\overline{F_i(t)}\right)^{\rho}$. The revised expression is as follows:

$$R_{k_i}^R(t) = \sum_{l_1=0}^{m_1} \sum_{l_2=0}^{m_2} \dots \sum_{l_i=0}^{m_i-k_i} \sum_{l_{i+1}=0}^{m_{i+1}} \dots \sum_{l_n=0}^{m_n} \sum_{l_{n+1}=0}^{k_i} \Bigg[\Phi(l_1, l_2, \dots, l_n, l_{n+1}) \left(\prod_{s=1,\ s\neq i}^{n} \binom{m_s}{l_s} \left[1 - \overline{F_s(t)}\right]^{m_s-l_s} \left[\overline{F_s(t)}\right]^{l_s} \right) \binom{m_i - k_i}{l_i} \left[1 - \overline{F_i(t)}\right]^{m_i-k_i-l_i} \left[\overline{F_i(t)}\right]^{l_i} \binom{k_i}{l_{n+1}} \left[1 - \left(\overline{F_i(t)}\right)^{\rho}\right]^{k_i-l_{n+1}} \left[\left(\overline{F_i(t)}\right)^{\rho}\right]^{l_{n+1}} \Bigg] \tag{7}$$

The MTTF of the system, enhanced through the reduction method, can be calculated using Equation (7), as detailed below:

$$MTTF_{k_i}^R = \int_0^{\infty} \sum_{l_1=0}^{m_1} \sum_{l_2=0}^{m_2} \dots \sum_{l_i=0}^{m_i-k_i} \sum_{l_{i+1}=0}^{m_{i+1}} \dots \sum_{l_n=0}^{m_n} \sum_{l_{n+1}=0}^{k_i} \Bigg[\Phi(l_1, l_2, \dots, l_n, l_{n+1}) \left(\prod_{s=1,\ s\neq i}^{n} \binom{m_s}{l_s} \left[1 - \overline{F_s(t)}\right]^{m_s-l_s} \left[\overline{F_s(t)}\right]^{l_s} \right) \binom{m_i - k_i}{l_i} \left[1 - \overline{F_i(t)}\right]^{m_i-k_i-l_i} \left[\overline{F_i(t)}\right]^{l_i} \binom{k_i}{l_{n+1}} \left[1 - \left(\overline{F_i(t)}\right)^{\rho}\right]^{k_i-l_{n+1}} \left[\left(\overline{F_i(t)}\right)^{\rho}\right]^{l_{n+1}} \Bigg] dt \tag{8}$$

When the reduction strategy is applied to enhance all components of type *i* in the system, the only change in the modified system, which otherwise maintains all original characteristics, is the adoption of a new survival function of type *i* components, shifting from $\overline{F_i(t)}$ to $\left(\overline{F_i(t)}\right)^{\rho}$, thus indicating their improvement. Utilizing Equation (5), the survival function of the improved system is expressed as,

$$R_i^R(t) = \sum_{l_1=0}^{m_1}\sum_{l_2=0}^{m_2}\dots\sum_{l_{i-1}=0}^{m_{i-1}}\sum_{l_i=0}^{m_i}\dots\sum_{l_{n-1}=0}^{m_{n-1}}\sum_{l_n=0}^{m_n}\left[\Phi(l_1, l_2, \dots, l_n)\left(\prod_{s=1,\ s\neq i}^{n}\binom{m_s}{l_s}\ \left[1-\overline{F_s(t)}\right]^{m_s-l_s}\left[\overline{F_s(t)}\right]^{l_s}\right)\binom{m_i}{l_i}\left[1-\left(\overline{F_i(t)}\right)^{\rho}\right]^{m_i-l_i}\left[\left(\overline{F_i(t)}\right)^{\rho}\right]^{l_i}\right] \quad (9)$$

If multiple types of system components are to be improved, Equation (9) is modified by altering the survival function for each type of component enhancement. Moreover, Equation (9) can also be used to estimate the MTTF by taking into consideration the attributes of the improved system, as shown in the following equation:

$$MTTF_i^R = \int_0^{\infty}\sum_{l_1=0}^{m_1}\sum_{l_2=0}^{m_2}\dots\sum_{l_{i-1}=0}^{m_{i-1}}\sum_{l_i=0}^{m_i}\dots\sum_{l_{n-1}=0}^{m_{n-1}}\sum_{l_n=0}^{m_n}\left[\Phi(l_1, l_2, \dots, l_n)\left(\prod_{s=1,\ s\neq i}^{n}\binom{m_s}{l_s}\ \left[1-\overline{F_s(t)}\right]^{m_s-l_s}\left[\overline{F_s(t)}\right]^{l_s}\right)\binom{m_i}{l_i}\left[1-\left(\overline{F_i(t)}\right)^{\rho}\right]^{m_i-l_i}\left[\left(\overline{F_i(t)}\right)^{\rho}\right]^{l_i}\right] dt \quad (10)$$

In the situation where the system has *i.i.d.* components and all components are enhanced by weakening the hazard rate by ρ, the reliability function for the enhanced system, can be

$$R^R(t) = \sum_{l=0}^{n}\Phi(l)\binom{n}{l}\left[1-\left(\overline{F(t)}\right)^{\rho}\right]^{n-l}\ \left[\overline{F(t)}\right]^{\rho l} \quad (11)$$

where the survival signature $\Phi(l)$ remains the same as for the standard system. However, if a certain number of components k, with $k < m$, are improved, the system will then consist of two types of components. By utilizing Equation (5), the survival function of the improved system can be calculated as follows:

$$R_{k,m}^R(t) = \sum_{l_1=0}^{m-k}\sum_{l_2=0}^{k}\Phi(l_1, l_2)\binom{m-k}{l_1}\left[1-\overline{F(t)}\right]^{m-k-l_1}\left[\overline{F(t)}\right]^{l_1}\binom{k}{l_2}\left[1-\left(\overline{F(t)}\right)^{\rho}\right]^{k-l_2}\ \left[\overline{F(t)}\right]^{\rho l_2} \quad (12)$$

3.2 Duplication Method

The duplication method offers a range of strategies for enhancing system reliability, serving as an alternative approach to system improvement. These strategies include hot duplication, cold duplication, and warm duplication, each with its unique advantages and considerations. In our study, we have focused on warm duplication strategies, which include both perfect and imperfect switches. Warm duplication strategies enable the system to maintain operations in the event of a component failure, and the use of imperfect switches can contribute to cost reduction.

3.2.1 Warm Duplication with Perfect Switch

Considering the same coherent system, which is composed of m components categorized into n distinct types, we now examine the implementation of standby components. This method involves connecting a standby component directly to an active component, ensuring the immediate activation of the standby component upon the failure of the active one. There is a possibility for both the active and standby components to fail simultaneously. It is assumed that, despite their independent failure rates, the standby components have a lower failure rate than the active components. Each type i $(i = 1, 2, \dots, n)$ includes m_i components, denoted as $j = 1, 2, \dots, m_i$. These components are characterized by known survival function

$\overline{F_i(t)}$ and probability density function $f_i(x)$. Let $R_{i'}(x)$ and $R_{i'd}(x)$ represent the survival functions of the standby component in active mode and standby mode, respectively, assuming independence. In the context of warm duplication with a perfect switch, this enhancement aligns with a partially renewed process. Subsequently, the survival function for the component subjected to warm duplication with a perfect switch denoted as $\overline{F_{ji}{}^{p}(t)}$, is expressed as follows (Kuo and Zhu, 2012):

$$\overline{F_{ji}{}^{p}(t)} = \overline{F_i(t)} + \int_0^t f_i(x)\, R_{i'd}(x) R_{i'}(t-x)\, dx\ ;\ 0 \le x \le t \tag{13}$$

Enhancing such k_i components, where $k_i \in \{1, \dots, m_i - 1\}$, through warm duplication with a perfect switch, and employing the survival signature approach, leads to the following expression for the reliability function of the improved system:

$$R_{k_i}^{W_p}(t) = \sum_{l_1=0}^{m_1} \sum_{l_2=0}^{m_2} \dots \sum_{l_i=0}^{m_i-k_i} \sum_{l_{i+1}=0}^{m_{i+1}} \dots \sum_{l_n=0}^{m_n} \sum_{l_{n+1}=0}^{k_i} \Bigg[\Phi(l_1, l_2, .., l_n, l_{n+1}) \left(\prod_{s=1,\ s\neq i}^{n} \binom{m_s}{l_s} \left[1 - \overline{F_s(t)}\right]^{m_s - l_s} \left[\overline{F_s(t)}\right]^{l_s} \right) \binom{m_i - k_i}{l_i} \left[1 - \overline{F_i(t)}\right]^{m_i - k_i - l_i} \left[\overline{F_i(t)}\right]^{l_i} \binom{k_i}{l_{n+1}} \left[1 - \left(\overline{F_i(t)} + \int_0^t f_i(x)\, R_{i'd}(x) R_{i'}(t-x)\, dx\right)\right]^{k_i - l_{n+1}} \left[\overline{F_i(t)} + \int_0^t f_i(x)\, R_{i'd}(x) R_{i'}(t-x)\, dx\right]^{l_{n+1}} \Bigg] \tag{14}$$

For optimal performance, we can apply this technique to every component of a specific type $i \in \{1, 2, \dots, n\}$. Implementing Equations (5) and (13), the enhanced survival function, denoted as $\overline{F_{ji}{}^{p}(t)}$, methodically replace the original survival function $\overline{F_i(t)}$ for each component of type i. As a result, the survival function of the system, reflecting these comprehensive upgrades, is represented by the following equation:

$$R_{i}^{W_p}(t) = \sum_{l_1=0}^{m_1} \sum_{l_2=0}^{m_2} \dots \sum_{l_{i-1}=0}^{m_i-1} \sum_{l_i=0}^{m_i} \dots \sum_{l_{n-1}=0}^{m_{n-1}} \sum_{l_n=0}^{m_n} \Bigg[\Phi(l_1, l_2, .., l_n) \left(\prod_{s=1,\ s\neq i}^{n} \binom{m_s}{l_s} \left[1 - \overline{F_s(t)}\right]^{m_s - l_s} \left[\overline{F_s(t)}\right]^{l_s} \right) \binom{m_i}{l_i} \left[1 - \left(\overline{F_i(t)} + \int_0^t f_i(x)\, R_{i'd}(x) R_{i'}(t-x)\, dx\right)\right]^{m_i - l_i} \left[\left(\overline{F_i(t)} + \int_0^t f_i(x)\, R_{i'd}(x) R_{i'}(t-x)\, dx\right)\right]^{l_i} \Bigg] \tag{15}$$

The MTTF for the specifically improved system can be further evaluated using Equations (14) and (15). Equation (15) is iteratively applied for each enhancement when different types of components are to be improved. This iterative process ensures complete coverage when addressing enhancements across different component types. In cases where all components are *i.i.d.*, we consider their survival function as $\overline{F(t)}$ and their probability density function as $f(t)$. Under Case (I) where all components undergo improvement through the warm duplication approach with a perfect switch, it is assumed that each standby component has a reliability function $R_a(t)$ in active mode and $R_d(t)$ in standby mode. Assuming $R_a(t)$ is independent of $R_d(t)$, the improved system survival function, using Equations (2) and (13), can be written as,

$$R^{W_p}(t) = \sum_{l=0}^{m} \Phi(l) \binom{m}{l} \left[1 - \left(\overline{F(t)} + \int_0^t f(x) R_d(x)\, R_a(t-x)\, dx\right)\right]^{m-l} \left[\overline{F(t)} + \int_0^t f(x) R_d(x)\, R_a(t-x)\, dx\right]^{l} \tag{16}$$

If, under Case (II), some components are improved, assuming k out of m components ($k \neq m$), then the reliability of the improved system, which is enhanced through warm duplication with a perfect switch, can be expressed as follows:

$$R_{k,m}^{W_p}(t) = \sum_{l_1=0}^{m-k} \sum_{l_2=0}^{k} \Phi(l_1, l_2) \binom{m-k}{l_1} \left[1 - \overline{F(t)}\right]^{m-k-l_1} \left[\overline{F(t)}\right]^{l_1} \binom{k}{l_2} \left[1 - \left(\overline{F(t)} + \right.\right.$$

$$\int_0^t f(x)R_d(x)\,R_a(t-x)\,dx\Big)\Big]^{k-l_2}\ \Big[\Big(\overline{F(t)}+\int_0^t f(x)R_d(x)\,R_a(t-x)\,dx\Big)\Big]^{l_2} \tag{17}$$

3.2.2 Warm Duplication with Imperfect Switch

In warm standby duplication, the switch facilitating the transition between backup and primary components may be imperfect, introducing a likelihood of switch failure. Imperfection in the switch mechanism, due to manufacturing defects, environmental conditions, or wear over time, can impact the effectiveness of the standby system. Considering, *m*-order coherent system, with *n* types of components, the survival function for the j^{th} $(j = 1, 2, \ldots, m_i)$ component of i $(i = 1, 2, \ldots, n)$ type, optimized through warm duplication with an imperfect switch, is expressed as

$$\overline{F_{ji}}^{\bar{p}}(t) = \overline{F_i(t)} + \int_0^t f_i(x)\,R_{i'd}(x)\,R_{sw}(x)\,R_{i'}(t-x)\,dx\,;\ 0 \le x \le t \tag{18}$$

where, $R_{sw}(x)$, $R_{i'}(x)$, $R_{i'd}(x)$ represent the reliability functions for the switching mechanism, the standby component in active mode, and standby mode, respectively. Consequently, based on the concept of the survival signature and considering the scenario of an imperfect switch, Equations (14)-(17) are transformed into the following set of equations:

$$R_{k_i}^{W_{\bar{p}}}(t) = \sum_{l_1=0}^{m_1}\sum_{l_2=0}^{m_2}\ldots\sum_{l_i=0}^{m_i-k_i}\sum_{l_{i+1}=0}^{m_{i+1}}\ldots\sum_{l_n=0}^{m_n}\sum_{l_{n+1}=0}^{k_i}\Bigg[\Phi(l_1,l_2,..,l_n,l_{n+1})\Big(\prod_{s=1,\ s\neq i}^{n}\binom{m_s}{l_s}\ \big[1-\overline{F_s(t)}\big]^{m_s-l_s}\big[\overline{F_s(t)}\big]^{l_s}\Big)\binom{m_i-k_i}{l_i}\big[1-\overline{F_i(t)}\big]^{m_i-k_i-l_i}\big[\overline{F_i(t)}\big]^{l_i}\binom{k_i}{l_{n+1}}\Big[1-\Big(\overline{F_i(t)}+\int_0^t f_i(x)\,R_{i'd}(x)R_{sw}(x)R_{i'}(t-x)\,dx\Big)\Big]^{k_i-l_{n+1}}\Big[\overline{F_i(t)}+\int_0^t f_i(x)\,R_{i'd}(x)R_{sw}(x)R_{i'}(t-x)\,dx\Big]^{l_{n+1}}\Bigg] \tag{19}$$

$$R_{i}^{W_{\bar{p}}}(t) = \sum_{l_1=0}^{m_1}\sum_{l_2=0}^{m_2}\ldots\sum_{l_{i-1}=0}^{m_i-1}\sum_{l_i=0}^{m_i}\ldots\sum_{l_{n-1}=0}^{m_{n-1}}\sum_{l_n=0}^{m_n}\Bigg[\Phi(l_1,l_2,..,l_n)\Big(\prod_{s=1,\ s\neq i}^{n}\binom{m_s}{l_s}\ \big[1-\overline{F_s(t)}\big]^{m_s-l_s}\big[\overline{F_s(t)}\big]^{l_s}\Big)\binom{m_i}{l_i}\Big[1-\Big(\overline{F_i(t)}+\int_0^t f_i(x)\,R_{i'd}(x)R_{sw}(x)\,R_{i'}(t-x)\,dx\Big)\Big]^{m_i-l_i}\Big[\Big(\overline{F_i(t)}+\int_0^t f_i(x)\,R_{i'd}(x)R_{sw}(x)\,R_{i'}(t-x)\,dx\Big)\Big]^{l_i}\Bigg] \tag{20}$$

$$R^{W_{\bar{p}}}(t) = \sum_{l=0}^{m}\Phi(l)\binom{m}{l}\Big[1-\Big(\overline{F(t)}+\int_0^t f(x)R_d(x)R_{sw}(x)\,R_a(t-x)\,dx\Big)\Big]^{m-l}\Big[\overline{F(t)}+\int_0^t f(x)R_d(x)R_{sw}(x)\,R_a(t-x)\,dx\Big]^{l} \tag{21}$$

and

$$R_{k,m}^{W_{\bar{p}}}(t) = \sum_{l_1=0}^{m-k}\sum_{l_2=0}^{k}\Phi(l_1,l_2)\binom{m-k}{l_1}\big[1-\overline{F(t)}\big]^{m-k-l_1}\big[\,\overline{F(t)}\,\big]^{l_1}\binom{k}{l_2}\Big[1-\Big(\overline{F(t)}+\int_0^t f(x)R_d(x)\,R_{sw}(x)\,R_a(t-x)\,dx\Big)\Big]^{k-l_2}\ \Big[\Big(\overline{F(t)}+\int_0^t f(x)\,R_d(x)\,R_{sw}(x)\,R_a(t-x)\,dx\Big)\Big]^{\rho l_2} \tag{22}$$

where, the functions $R_{k_i}^{W_{\bar{p}}}(t)$, $R_{i}^{W_{\bar{p}}}(t)$, $R^{W_{\bar{p}}}(t)$, $R_{k,m}^{W_{\bar{p}}}(t)$ represent the reliability functions of the improved system when certain *k* components of type *i*, all components of type *i*, all *m i.i.d.* components, and *k* components out of *m i.i.d.* components are improved, respectively. These Equations (19)-(22) can also be used to calculate the MTTF of the improved system.

4. Reliability Equivalence Factor

The concept of REF is instrumental in evaluating and comparing different system designs. Acting as a scaling parameter, its application to specific component characteristics within a system design brings it in line with a benchmark superior design. REFs enable comparison across various system designs, especially regarding crucial reliability aspects such as the survival function and MTTF. In this study, we focus on computing two distinct types of REFs. The first type pertains to SREFs, which are derived from the survival functions of the improved systems. The second type, MREFs, are calculated based on the MTTF of these systems.

4.1 Survival Reliability Equivalence Factor
We determined SREFs for coherent system by solving the given equations,

$$R^R(t) = R^W(t) = \omega \tag{23}$$

where, $R^R(t)$ and $R^W(t)$ respectively refer to the reliability of the improved system due to the reduction method and the warm duplication method, and ω indicates the targeted reliability for the enhanced system. These equations serve as a fundamental tool for comparing the survival functions of systems that have been improved through various design strategies, and for deriving the SREFs. This quantifies the reliability improvement achieved by a particular strategy in relation to another. In the evaluation of the SREFs, the survival signature-based reliability functions presented in section 3 are utilized.

4.2 Mean Reliability Equivalence Factor
For the evaluation of the MREF, the following equations are solved,

$$MTTF^R = MTTF^W \tag{24}$$

where, $MTTF^R$ and $MTTF^W$ represent MTTF for a system optimized through the reduction method and warm standby redundancy, respectively. These equations enable the quantification of how the MTTF of a system improved by the reduction strategy compares to a system improved through duplication strategies. The MREF provides data on the relative improvement in reliability achieved through different design approaches. Again, in calculating the MREFs, the system's MTTF is computed by employing the concept of the survival signature.

5. Numerical Example
The theory discussed in previous sections is applied to a bridge system, as illustrated in Figure 1, which comprises six components categorized into two types.

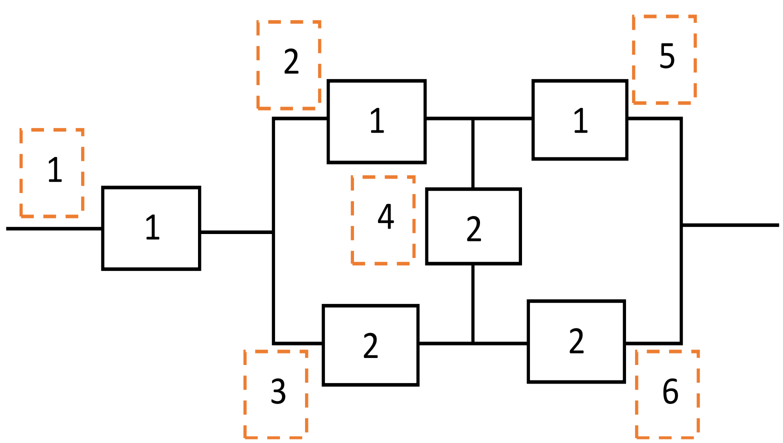

Figure 1. The bridge system with two types of components.

The characteristics of these components are outlined in Table 1, which specifies that components 1, 2, and 5 are exponentially distributed, while components 3, 4, and 6 are governed by a Weibull distribution. The system's reliability function is assessed using the survival signature provided in Table 2. This table presents data in three rows: l_1 and l_2, which denote the number of functioning components of the two types within the system, and $\Phi(l_1, l_2)$ which indicates the probability of the system working for each corresponding combination of l_1 and l_2. Notably, Table 2 includes only combinations with non-zero survival signature values, thus emphasizing the analysis on scenarios in which the system retains some operational capacity.

Table 1. Description of the system.

Component type	Component lifetime distribution	Component reliability in the reduction method
Type 1	Exponential ($\lambda = 0.5$)	$e^{-\rho\lambda t}$
Type 2	Weibull ($\alpha = 0.25, \beta = 2.0$)	$e^{-\rho\alpha t^{\beta}}$

Table 2. Survival signature of the original system.

$\boldsymbol{l_1}$	1	1	2	2	3	3	3	3
$\boldsymbol{l_2}$	2	3	2	3	0	1	2	3
$\boldsymbol{\Phi(l_1, l_2)}$	1/9	1/3	4/9	2/3	1	1	1	1

Building on the assessment of the system's reliability function through the survival signature, we employ Equations (7), (14), (19), and (23) to compute the SREFs for each component enhancement under various scenarios with ω values of 0.2, 0.6, and 0.9. In the case of an imperfect switch, we assume a constant failure rate of μ = 0.2. The data in Table 3 demonstrate that reducing the hazard rate of component 1, indicated by $\rho = 0.4690$, is equivalent to introducing an additional component in parallel to component 1, implementing warm redundancy with a perfect switch, and achieving a system reliability of ω = 0.2. Similarly, Table 4 shows that decreasing the failure rate of component 2 by a factor of ρ = 0.1031 leads to enhanced system reliability. This improvement is comparable to adding an extra element to component 2 through a warm duplication technique with an imperfect switch, aiming for system reliability of ω = 0.9.

Table 3. Warm SREF with perfect switch (component improvement).

Component	$\boldsymbol{\omega = 0.2}$	$\boldsymbol{\omega = 0.6}$	$\boldsymbol{\omega = 0.9}$
1	0.4690	0.3437	0.2070
2	0.4458	0.2757	0.0855
3	0.2816	0.0925	0.0061
4	0.2589	0.0822	0.0059
5	0.4458	0.2757	0.0855
6	0.2816	0.0925	0.0061

Table 4. Warm SREF with imperfect switch (component improvement).

Component	$\boldsymbol{\omega = 0.2}$	$\boldsymbol{\omega = 0.6}$	$\boldsymbol{\omega = 0.9}$
1	0.5287	0.3923	0.2376
2	0.5066	0.3224	0.1031
3	0.3867	0.1801	0.0331
4	0.3683	0.1678	0.0320
5	0.5066	0.3224	0.1031
6	0.3867	0.1801	0.0331

Employing Equations (9), (15), (20), and (23), SREFs are computed for the enhancement of all components within a single type. An enhancement in system reliability, reaching ω = 0.6 as indicated in Table 5, is observed by weakening the hazard rate associated with each component within the first type, using a scaling factor of ρ = 0.3881. This enhancement involves adding an additional component alongside each component of type 1, employing warm duplication methodology with a perfect switch. Analogous interpretations can be drawn from the other results presented in Table 6.

Table 5. Warm SREF with perfect switch (type improvement).

Component types	$\omega = 0.2$	$\omega = 0.6$	$\omega = 0.9$
Type 1	0.5172	0.3881	0.2399
Type 2	0.3352	0.1134	0.0062

Table 6. Warm SREF with imperfect switch (type improvement).

Component types	$\omega = 0.2$	$\omega = 0.6$	$\omega = 0.9$
Type 1	0.5685	0.4323	0.2672
Type 2	0.4266	0.2023	0.0335

Transitioning to the analysis of the MREF, it is observed that the MTTF of the original system is 0.9057. Utilizing Equations (8), (14), (19), and (24), we evaluate the MREF and MTTF for the system components in the context of warm duplication with both perfect and imperfect switches, as illustrated in Table 7 and Table 8. The data in Table 7 reveal that enhancing component 1 through warm duplication leads to an increased MTTF, achieving a value of 1.4143. Remarkably, an equivalent MTTF is attainable by reducing the failure rate of the same component, with ρ set to 0.3715, as detailed in Table 7. The most significant component improvement is realized by enhancing the performance of component 1, as evidenced in Table 7 and Table 8. Further effective improvements are noted in the optimization of components 3 and 6, followed by components 2 and 5. Conversely, the least enhancement in system reliability is observed with improvements to component 4.

Table 7. Warm MREF with perfect switching (component improvement).

Component	MREF	MTTF
1	0.3715	1.4143
2	0.4697	1.1556
3	0.2410	1.1732
4	0.2715	1.1049
5	0.4697	1.1556
6	0.2410	1.1732

Table 8. Warm MREF with imperfect switching (component improvement).

Component	MREF	MTTF
1	0.4219	1.3814
2	0.5272	1.1452
3	0.3024	1.1591
4	0.3396	1.1009
5	0.5272	1.1452
6	0.3024	1.1591

Additionally, Table 9 and Table 10 provide data on MREFs for different component types, calculated using Equations (10), (15), (20), and (24). Optimizing the performance of each component within type 1 is shown to be the most effective approach for type improvement, as demonstrated in Table 9 and Table 10.

Table 9. Warm MREF with perfect switching (type improvement).

Component type	MREF	MTTF
Type 1	0.4856	1.7156
Type 2	0.3349	1.3952

Table 10. Warm MREF with imperfect switching (type improvement).

Component type	MREF	MTTF
Type 1	0.5290	1.6154
Type 2	0.3993	1.3249

Figure 2 illustrates the survival functions of the original system alongside those improved via the warm duplication technique applied sequentially to individual components. The graph clearly outlines the improvement in system reliability when the warm duplication technique is applied, particularly to Component 1, which shows the most significant increase in survival probability over time. This indicates that it is the most beneficial target for this enhancement method. Components 3 and 6 also show notable improvements, suggesting their effective contribution to system reliability when duplicated. Components 2 and 5 exhibit a less pronounced but still meaningful improvement, as depicted by their survival functions. The effectiveness of warm duplication on these components is validated by the data in Figure 2 and Table 7 and Table 8, enabling a thorough comparative analysis.

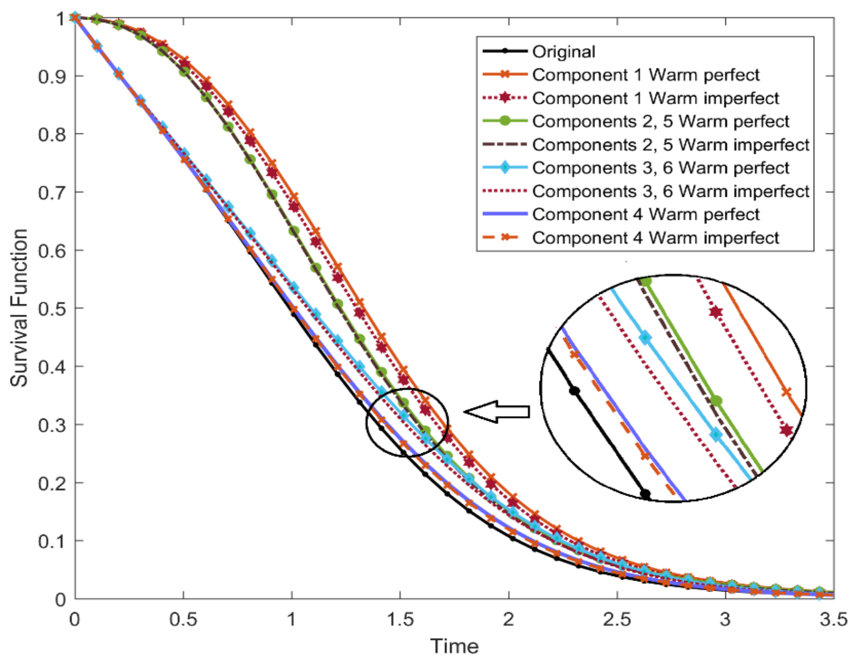

Figure 2. Survival function vs. time for each component's improvement.

Furthermore, Figure 3 illustrates the comparative benefits of warm duplication across different types. It is evident that Type 1 components reap the most significant advantages from warm duplication, showcasing a higher survival function curve than Type 2. This distinction highlights Type 1 as the preferred category for implementing warm duplication, with Type 2 also benefiting from the technique but to a lesser extent.

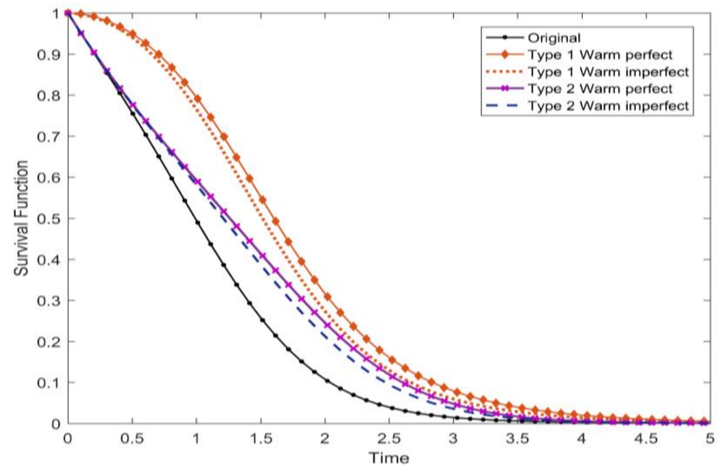

Figure 3. Survival function vs. time with improvements across all components of each type.

Subsequently, Figure 4 presents a quantitative analysis of system reliability by examining the MTTF as a function of the factor ρ, which scales the failure rate of system components. The graph shows an inverse relationship between ρ and MTTF across different components. Component 1 exhibits a significant improvement in MTTF with reductions in ρ, highlighting a substantial enhancement in reliability compared to other components. Conversely, Components 2 and 5, as well as 3 and 6, display a parallel trend of improvement, signifying their similar contributions to the overall system reliability. The increase in MTTF for these components, while not as pronounced as for Component 1, is nevertheless meaningful to system performance. Component 4 displays minimal sensitivity to changes in ρ, with an almost constant MTTF trend, suggesting that modifications to its failure rate have a negligible impact on system performance. Lastly, Figure 5 graphically demonstrates that reducing the failure rates of all components within Type 1 leads to a considerable increase in MTTF, highlighting their critical role in system reliability. In contrast, the more subtle response of Type 2 components suggests their lesser yet still significant effect, emphasizing the importance of prioritizing maintenance for Type 1 components to maximize system uptime.

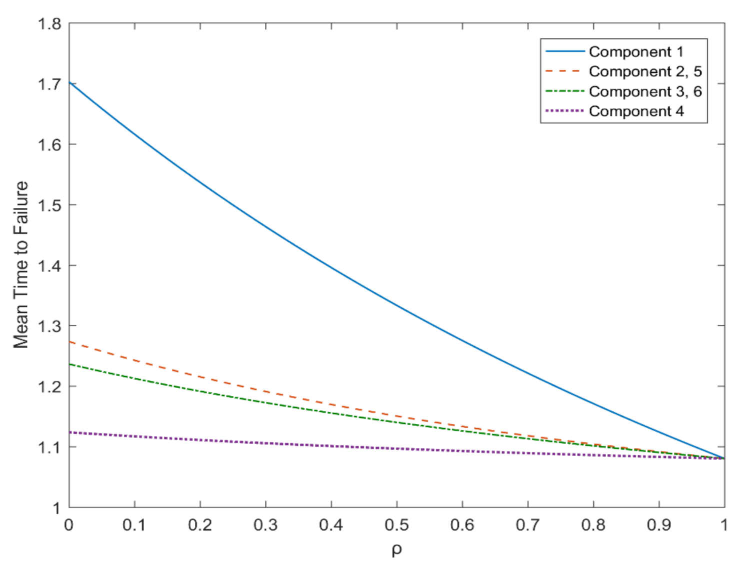

Figure 4. For the components, MTTF vs. ρ.

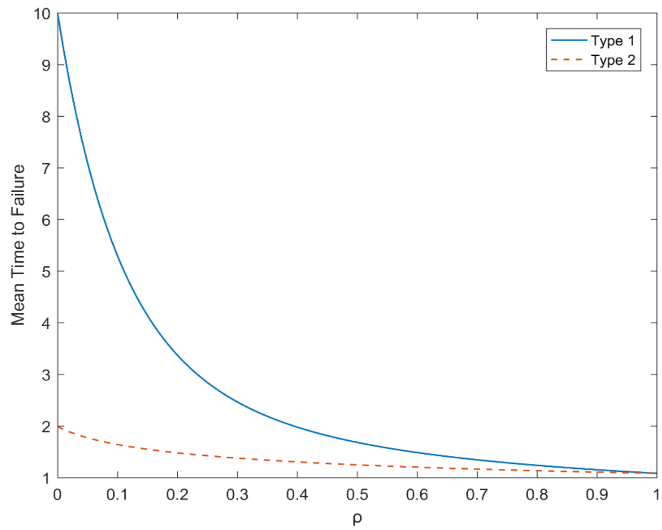

Figure 5. For the component types, MTTF vs. ρ.

6. Conclusion

The survival signature is a crucial tool with numerous advantages, ranging from its applicability to systems involving various types of components to enabling systems comparison, assessing component importance, and facilitating sensitivity analysis. In this study, we have explored the application of the survival signature as an indispensable tool for determining REFs, emphasizing its essential benefits in enhancing system reliability. The derivation and application of REFs play a pivotal role in the architectural enhancement of systems, facilitated here by the adoption of reduction and warm duplication methodologies within the framework of coherent systems. A numerical example focusing on a bridge system, utilizing survival signature analysis, and employing a detailed assessment of component and type improvements through both perfect and imperfect switching, has demonstrated quantifiable enhancements in the system's MTTF. Specifically, the optimization of Component 1, alongside systematic enhancements within Type 1 components, has emerged as the most impactful strategy for reliability improvement. These findings not only validate the survival signature method as a robust tool for reliability assessment but also emphasize the necessity for targeted reliability enhancements within system design and maintenance practices. The study finds an inverse relationship between scaling factor ρ and the MTTF across different components, indicating that as ρ increases, the MTTF decreases, and vice versa. The proposed methodology is not distribution-specific and can be applied to systems with multiple types of units. The approaches and methods outlined in this study are applicable to realistic systems to improve their reliability. However, it is important to note that while our study is comprehensive, it does not encompass closed-form solutions for equivalence factors, focusing instead on numerical computation. This limitation highlights a significant opportunity for future research to develop analytical methods or closed-form solutions, enhancing the applicability and ease of implementation of REFs. Furthermore, our research did not evaluate α-fractiles of the original and improved systems, representing another area for further investigation. The future scope of this research includes expanding these methodologies to a wider range of applications, exploring the development of availability equivalence factors, and refining strategies for enhancing the reliability of coherent systems in diverse engineering contexts. Given the dependance on numerical computation and the absence of closed-form solutions for equivalence factors in the current study, this area presents a clear direction for future work. Moreover, the study could be extended to multi-state systems to further determine REFs and improve system performance.

Conflict of Interest

The authors declare that they do not have any conflicts of interest.

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