On Non-Homogeneous Markov Reward Model to Availability and Importance Analysis for CNC Machine Tools

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Abstract
This paper exploits the Markov reward method to availability and importance evaluation for CNC machine tools. The presented paper regards CNC machine tools as a multi-state system that state transitions process caused by elements and subsystem's failures and corresponding maintenance activities during the lifetime. Simultaneously, considering the time-varying characteristics of the failure rates of subsystems, the non-homogeneous Markov reward model is introduced for evaluation of availability and subsystem importance identification for aging multi-state CNC machine tools under minimal repair. Corresponding procedures for the failure rates, state transition matrix, and reward matrix definition are suggested for the availability and importance measures. A numerical example is presented in order to illustrate the approach.

1. Introduction

Facing the fierce market competition, the items related to the reliability of CNC machine tools are increasingly valued by manufacturers (Zhang et al., 2016). Naturally, availability as the crucial attribute of CNC machine tools can demonstrate if the system can be used when it is desired. By combining the definitions taken from both the IEC international vocabulary (IEC 2001) and the IFIP WG10.4 working group (Avizienis et al., 2004), availability can be defined as the ability to be in a state to perform a required function, under given conditions, at a given instant of time, or after enough time has elapsed, assuming that the required external resources are provided. Although there are a large number of studies and applications on system availability (Trivedi and Bobbio, 2017), rarely found research on the availability of CNC machine tools and most research focuses on building the specific aspect of reliability or maintainability, including functional component models, component-based system models, and fault-based dynamic system models (Ran et al., 2018).

Keller et al. (1982) first systematically studied the reliability and maintainability of CNC machine tools, which used the Weibull distribution and lognormal distribution to analysis the mean time to failure (MTBF) and mean time to repair (MTTR) based on the field fault data collected on 35 CNC machine tools in 3 years. Kim et al. (2005) developed a web-based analysis system to establish the reliability assessment model of machine tools, which could study the failure mode of system components. Sung and Lee (2011) presented the failure analysis results and major reliability parameters for improving the reliability of machine tool changing servo motor. Lanza et al. (2009) presented a method which calculates the optimal time for preventive maintenance and spare part provision by a stochastic optimization algorithm based on a load-dependent reliability model. Shen et al. (2017a, b) explored the relationships between CNC machine tools and corresponding subsystems, and presented the maintenance characteristic of system from the perspective of independent or correlated subsystems, respectively. Zhang et al. (2019) proposed a fault diagnosis strategy based on cascading failure to ensure the safe operation of CNC machine tools. Yang et al. (2011, 2017) established a comprehensive reliability distribution model for CNC machine tools, which assembled the interval analysis, fuzzy comprehensive evaluation and analytic hierarchy process to study the reliability evaluation, allocation and maintainability of the system. Based on the stress-strength interference theory, a general method for modeling and analyzing inaccurate reliability of machine tool components is proposed by Huang et al. (2018) for the characteristics of heavy-duty CNC machine tool uncertainty analysis. Moreover, a meta-action decomposition method proposed by Ran et al. (2016, 2017) for reliability modeling and quality characteristic correlation analysis of CNC machine tool assembly process.

Currently, most research on availability of CNC machine tools concentrated the inherent availability, which calculated through the mean time between failure (MTBF) and mean time to repair (MTTR). Simultaneously, some researchers have contributed to the study of instantaneous availability of CNC machine tools. Wei et al. (2011) used the Petri net to establish the state of the CNC machine tools and corresponding dynamic process model, then the system availability simulated by the Monte Carlo method. Li et al. (2014) proposed the linear model of instantaneous availability of CNC machine tools based on the functional data analysis method. Zhang et al. (2016) from the client's perspective, consider the correlation between the user's requirements for the

availability of CNC machine tools (precision accuracy, failure rate, difficulty of diagnosis, maintenance difficulty, maintenance cost, etc.) and market competition factors to determine the importance of each demand indicator.

Therefore, a novel and concise instantaneous availability evaluation approach of CNC machine tools proposed in current paper that considers the system as a multi-state system (MSS), which formed by the changes in subsystems' state. Thus, the state changes process of CNC machine tools can be regarded as a discrete-state continuous time process (DSCT). Additionally, incorporating the time-varying characteristics of failure rates of subsystems, the non-homogeneous Markov reward model (NHMRM) and Howard differential equation are introduced to determine the availability of the CNC machine tools under minimum repair. Moreover, a sensitivity analysis is performed on each subsystem to determine the failure criticality importance (FCI) of each subsystem. A numerical example is presented in order to illustrate the approach.

2. CNC Machine Tools Availability Evaluation Procedure Based on MHMRM

CNC machine tools belong to complex electromechanical products. When evaluating the availability, the system needs to be divided into different subsystems according to functions, principles and structural features, as shown in Figure 1. The system state changes from normal to fault in the event of any subsystem failure and transferred from faulty to normal after the corresponding repair completed. Therefore, we believe that the state change of the CNC machine tools throughout its life cycle is a multi-state system. Due to the time-varying characteristics of failure rates of subsystems (Wang et al., 2001), it is judged that the CNC machine tools failure events are described by the non-homogeneous Poisson process. Therefore, the presented paper introduces the non-homogeneous Markov reward model to determine the instantaneous availability of CNC machine tools, the specific steps shown as follow.
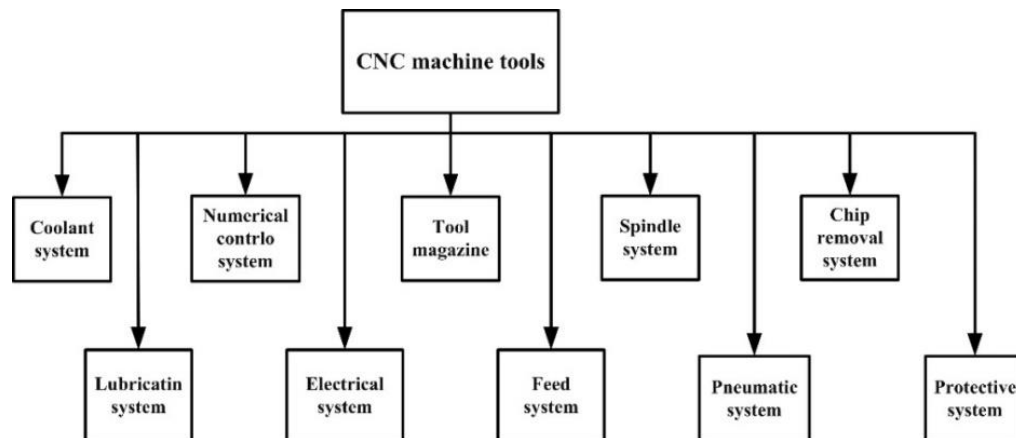

Figure 1. Subsystems division diagram of CNC machine tools.

2.1 Construction of State Transition Matrix

In this paper, the normal operation of CNC machine tools is defined as an acceptable state, and the system shutdown caused by subsystem failures is defined as an unacceptable state. The following assumptions are made:

- The failures between the machine tool subsystems are independent, that is, system shutdown is caused by one subsystem fault.
- The subsystems are binary-state system, normal operation and failure, which means the maintenance activity begins with the discovery of system failure.
- As a complex system composed by a large number of components, we define the maintenance effect of CNC machine tools as "minimal repair".
- The probability of failure and the repair rate of subsystems are independent.

Thus, as shown in Figure 1, CNC machine tools are divided into n subsystems, when a subsystem failure leads to the system shut down, it is considered that the machine tool is transferred from the acceptable state to an unacceptable state caused by the corresponding subsystem. Conversely, after the successful maintenance, it is considered that the machine tool is transferred from the unacceptable state to the acceptable state. Therefore, the operation process of the CNC machine tools can be divided into $(n + 1)$ states. The system state transfer process is shown in Figure 2.
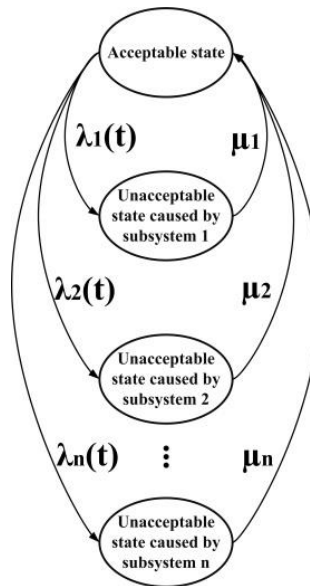
Figure 2. State transition diagram of CNC Machine tools.

The state transition process of CNC machine tools only related to the current state and has no relationships with historical states. It can be judged that the machine tool is a discrete-state continuous time system. According to the characteristic of system performance changing with time, combined with practice, it is determined that the failure rate is time-varying functions, which are expressed in λ_{ij} and the repair process of each subsystem conforms to the exponential distribution, designed as μ_i. Additionally, based on the characteristics of such systems, the transition rates a_{ij}

between state i and j （$i, j = 0,1,2, \cdots, n$）can be expressed by the corresponding failure rate λ_{ij} and repair rate μ_i of the system (Lisnianski et al., 2010), where 0 states denote the acceptable state. Consequently, according to reference (Gertsbakh, 2000; Xie et al., 2004), the state transition process of CNC machine tools can be regarded as a Markov model with time-varying transition intensities $a_{ij}(t)$, namely the non-homogeneous Poisson process. Therefore, the state transition intensities matrix $\boldsymbol{a}$ can be constructed for CNC machine tools as follows:

$$
\boldsymbol{a} = \begin{bmatrix} a_{0,0} & \cdots & a_{0,n} \\ \vdots & \ddots & \vdots \\ a_{n,0} & \cdots & a_{n,n} \end{bmatrix}.
$$

Taking time-varying failure rate $\lambda_{ij}(t)$ and repair rate μ_i into matrix $\boldsymbol{a}$, $i, j = 0,1,2, \dots, n$. For convenience of omitting 0 in $\lambda_{ij}(t)$ and μ_i subscripts and based on the mathematical theory of reliability (Cao and Chen, 2006), the state transition intensities matrix $\boldsymbol{a}$ is obtained as follows:

$$
\boldsymbol{a} = \begin{matrix} 0 \\ 1 \\ 2 \\ \vdots \\ n \end{matrix} \begin{bmatrix} -\sum \lambda_i(t) & \lambda_1(t) & \lambda_2(t) & \dots & \lambda_n(t) \\ \mu_1 & -\mu_1 & 0 & \dots & 0 \\ \mu_2 & 0 & -\mu_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mu_n & 0 & 0 & \dots & -\mu_n \end{bmatrix}.
$$

2.2 Key Subsystems Identification and Corresponding Failure Rates Calculation

Due to the complexity of structure and function, the CNC machine tools contain a variety of fault modes and cause. For saving the cost of experiment and calculation, the key subsystems of machine tool are determined according to the frequency of subsystem fault occurrence. Then, the failure rates of key subsystems should be determined. Moreover, the procedure of subsystem failure modeling includes three steps: fault data rank correction, parameter estimation and hypothesis test, which shown as follows:

(1) *Subsystem Fault Data Rank Correction*
In order to save the test time and cost, the reliability tests of CNC machine tools are carried out by means of timing truncation. Therefore, at the end of each test will produce a truncated data. Furthermore, when a subsystem fails, a failure time of its own is generated and the time for all other subsystems is truncated. Because the existence of multiple truncated data, the order of subsystems failures is changed, so the rank increment is introduced to correct this rank change by using Johnson method in this paper.

The new rank m_i of the ith failure of any subsystem is calculated as follows:
$$
m_i = m_{i-1} + \frac{(n+1-m_{i-1})}{N+2-j}
\tag{1}
$$

where

m_{i-1} : rank of the $(i-1)^{th}$ fault, $i = 1,2, \cdots, n$, n represents total number of failures, define $m_0 = 0$;

   N: total number of fault data and truncated data;

j: all fault data rank from small to large, $j = 1, 2, \cdots, N$.

The empirical value of the fault distribution function $F(t)$ of the subsystem is calculated by approximate median rank method. The formula is

$$\hat{F}(t) \approx \frac{m_i - 0.3}{n + 0.4} \tag{2}$$

(2) *Parameter Estimation.*
In this paper, the two-parameter Weibull distribution is taken as the hypothetical model, and the model parameters of the subsystem are estimated by the least square method based on the fault data obtained from reliability test.

The two-parameter Weibull distribution function is

$$F(t) = 1 - exp[(-\left(\frac{t}{\alpha}\right)^{\beta}], \qquad t \geq 0 \tag{3}$$

where α and β are the scale parameter and shape parameter, respectively.

Assume linear regression equations as

$$\hat{y} = A + Bx \tag{4}$$

where A is the intercept of the straight line and B is the slope. The estimated values of parameters A and B based on the least square method are as follows:

$$\hat{B} = \frac{\sum_{i=1}^{n} x_i y_i - n\overline{xy}}{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}, \quad \hat{A} = \bar{y} - \hat{B}\bar{x} \tag{5}$$

The logarithm of the two ends of formula (3) is obtained:

$$ln\left[ln\left(\frac{1}{1-F(t)}\right)\right] = \beta lnt - \beta ln\alpha \tag{6}$$

Setting $y = ln\left[ln\left(\frac{1}{1-F(t)}\right)\right]$, $x = lnt$ and based on formula (4) and (5), the parameters estimation of Weibull distribution model is

$$\hat{\beta} = \hat{B}, \quad \hat{\alpha} = exp\left(-\frac{\hat{A}}{\hat{B}}\right) \tag{7}$$

(3) *Hypothesis Test*
In this paper, the Kolmogorov–Smirnov test is used to inspect the goodness of fit for the fault interval time distribution function of subsystems.

Kolmogorov–Smirnov test is to arrange the test data in the order from small to large. According to the hypothetical distribution, the corresponding $F_0(x_i)$ of each data is calculated and compared with the empirical distribution function $F_n(x_i)$, where the maximum absolute value of the difference is the observation value of the test statistic D_n. And D_n is compared with the critical value $D_{n,\alpha}$. If the following conditions are satisfied, the original hypothesis is accepted, otherwise the original hypothesis is rejected.

$$D_n = \sup_{-\infty < x < \infty} |F_n(x_i) - F_0(x_i)| = max\{d_i\} < D_{n,\alpha} \tag{8}$$

where, $D_{n,\alpha}$ is the critical value, $F_0(x_i)$ is original hypothetical distribution function value and $F_n(x_i)$ is an empirical distribution function of sample size n, that is,

$$F_n(x_i) = \begin{cases} 0, x < x_i \\ \frac{m_i - 0.3}{n + 0.4}, x_i < x < x_{i+1} \\ 1, x \geq x_N \end{cases} \tag{9}$$

2.3 Build Reward Matrix and Calculate the Availability of CNC Machine Tools

Howard (1960) proposed the Markov reward model, which was widely used in various theoretical and practical studies (Reibman et al., 1989, Trivedi and Bobbio, 2017). The core idea of the Markov reward model is as follows: a continuous Markov chain with K different states and a transition matrix between states is $\boldsymbol{a}=[a_{ij}]$, $i, j = 1, 2, ..., K$. It is assumed that if the process stays in any state i during unit time, a certain amount of reward r_{ii} is achieved. Similarly, it is also assumed that each time the process transits from state i to state j an additional amount of reward r_{ij} is achieved. The rewards may also be negative when it characterizes a loss or penalty. Consequently, for such processes, it is crucial to determine a reward matrix $\boldsymbol{r}=[r_{ij}]$, $i, j = 1, 2, ..., K$ in additional to a transition intensity matrix. Then, many important reliability indexes can be determined by comprehensively applying and calculating these two matrices during the operation of the system. Because the machine tool state transition process is a non-homogeneous Poisson process, the Markov reward model related to its state transition is called the non-homogeneous Markov reward model.

Let $V_i(t)$ be the expected total reward accumulated up to time t, given the initial state of the process as time instant $t = 0$ is in state i. Howard differential equations with time-varying transition intensities a_{ij} should be solved under specified initial conditions to find the total expected rewards.

$$\frac{dV_i(t)}{dt} = r_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{K} a_{ij} r_{ij} + \sum_{j=1}^{K} a_{ij} V_j(t), i, j = 1, 2, ..., K \tag{10}$$

In the most common case, the MSS begins to accumulate reward after time instant $t = 0$, thus, the initial conditions are

$$V_i(0) = 0, i = 1, 2, ..., K \tag{11}$$

For instance, if the state K with the highest performance level is defined as the initial state, the value $V_K(t)$ should be found as a solution of system (10).

Therefore, for determining the availability of CNC machine tools, it is necessary to build the reward matrix $\boldsymbol{r_1}$, and the corresponding rewards in $\boldsymbol{r_1}$ can be determined in the following manner:

(1) All rewards that indicate acceptable are defined as 1.
(2) The rewards associated with all unacceptable states should be zeroed as well as the rewards associated with all transitions.

Thus, the reward matrix r_1 of the CNC machine tools depicted in Figure 2 is

$$r_1 = \begin{matrix} 0 \\ 1 \\ 2 \\ \vdots \\ n \end{matrix} \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}.$$

In this paper, the cumulative reward $V_0(t)$ of the CNC machine tools in interval $[0, t]$ define a time that the system will be in the set of acceptable state in the case where state 0 is the initial state. After obtaining the $V_0(t)$, the CNC machine tools instantaneous availability $A_0(t)$ can be expressed as follows:

$$A_0(t) = \frac{V_0(t)}{t} \tag{12}$$

2.4 Reward Matrix Creation for the Calculation of FCI of Subsystems

After determining the system availability, it is significant to compare the contribution of each unacceptable state to the system downtime, which helps optimize the production cost. Therefore, the failure criticality importance (FCI) (Wang et al., 2004; Toledano et al., 2016) is introduced to calculate the percentage of each failure state to the system downtime. The FCI can be obtained as follows:

$$I_{FCI}(j; t) = \frac{\begin{pmatrix} number\ of\ system\ failures, \\ caused\ by\ subsystem\ j\ in\ [0,t] \end{pmatrix}}{(number\ of\ system\ failures\ in\ [0,t])} \tag{13}$$

where $I_{FCI}(j; t)$ is the failure criticality importance of the jth failure state in $[0, t]$. Subsequently, the key is to evaluate the number of system failures and the number of the system entrances to each down state, which can be obtained through the Markov reward approach. In this case, the mean accumulated reward $V_i(t)$ obtained by solving (10) provides the mean number of entrances into the failure state in the time interval $[0, t]$.

The reward matrix formulated under the following rules:

(1) All rewards that indicate an acceptable state transfer to an unacceptable state are defined as 1;
(2) The other awards in the matrix are defined as 0.

Then the mean number of system transfer from normal to unacceptable state caused by subsystem j can be obtained through the Howard differential equations, defined as $V_{0j}(t), j = 1,2, \dots, n$. Correspondingly, the criticality importance of jth subsystem is defined as following:

$$I_{FCI}(j; t) = \frac{V_{0j}(t)}{\sum_{j=1}^{n} V_{0j}(t)}, j = 1,2, \dots, n \tag{14}$$

3. Numerical Example

The data used in this paper are obtained by a field test of 67 CNC machine tools and the last fault occurrence time is recorded as truncation time. The failure interval time is designated as t and we realized that faults of this type of CNC machine tools are mainly concentrated in the feed system, tool magazine and the spindle system. Moreover, the mean time to repair of each subsystem also collected in the field. Thus, three key subsystems and the corresponding specific data are shown in Table 1.

Table 1. Subsystem failure interval time and mean time to repair.

Fault subsystem	Failure interval time (t/hr)	Mean time to repair (hr)
Feed system	1419.0，766.1，212.4，925.7，604.6，694.1，139.8，209.5，49.2，37.5，1217.7，545.0，131.4，342.9，98.5，852.5，333.6，52.6，144.9，1775.6，211.6，1186.9，61.8，321.8，838.6，2443.9，27.4，15.7，840.9，766.9，49.4	22
Tool magazine	533.3，16.3，186.2，395.4，211.9，26.6，755.4，579.4，532.8，446.2，493.0，888.6，433.2，206.6，52.3，1074.4，865.2，89.6，122.8，37.8，163.0，110.4，258.5，38.2，335.7，604.4，12.6	35
Spindle system	382.9，821.6，974.2，1260，296.8，1960，147.1，210.3，77.0，775.7，446.6，88.2，939.1，122.7	40

The failure rate functions of the key subsystems are established by using a Weibull distribution, and the linear correlation coefficient and the KS-test were carried out. The results are shown in Table 2 and Figure 3, respectively.

Table 2. Failure rate functions of key subsystems.

Fault subsystem	Failure rate functions	Linear correlation coefficient	KS-test observations	Critical value $D_{n,0.01}$
Feed system	$\lambda_3(t) = \dfrac{0.9035}{2002.8}\left(\dfrac{t}{2002.8}\right)^{-0.0965}$	0.9883	0.03593	0.290
Tool magazine	$\lambda_2(t) = \dfrac{0.8482}{2272.2}\left(\dfrac{t}{2272.2}\right)^{-0.1518}$	0.9819	0.02385	0.317
Spindle system	$\lambda_1(t) = \dfrac{1.1362}{3156.2}\left(\dfrac{t}{3156.2}\right)^{0.1362}$	0.9239	0.04748	0.418
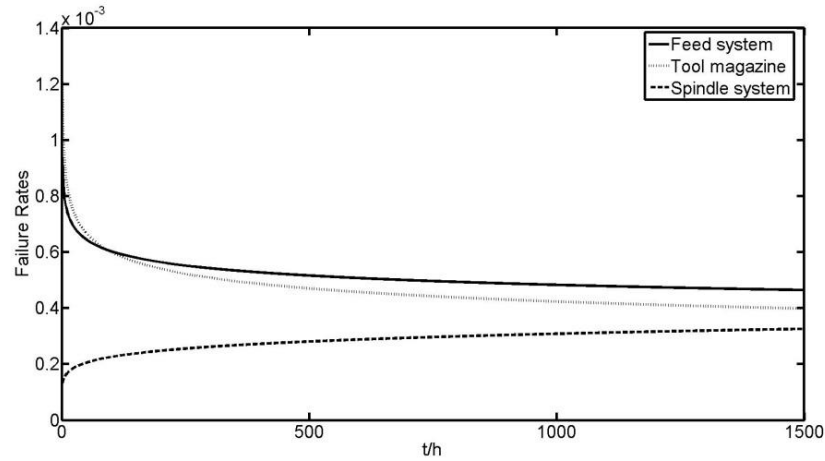
Figure 3. Failure rates $\lambda(t)$ curve of key subsystems.

According to the identified key subsystems, the state transition process of CNC machine tools can be determined as shown in Figure 4:
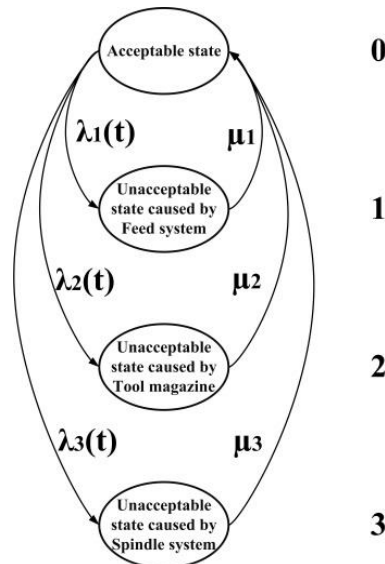
**Figure 4.** State transition diagram of CNC Machine tools.

Therefore, according to the construction rules of reward matrix r_1 and Figure 4, we can determine the state transition matrix a and the reward matrix r_1 are:

$$a=\begin{bmatrix} -\sum_{i=1}^{3}\lambda_i(t) & \lambda_1(t) & \lambda_2(t) & \lambda_3(t) \\ \mu_1 & -\mu_1 & 0 & 0 \\ \mu_2 & 0 & -\mu_2 & 0 \\ \mu_3 & 0 & 0 & -\mu_3 \end{bmatrix}, \quad r_1=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{15}$$

The cumulative reward differential equations associated with the availability are:

$$\begin{cases} \frac{dV_0(t)}{dt} = 1 + \lambda_1(t)V_1(t) + \lambda_2(t)V_2(t) + \lambda_3(t)V_3(t) - \sum_{i=1}^{3}\lambda_i(t)\,V_0(t) \\ \frac{dV_1(t)}{dt} = \mu_1 V_0(t) - \mu_1 V_1(t) \\ \frac{dV_2(t)}{dt} = \mu_2 V_0(t) - \mu_2 V_2(t) \\ \frac{dV_3(t)}{dt} = \mu_3 V_0(t) - \mu_3 V_3(t) \end{cases} \tag{16}$$

According to the formula (12), the availability of this type of CNC machine tools can be obtained by using MATLAB® solution, and the resulting curve is shown in Figure 5.
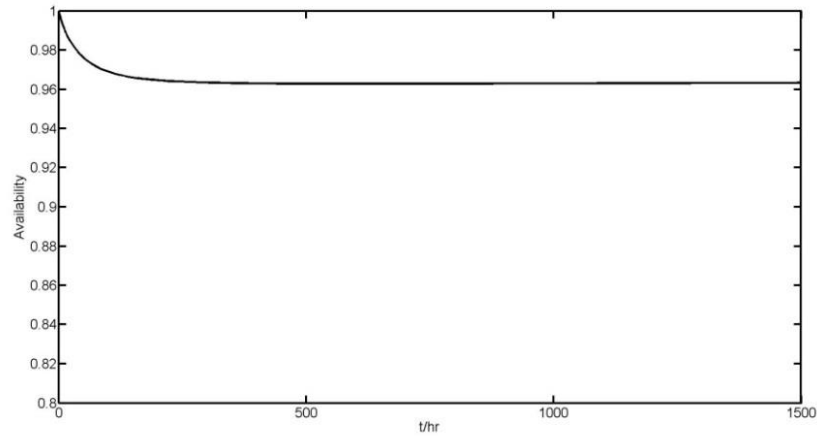
**Figure 5.** Availability of CNC Machine tools.

Similarly, the reward matrix of key subsystems based on the FCI rules tools are as follows:

$$r_{21}=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, r_{22}=\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, r_{23}=\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{17}$$

Taking the feed system as an example, the Howard differential equations determined as following. Using MATLAB® solution, the mean number of failures $V_{01}(t)$ of the feed system can be obtained through calculating the following system.

$$\begin{cases} \frac{dV_{01}(t)}{dt} = \lambda_1(t) + \lambda_1(t)V_1(t) + \lambda_2(t)V_2(t) + \lambda_3(t)V_3(t) - \sum_{i=1}^{3} \lambda_i(t)\, V_{01}(t) \\ \frac{dV_1(t)}{dt} = \mu_1 V_{01}(t) - \mu_1 V_1(t) \\ \frac{dV_2(t)}{dt} = \mu_2 V_{01}(t) - \mu_2 V_2(t) \\ \frac{dV_3(t)}{dt} = \mu_3 V_{01}(t) - \mu_3 V_3(t) \end{cases} \tag{18}$$

Therefore, according to the formula (14) the "failure criticality importance" of Feed system, Tool magazine and Spindle system are shown in Figure 6.
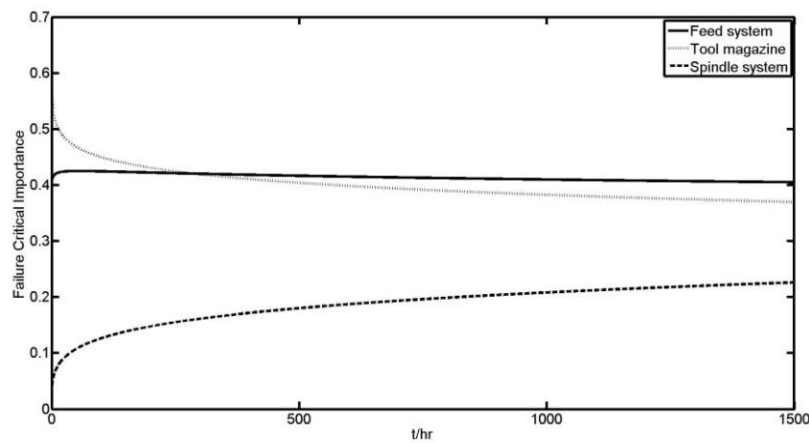
Figure 6. Comparison of key components Failure criticality importance.

Here one can see that the importance of the Tool magazine is a decreasing function over time, the importance of the Spindle system is mostly increasing, and the importance of the Feed system is increasing over time. During the first 300 hours of usage, the Tool magazine has greater importance than the Feed system, and after 300 hours, the importance of the Feed system becomes greater.

4. Conclusions
(1) The CNC machine tools during the life cycle were considered as a discrete-state continuous time process from the perspective of state transition of subsystems. Incorporating the time-varying characteristic of subsystems' failure rate, then the non-homogeneous Markov reward model was developed for the calculation of instantaneous availability for aging multi-state CNC machine tools. Furthermore, the failure criticality importance (FCI) of subsystems was evaluated by the proposed approach, which could identify the contribution of each subsystem to system downtime.
(2) The method proposed in this paper is a concise and rational solution for the non-homogeneous Poisson process in the field of CNC machine tools reliability research. More importantly, the suggested approach is well formalized and suitable for practical application in reliability engineering. In fact, the presented work can be applied to any system. The numerical example was presented in order to illustrate the suggested approach.

Conflict of Interests
The authors declared no potential conflicts of interest with respect to the research, authorship and/or publication of this article.

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