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A SIGNALING THEORY OF UNEMPLOYMENT

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ABSTRACT

This paper presents a signaling explanation for unemployment. The basic idea is that employment at an unskilled job may be regarded as a bad signal. Therefore, good workers who are more likely to qualify for employment at a skilled job in the future are better off being unemployed than accepting an unskilled job. We present conditions under which all equilibria satisfying the Cho-Kreps intuitive criterion involve unemployment. However, there always exist budget balancing wage subsidies and taxes that eliminate unemployment. Also, for any unemployment equilibrium, either there always exists a set of Pareto improving wage taxes and subsidies, or we give conditions under which there exists a set of Pareto improving wage taxes and subsidies.

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Introduction

Several information theoretic (or efficiency wage) models have recently been proposed to explain persistent involuntary unemployment. They explain downward wage rigidities by the productivity enhancing effects of high wages. In these models wage cuts lower the quality of the firm's workers (the adverse selection effect), decrease their output, increase their quit rate (the incentive effect), or in other ways cause workers to act against the firm's interest.[1]

We offer an alternative information theoretic explanation of unemployment in which employment at an unskilled job serves as a bad signal to managers seeking to hire workers for skilled jobs. Workers may therefore refuse unskilled jobs to avoid the associated bad label.[2]

In previous efficiency wage models, workers never refuse jobs. Those models can be extended to encompass voluntary unemployment if one of the following auxiliary assumptions is made: Either the cost of finding a new job is higher for the employed than for the unemployed, or the jobs that are being rejected are offering wages below the reservation wage of the unemployed.

There are difficulties with each of these assumptions. First, our reading of the evidence on the cost and efficacy of search suggests that search costs are no higher for employed than for unemployed workers. For instance, surveys of workers indicate that the best source for information about jobs is

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[1] See Weiss (1990) or Katz (1986) for surveys of efficiency wage models of unemployment.

[2] Although this unemployment is "voluntary", it may nonetheless be socially devastating. Even among members of groups that have the worst labor market experience, a significant proportion of total unemployment appears to be voluntary. For instance 54% of black males aged 16–19 in poverty neighborhoods report that "The people [they] know who are unemployed could find work if they really wanted." See Daitcher-Loury and Loury (1986). We are not, however, arguing that our model explains the high unemployment rates of black youth. Rather we claim that our model provides insight into some aspects of unemployment.

contacts at work.[3] Second, the argument that unemployed workers have reservation wages that are higher than offered wages is not an explanation of unemployment. The relevant question is <u>why</u> the unemployed have reservation wages above current wages.

In our model there are at least some equilibria in which taking an unskilled job is sufficiently damaging to the future employment prospects of a (skilled) worker that he will choose unemployment even if there is no disutility from work. The key assumption in the model is that workers possess private information about their own abilities which is correlated with employers' evaluations. There is a potential gain to employers from using this private information, along with their own evaluations, to select among job applicants who are observationally indistinguishable. Therefore workers with favorable private information wish to signal this to employers. One way of signaling favorable information is for a worker to take a costly action for which the expected benefit is positively correlated with his private information. Unemployment has two attributes that may make it a good signaling device. First, it is costly. Second, the expected (future) benefit from unemployment may be greater for workers with good messages. Suppose employers screen applicants,[4] and the probability of passing this screening is positively correlated with workers' private information. Then unemployment can be an effective signal because the benefit from unemployment only accrues to workers who pass the hiring exam, and passing is more likely for workers with good messages. Because this signaling property raises the cost of taking a low

[3] Corcoran, Daitcher, and Duncan (1980) report that roughly half of all workers know someone who worked for their current employers before they got their first job there. Rosenfeld (1977) found that roughly the same number of employed people were actively searching for (other) jobs as were unemployed workers.

[4] Barron, Black and Loewenstein (1987) report that for the 2336 firms in the EOPP survey the mean number of screened applicants per acceptable applicant is 9.

wage (unskilled) job for all workers, it can lead to equilibria in which, among a group of observationally indistinguishable workers, all (or some) choose to be unemployed rather than taking unskilled jobs.

Even if the probability of passing the hiring test is uncorrelated with workers' private information, unemployment can still serve as an effective signal. This would be the case if the cost of unemployment were negatively correlated with the productivities of workers. For example, the productivity in home production may be positively correlated with productivities in the workplace and with workers' messages. As a second alternative, we could have allowed for risk averse workers and assumed that more productive workers either have more wealth, better access to credit, or higher earning spouses. We have ignored these effects; including them would broaden the range of parameter values at which socially inefficient unemployment might occur.

Equilibria may be characterized by workers with good messages (or some proportion of those workers) choosing unemployment and workers with bad messages choosing unskilled jobs. Indeed, depending on the parameter values of the model, sequential equilibria exist in which all workers choose unemployment, all workers accept unskilled jobs, or some workers choose unemployment while others with the same private information choose unskilled jobs. Perhaps surprisingly, for some parameter values of the model, the only sequential equilibria that satisfy the Cho-Kreps intuitive criterion entail (some) unemployment.[5]

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[5]In the context of our model, the intuitive criterion operates as follows. Starting from a full employment equilibrium a worker choosing unemployment is implicitly making the following statement: "I must have received a good message because persons receiving the bad message have such low probabilities of passing the hiring exam that they wouldn't have chosen unemployment, even if you believed that only those who had received good messages chose unemployment." The main problem is that the availability of this implicit message could cause a worker with the bad message to believe that he would be labeled as having bad private information if he did not send the message. This could induce him to mimic the good message worker. In that case the statement loses its force. In the context of our model, with many workers choosing unskilled jobs in the putative full employment equilibrium, this objection has less force than in the typical one person signaling games.

The main policy implication of our model is that there always exist budget balancing wage subsidies and taxes that eliminate unemployment. We construct tax-subsidy schedules that induce full employment as a unique sequential equilibrium without taxing unemployment or making taxes and subsidies contingent on employment histories. The full employment equilibrium also satisfies the intuitive criterion. Full employment can be induced through a tax-subsidy scheme that does not collect any taxes in equilibrium, or one that redistributes money from higher income to lower income workers. The basic intuition is that sufficiently progressive taxes at high wages destroy the incentive for workers with good (private) messages to separate themselves by choosing unemployment. If workers with good messages have no incentive to separate themselves, neither will workers with bad messages, so the intuitive criterion is also satisfied.

We also prove that starting from any symmetric equilibrium with unemployment, the government can introduce a tax-subsidy scheme that improves the expected utilities of each type of workers, where a type is defined according to the message received. (By a symmetric equilibrium, we mean one in which all workers of the same type choose the same action.) These taxes and subsidies also eliminate unemployment.

We conclude by extending our model to a setting where good jobs are offered in every period. In many equilibria of this richer model, workers who have experienced a spell of unemployment have lower expected wages than those who have not. The average productivity of workers who worked continuously may be higher (or lower) than that of those who experienced some unemployment.

2. A Signaling Model

We use the following model to explain why some (skilled) workers choose unemployment when (unskilled) jobs are available to them. The essential features of the model are that workers have private information and firms have an imperfect testing technology.

In period 1, each worker receives a "good" message, G, or a "bad" message, B. We label a worker that receives a "t" message as being of type "t", t = B,G. Note, however, that not all workers of the same type are identical. One may think of this message as the worker's own (possibly inaccurate) assessment of his ability.

In the second period workers enter the labor market due to an exogenous event, such as a plant closing, or graduation from school, that does not discriminate among workers with different abilities. In that period, workers choose either to get an unskilled job or become unemployed. In Section 6, we allow (some) workers to be offered jobs in the skilled sector in the second period. (In this extension of our model, workers that obtain jobs without an intervening spell of unemployment may be better paid than those who obtain jobs after a spell of unemployment.) We normalize the current income of an unemployed worker at zero. The productivity of a worker at an unskilled job is $M > 0$ and is independent of his type and of the number of workers choosing unskilled jobs. Firms that offer unskilled jobs act competitively. Therefore the wage at an unskilled job is also M.

In the third period, workers are evaluated by firms that have minimum hiring standards. We refer to these firms as being in the skilled sector. These evaluations are known by at least two firms. Evaluations are zero-one (or pass-fail): workers are determined to be either qualified or unqualified.[6] Workers with a good message are more likely to be qualified than are workers with a bad message. Specifically, a type t worker will be evaluated as qualified with probability p_t, t = B,G, and $1 > p_G > p_B > 0$.

[6]We are implicitly assuming that when workers were hired for the jobs they held in period 1, a different "test" was used to determine whether they were qualified. We are also assuming that all firms hiring skilled workers in the third period use the same qualification test.

The (expected) productivity of a qualified type G (resp. B) worker is denoted by G (resp B). We initially assume that $0 < M < B < G$. In Appendix A, we investigate the case of $M > B$. The (expected) productivity of an unqualified worker of either type is less than M.[7]

Workers are risk neutral and have time separable utility functions. For notational simplicity we assume that workers have no disutility from work and do not discount future wage payments. We further assume that there is a continuum of workers of each type. The cumulative distribution of all workers is normalized to 1 and the proportion of workers who receive "good" messages (type G workers) is α, where $0 < \alpha < 1$. Firms are profit maximizers with constant returns to scale in production.

Firms know the population parameters: B, G, p_G, p_B, α and M. They do not know whether a particular worker is type B or G. We assume that in period 3, firms observe the (period 2) employment histories of job applicants.[8] After evaluating workers, firms in the skilled sector may make wage offers on the basis of employment histories, and evaluation results of workers. Firms in the unskilled sector continue to offer M. Workers accept the highest wage offered. If two or more firms make the same wage offer to a worker, he accepts each offer with equal probability. Firms hire all workers that accept their wage offers.

[7] One can think of the productivity of a worker as being the product of two attributes, one of which takes the values G or B and the other takes the values 0 or 1. Workers know the first attribute, firms can test for the second.

[8] A worker may attempt to hide his employment history, but in practice, this may not be easy. It is very difficult for a worker to falsely claim that he is working at a certain firm, since by a simple phone call a potential employer can find out that he is not. An applicant who is working may also find it difficult to claim that he is unemployed. He will not be able to receive phone calls at home during normal business hours, or to go for lengthy interviews. However, even if these factors are not operative, few workers would lie on the application forms. Not only are those lies grounds for immediate dismissal, but they violate social conventions. These conventions seem to have particularly great force in long term relationships such as employee-firm.

Workers choose either unemployment, U, or the unskilled job, L. (In the last period workers automatically work for the firm offering them the highest wage.) Firms choose wage offers conditional on employment histories and evaluation results of workers. (We could also allow firms to condition wages on the distribution of workers choosing U and M. However, since this assumption would require additional notation and would not change any of our results, we shall assume that this distribution is not observed.[9])

A sequential equilibrium in this model is a strategy combination of workers and firms and a belief structure of firms such that a worker cannot increase his total expected lifetime income by changing his second period choice of U or L given the wage schedules being offered, and a firm cannot increase its expected profit by offering a different contingency wage schedule given workers' strategies and its beliefs. Without loss of generality, we need only consider pure strategy sequential equilibria in our model.[10] In the context of our model the restrictions imposed by sequentiality are weak. They simply preclude bizarre responses to out-of-equilibrium moves such as firms offering wages less than B or more than G to workers who pass the screening test. (See Kreps-Wilson (1982) for a formal definition of sequential equilibrium.) In the next section we shall analyze two classes of equilibria: symmetric equilibria in which workers with the same private information choose identical strategies, and asymmetric equilibria in which workers with the same private information choose different actions.

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[9] The reason our substantive results are not changed is that we have assumed that there is a continuum of workers. Hence, no single worker can affect the distribution of employed and unemployed workers observed by firms. Had we let firms condition their wage offers on the distribution of workers choosing employment and unemployment, this would give a stronger justification for assuming that firms play their Nash equilibrium strategy when there are multiple equilibria. By observing the distribution of actions workers choose, firms would be able to deduce the equilibrium being played.

[10] Since there is a continuum of workers of each type, and we allow for asymmetric equilibria, this restriction has no force.

3. Characterization of Equilibria

Recall that a worker's productivity in the skilled sector exceeds M only if the worker passes the evaluation test. We can simplify a firm's strategy by assuming that it offers zero wages to workers who do not pass the test, regardless of the firm's belief about the worker's type. Workers who fail the test are unqualified, and are employed in the unskilled sector. Correspondingly if $W(L)$ and $W(U)$ denote a firm's wage offer to a successful worker who has an employment history of L and U respectively, they also summarize the strategy of a firm.

The following definition will be useful in the characterization. Let

$$\omega = \frac{\alpha\, p_G\, G + (1-\alpha)\, p_B\, B}{\alpha\, p_G + (1-\alpha)\, p_B}\ .$$

ω is the average productivity of a worker who passes the evaluation test. The first proposition shows that there is always an equilibrium in which all workers choose to get unskilled jobs in the second period.

Proposition 1: There is always a full employment sequential equilibrium characterized by $W(L) = \omega$.

Proof: If all workers are employed, $W(U)$ must satisfy

$$(1) \qquad M + p_G\, \omega + (1-p_G)\, M \geq p_G\, W(U) + (1-p_G)\, M \qquad \text{and}$$

$$(2) \qquad M + p_B\, \omega + (1-p_B)\, M \geq p_B\, W(U) + (1-p_B)\, M\ .$$

Inequalities (1) and (2) require that type G and B workers prefer the unskilled job to unemployment when $W(L) = \omega$. Since all workers choose to work in the unskilled sector, firms will offer $W(L) = \omega$. Thus, if (1) and (2) hold, there exists a full employment equilibrium. For $W(U) = \omega$, (1) and (2) hold. Firms can have consistent and rational beliefs that if the out-of-equilibrium action U were to be observed, it would have been taken by a randomly selected worker. Therefore they can set $W(U) = \omega$. ||

We now characterize two other possible symmetric equilibria.

Proposition 2A: The necessary and sufficient condition for a sequential equilibrium in which all workers choose to be unemployed is

$$\frac{M}{\omega - B} \leq p_B \ . \tag{3}$$

Proof: Because $\omega > B$, (3) can be written as

$$p_B\ \omega + (1 - p_B)\ M \geq M + p_B\ B + (1 - p_B)\ M \ . \tag{4}$$

Inequality (4) and $p_G > p_B$ imply

$$p_G\ \omega + (1 - p_G)\ M \geq M + p_G\ B + (1 - p_G)\ M \ . \tag{5}$$

We first prove that (4) is necessary. In the sequential equilibrium we are considering, competition among firms in the skilled sector means that they must offer $W(U) = \omega$. Consistency requires $W(L) \geq B$. Therefore if (4) failed to hold, type B workers would prefer the unskilled job to unemployment.

To show that (4) is sufficient, note that if (4) and (5) hold, both types of workers would choose unemployment in the second period. If all workers choose unemployment in the second period, $W(U) = \omega$, and observing an out-of-equilibrium employment history of L from a worker, it is possible that firms would believe that they were observing a type B worker. Those beliefs would lead firms to offer $W(U) = B$.||

The equilibria described in the last two propositions are pooling equilibria; in the third period, firms cannot correctly infer workers' private information based on their choices in the second period. We now turn to separating equilibria.

Proposition 2B: The necessary and sufficient condition for a sequential equilibrium in which all type G workers choose unemployment and type B workers accept jobs in the unskilled sector is

$$p_B \leq \frac{M}{G - B} \leq p_G \ . \tag{6}$$

In the last period, firms set $W(U) = G$ and $W(L) = B$.

Proof: Given workers' choices in the second period, firms offer $W(U) = G$ and $W(L) = B$ in equilibrium: they infer workers' private information from their strategies. Furthermore, for workers to adopt the strategy described in this proposition, it is necessary and sufficient that

$$p_G\ G + (1 - p_G)\ M \geq M + p_G\ B + (1 - p_G)\ M \qquad \text{and}$$

$$M + p_B\ B + (1 - p_B)\ M \geq p_B\ G + (1 - p_B)\ M\ ,$$

which reduce to (6).||

The equilibrium in Proposition 2B is the unique (completely) separating sequential equilibrium in our model. It is never incentive compatible for type B workers to choose unemployment while type G workers accept unskilled jobs.

We now characterize two asymmetric (semi-separating) equilibria. In these equilibria a firm is only able to infer perfectly the private information of one type of workers from their strategies. The following definitions will be used in the next two propositions. For $\beta, \gamma \in [0,1]$, let

$$\hat{\omega}(\beta) = \frac{\alpha\ p_G\ G + \beta\ (1 - \alpha)\ p_B\ B}{\alpha\ p_G + \beta\ (1 - \alpha)\ p_B} \qquad \text{and} \tag{7}$$

$$\tilde{\omega}(\gamma) = \frac{\gamma\ \alpha\ p_G\ G + (1 - \alpha)\ p_B\ B}{\gamma\ \alpha\ p_G + (1 - \alpha)\ p_B}\ . \tag{8}$$

These definitions have obvious interpretations: $\hat{\omega}(\beta)$ is the expected productivity of a randomly selected successful worker from the sub-population of all type G workers and a proportion β of type B workers; $\tilde{\omega}(\gamma)$ is the expected productivity of a randomly selected successful worker from the sub-population of all type B workers and a proportion γ of type G workers. Notice that as β increases from 0 to 1, $\hat{\omega}(\beta)$ decreases from G to ω, and as γ increases from 0 to 1, $\tilde{\omega}(\gamma)$ increases from B to ω.

Proposition 3A: The necessary and sufficient condition for a sequential equilibrium in which all type G workers choose unemployment and a proportion, $\hat{\beta}$, of type B workers choose unemployment while all other type B workers choose unskilled jobs is

$$\frac{M}{G - B} \le p_B \le \frac{M}{\omega - B} , \tag{9}$$

and $\hat{\beta}$ is given by the (unique) solution to the following equation:

$$\frac{M}{\hat{\omega}(\hat{\beta}) - B} = p_B . \tag{10}$$

Proof: We first show that (9) is sufficient for the equilibrium just described. Suppose (9) is true. Since $\hat{\omega}(\beta)$ has a range $[\omega, G]$ and is continuous, there must exist $\hat{\beta} \in [0,1]$ that satisfies (10). Since $p_G > p_B$, we have

$$\frac{M}{\hat{\omega}(\hat{\beta}) - B} < p_G . \tag{11}$$

(10) and (11) can be written as

$$p_B\ \hat{\omega}(\hat{\beta}) + (1 - p_B)\ M = M + p_B\ B + (1 - p_B)\ M \qquad \text{and} \tag{12}$$

$$p_G\ \hat{\omega}(\hat{\beta}) + (1 - p_G)\ M > M + p_G\ B + (1 - p_G)\ M . \tag{13}$$

(12) and (13) imply that if firms offer $W(U) = \hat{\omega}(\hat{\beta})$ and $W(L) = B$, type B workers will be indifferent between unemployment and taking unskilled jobs, while type G workers will strictly prefer unemployment. Suppose all type G workers and a proportion, $\hat{\beta}$, of type B workers choose unemployment while all other type B workers choose unskilled jobs, then firms offer $W(U) = \hat{\omega}(\hat{\beta})$ and $W(L) = B$. Therefore the strategies described in the proposition are equilibrium strategies.

We now show that (9) is necessary. Suppose there is an equilibrium with workers' strategies described in the proposition. Firms must offer $W(U) = \hat{\omega}(\hat{\beta})$ and $W(L) = B$. From (12) we see that (10) must be true, otherwise type B workers would choose only unemployment or unskilled jobs. But if (9) fails to hold, there is no value in the range of $\hat{\omega}(\beta)$ that satisfies (10).||

<u>Proposition 3B</u>: The necessary and sufficient condition for a sequential equilibrium in which all type B workers accept unskilled jobs and a proportion, $\tilde{\gamma}$, of type G workers choose unskilled jobs while all other type G workers choose unemployment is

$$\text{(14)} \qquad \frac{M}{G - B} \le p_G \le \frac{M}{G - \omega} ,$$

and $\tilde{\gamma}$ is given by the (unique) solution to the following equation:

$$\text{(15)} \qquad \frac{M}{G - \tilde{\omega}(\tilde{\gamma})} = p_G .$$

The proof of Proposition 3B is essentially the same as that of Proposition 3A and is therefore omitted. (Figure 1 illustrates the various equilibria and their necessary and sufficient conditions.)

The reader should note that the necessary and sufficient conditions in Propositions 2A, 2B, and 3A are mutually exclusive. Therefore at most two outcomes generating unemployment are feasible at the same time. If two unemployment outcomes are feasible, one must be an outcome generated by an asymmetric equilibrium described in Proposition 3B.[11]

We find the equilibria described in Proposition 3B unsatisfactory. They do not fit our intuitive understanding of the dynamic processes that our simple static model is supposed to represent. In those equilibria, type B workers are indifferent between unemployment and employment at the unskilled

[11] We are using "outcomes" to refer to the proportion of each type choosing unemployment or unskilled jobs. If outcomes were to refer to the choices of each worker, then there would be a continuum of outcomes in the asymmetric equilibria, since there is a continuum of workers.

sector. However, if in the past fewer than $\tilde{\gamma}$ of type G workers chose unemployment, the average productivity of qualified workers with history L would exceed $\tilde{\omega}(\tilde{\gamma})$. If this were reflected in an increase in W(L), more type G workers would choose the unskilled jobs and the equilibrium would not be sustained. Conversely, if more than $\tilde{\gamma}$ of type G workers chose unemployment, the average productivity of workers with history L would be less that $\tilde{\omega}(\tilde{\gamma})$. If this were reflected in a fall in W(L), more type G workers would choose unemployment. Thus it seems unlikely that an economy would ever converge to the equilibrium described in Proposition 3B.[12] If one rejects the equilibria in Proposition 3B, there are never more than two "outcomes": full employment and one in which some (or all) workers are unemployed for at least one period.

4. Equilibrium Refinements

In this Section, we apply the Cho-Kreps (1987) intuitive criterion to our model. We first define the intuitive criterion in the context of our model. (Footnote 5 showed informally how the criterion operates in the model.)

Definition of the intuitive criterion. Suppose that on observing an employment history, say $h \in \{L,U\}$, from a worker, firms believe with probability $0 \le \mu \le 1$ that this is a type G worker. Then let $BR(\mu,h)$ denote firms' best response. $BR(\mu,h)$ is a wage offer equal to the expected productivity of a randomly selected successful worker from a population consisting of a proportion μ of type G and a proportion $(1 - \mu)$ of type B workers. Furthermore, let $BR(\tau,h)$ represent the set of best responses when firms' probability assessments concentrate on the set $\tau \subset T = \{G,B\}$, or

$$BR(\tau,h) = \bigcup_{\{\mu:\mu(\tau)=1\}} BR(\mu,h) .$$

[12] On the other hand, the asymmetric equilibrium described in Proposition 3A appears stable. If the (indifferent) type B workers were to choose unemployment and unskilled jobs in a different ratio than that described in Proposition 3A, plausible responses by firms would lead them back to the postulated distribution of actions.

Let $u(t,h,w)$ be the expected life-time income of a type $t \in \{B,G\}$ worker with an employment history h when firms offer a wage w (conditional on a worker being successful in the test). Consider an equilibrium and let $u^*(t)$ be the equilibrium expected utility of a type t worker. For each out-of-equilibrium employment history h, denote by S(h) the set of all types t such that

$$u^*(t) > \max_{w \in BR(T,h)} u(t,h,w) \ .$$

If for any one history h, there is some type t' (necessarily not in S(h)) such that

$$u^*(t') < \min_{w \in BR(T \setminus S(h),h)} u(t',h,w) \ ,$$

then the equilibrium outcome is said to fail the Intuitive Criterion.

As we will show, the (full-employment) equilibrium in Proposition 1 may fail to satisfy the intuitive criterion, while all the unemployment equilibria always satisfy the intuitive criterion.

Proposition 4: The equilibria described in Propositions 2A, 2B, 3A, and 3B satisfy the intuitive criterion.

Proof: Since the equilibria described in Propositions 2B, 3A and 3B do not involve an unreached information set, they satisfy the intuitive criterion. The equilibria described in Proposition 2A also satisfy the criterion. For the equilibrium in Proposition 2A, S(L) is empty because $M > 0$. Therefore $\min_{w \in BR(T,L)} u(t,L,w) = M + p_t B + (1 - p_t)M$, which is smaller than the (equilibrium) expected utility for either type B or type G when the condition (see equation (3) above) for all workers to choose unemployment is satisfied. Hence, the equilibrium in Proposition 2A satisfies the intuitive criterion.||

In other words, in the equilibrium where all workers choose unemployment, choosing an unskilled job would be profitable for any worker if employers were to believe that only type G workers chose the unskilled jobs. Consequently the intuitive criterion does not restrict firms' beliefs when they observe the out of equilibrium employment history L. Thus, firms upon observing employment history L could believe that the worker with that history is of type B. This would dissuade workers from choosing L.

Proposition 5: The necessary and sufficient condition for the (full employment) sequential equilibrium in Proposition 1 to fail to satisfy the intuitive criterion is

$$(16) \qquad p_B < \frac{M}{G - \omega} < p_G .$$

Proof: We first show that (16) is sufficient. (16) can be written as

$$(17) \qquad p_B\, G + (1 - p_B)\, M < M + p_B\, \omega + (1 - p_B)\, M \qquad \text{and}$$

$$(18) \qquad p_G\, G + (1 - p_G)\, M > M + p_G\, \omega + (1 - p_G)\, M \quad .$$

(17) implies that $B \in S(U)$, while (18) implies that $G \notin S(U)$. Therefore $T\backslash S(U) = \{G\}$. (18) consequently implies that the full employment equilibrium fails to satisfy the intuitive criterion.

To show (16) is necessary, first note that $p_G > p_B$. If $p_B \geq M/(G - \omega)$, then $S(U)$ would be empty and $B \in BR(T\backslash S(U),U)$, which would deter a worker from deviating to L. If $p_G \leq M/(G - \omega)$, then a type G worker would not benefit from choosing unemployment even when correctly identified.||

When (16) is true, irrespective of firms' beliefs when they observe U, a type B worker is worse off choosing U than he was in the full employment equilibrium. However, if by choosing unemployment a type G worker can convince firms that he is indeed type G, he will obtain strictly more than the equilibrium utility. Hence when (16) is true, the full employment equilibrium fails to satisfy the intuitive criterion.

Propositions 4 and 5 together imply that when (16) holds, all equilibria that satisfy the intuitive criterion entail unemployment.

5. Taxes and Subsidies

We now show that there are tax-subsidy schemes that eliminate all equilibria with unemployment, and that induce full employment as the unique sequential equilibrium. The unique full employment equilibrium also satisfies the intuitive criterion.

Let $t(W)$ be a tax (or subsidy) paid by a worker who receives a wage W from employment.[13]

Proposition 6: Let the government choose

$$t(W) = \begin{cases} 0 & W < B \\ W - (\omega - \epsilon) & B \le W < \omega \\ W - \omega & \omega \le W, \end{cases}$$

where $M/\epsilon > p_G$, then full employment is the unique sequential equilibrium outcome. It satisfies the intuitive criterion, and equilibrium taxes and subsides are zero. (See Figure 2 for an illustration of a worker's after-tax income.)

Proof: The tax-subsidy scheme is obviously budget balancing if all workers choose L in period 1 and firms choose $W(L) = \omega$ in period 2, since taxes are zero.

We now show that full employment is the unique sequential equilibrium under this tax scheme. Given the tax-subsidy scheme defined above, suppose a type t worker chooses U in period 1. The highest possible (after-tax) lifetime income for this worker is $p_t\, \omega + (1 - p_t)M$. However, if this worker

[13] In our two period model, the government can actually introduce richer tax-subsidy schemes. In particular, taxes can also be made dependent on whether a worker is employed in the skilled sector or the unskilled sector, or was previously unemployed. Moreover, the tax scheme in period 2 may be different from the tax schedule in period 3. However, we do not need to use such tax schemes. In practice it might be difficult to implement taxes that are a function of a workers' previous employment history. It could be even less practical to condition taxes on whether a worker was currently employed in the skilled or unskilled sectors. Hence, we only consider tax schemes that are functions of workers' wages. We also restrict our attention to tax schemes that make after-tax income a non-decreasing function of wages.

chooses the unskilled job L in period 1, his lowest possible total utility is $M + p_t(\omega - \epsilon) + (1 - p_t)M$. Because ϵ is chosen to ensure that $M/\epsilon > p_G$ ($> p_B$), all workers are strictly better off choosing L in period 2. Given that all workers choose L in period 2, employers in the skilled sector offer a wage of ω conditional on a worker with an employment history L passing the test. The equilibrium satisfies the intuitive criterion. Regardless of the wage offer from a firm in the skilled sector in the second period, a worker can never benefit from deviating from L to U in the first period.||

The intuition for the above proposition is simple. The tax scheme is intended to eliminate the effect of workers' private information on wages offered by firms. A worker's maximal period 3 gain from taking a costly period 2 action (unemployment) is ϵp_t, which has been so chosen that it is always less than the benefit M from accepting employment in the unskilled sector in period 2. Without the need to signal his private information, a worker does not want to become unemployed. One surprising feature of this result is that equilibrium taxes and subsidies are zero. To eliminate unemployment, it is sufficient to eliminate the potential gain from unemployment (the high wage G). This result is sensitive to the assumption of $M < B$. In the case where $M > B$, if the government cannot impose sector specific tax schedule or make taxes a function of past employment histories, then the tax schedule needed to achieve full employment could entail nonzero taxes and subsidies in equilibrium. See Appendix A.

While the tax-subsidy schedule outlined in Proposition 6 eliminates all unemployment equilibria,[14] it does not guarantee that type G workers will (ex ante) benefit from the change. Suppose the economy has initially settled down in one of the equilibria that involves (some) unemployment. Type G workers may become worse off after the government has imposed the tax scheme described in

[14] Notice that the tax scheme used in Proposition 6 eliminates unemployment irrespective of the inefficient equilibria the economy starts from.

Proposition 6. This is because some type G workers may initially be earning a high wage G in period 3 (see Proposition 2B for example). After the tax scheme has been imposed, they only obtain ω in period 2. The gain in income M in period 2 may not be sufficient to offset this decrease. We now investigate whether there exist tax schemes that induce a Pareto improvement for each type of workers.[15]

There exists a balanced budget tax schedule that induces Pareto improvements (as well as eliminating unemployment) when the initial equilibrium is either a symmetric equilibrium (such as all workers choosing U (Proposition 2A), or all type B workers choosing L and all type G workers choosing U (Proposition 2B)), or the asymmetric one in which type B workers choose L, $\tilde{\gamma}$ of type G workers choose L and $(1 - \tilde{\gamma})$ type G workers choose U (Proposition 3B). We provide a necessary and sufficient condition for there to exist tax schedules that induce Pareto improvements if the initial equilibrium is the one described in Proposition 3A.

Proposition 7: Suppose the initial equilibrium is described by either Proposition 2A or Proposition 3B, if the tax schedule $t(\cdot)$ constructed in Proposition 6 is imposed, then all workers have higher (expected) equilibrium incomes.

Proof: First, suppose that the initial equilibrium of the economy is described by Proposition 2A, i.e., all workers choose U in period 2. In this equilibrium, a worker of type t receives expected income $p_t \omega + (1 - p_t) M$. After $t(\cdot)$ is imposed, a type t workers' equilibrium expected income is $M + p_t \omega + (1 - p_t) M$. Hence every type of worker becomes strictly better off.

Second, suppose that the initial equilibrium of the economy is described by Proposition 3B, i.e., all type B workers choose L, $\tilde{\gamma}$ of type G workers choose L and $(1 - \tilde{\gamma})$ of type G workers choose U in period 2. In this

[15]It is too much to hope for tax-subsidy schemes that would make every worker better off, ex post. In particular our criterion for a "Pareto improvement" only requires that the expected income of each type of worker be increased; not that workers who later pass the test—but who only knew their probability of passing at the time the taxes were levied—be made better off.

equilibrium, because type G workers are indifferent between L and U, each type t has expected income $M + p_t \tilde{\omega}(\tilde{\gamma}) + (1 - p_t)M$. Since $\tilde{\omega}(\tilde{\gamma}) < \omega$, every type of worker becomes strictly better off when the tax scheme $t(\cdot)$ is imposed.||

If the initial equilibrium is one described by Proposition 2B, then the tax-subsidy scheme used in Proposition 6 may not be sufficient to induce a Pareto improvement. (See the argument preceding Proposition 7.) The next proposition provides conditions under which the tax scheme used in Proposition 6 induces a Pareto improvement.

Proposition 8A: Suppose the initial equilibrium is described by Proposition 2B and $M \geq p_G (G - \omega)$. Then if the $t(W)$ schedule constructed in Proposition 6 is imposed, all workers will have higher (expected) equilibrium incomes.

Proof: In the equilibrium in Proposition 2B, type G workers choose U and type B workers choose L. Their equilibrium incomes are given by $p_G G + (1 - p_G)M$ and $M + p_B B + (1 - p_B)M$ respectively. Under $t(\cdot)$, the equilibrium taxes and subsidies are zero. Clearly, type B workers are better off; they gain by $p_B(\omega - B) > 0$. The difference between the equilibrium expected incomes for type G workers is $M - p_G(G - \omega)$, which is positive under the hypothesis of the proposition.||

The next proposition relaxes the assumption of $M \geq p_G (G - \omega)$. In this case, if the initial equilibrium is described by Proposition 2B, the government has to introduce a somewhat more complicated tax-subsidy scheme to induce a Pareto improvement. Equilibrium taxes and subsidies may have to be non-zero.

Proposition 8B: Suppose the initial equilibrium is described by Proposition 2B. There exists a budget-balanced tax-subsidy scheme under which the unique sequential equilibrium is the full employment equilibrium, and all workers have higher expected incomes. Moreover the equilibrium satisfies the Intuitive Criterion.

Proof: See Appendix B. ||

We next investigate the (asymmetric) equilibrium in Proposition 3A, where type G workers always choose U and a fraction of type B workers choose L. It turns out that we cannot always find a balanced budget tax scheme which makes each type of workers better off. First we state a more positive result.

Proposition 9A: Suppose the initial equilibrium is described by Proposition 3A and $M \geq p_G\ (\hat{\omega}(\hat{\beta}) - \omega)$. Then if $t(\cdot)$ as defined by that in Proposition 6 is imposed, full employment is the unique sequential equilibrium, and all workers will have higher (expected) equilibrium incomes.

The proof of Proposition 9A is omitted since it is similar to that in Proposition 8A.

Proposition 9B: Suppose the initial equilibrium is described by Proposition 3A. A necessary and sufficient condition for there to exist a tax scheme that induces full employment as the unique sequential equilibrium, and that generates higher expected incomes for all workers is

$$(19) \qquad -\frac{\hat{\omega}(\hat{\beta}) - \omega}{M} + \frac{2}{\alpha\ p_G + (1-\alpha)\ p_B} - \frac{1}{p_G} \geq 0 .$$

Proof: See Appendix B. ||

6. Job Applications upon Entry

In the model analyzed so far, a worker is not allowed to apply for jobs in the skilled sector in period 2 (the time they enter the labor market). This may not be a realistic assumption. However, it does make the analysis of the model and the intuition easier to understand. In this section, we relax this assumption. Here, we assume that workers obtain messages about their types in period 1, and then enter the labor market in period 2. In this period, they apply for jobs in the skilled sector. Firms evaluate workers using a screening test; the probability that a type t worker passes the test (qualifies for the job) is, as before, p_t, and test results are known by at least two firms. If a worker passes the test, he can choose between working in the skilled sector (J), working in the unskilled sector (L), or becoming unemployed (U). If a worker does not pass the test, then he can only choose between L and U. We assume that if a worker gets a skilled job in the second period, he keeps that job for at least two periods, i.e., he will not apply for a different skilled job in the third period.

In period 3, workers who have not been employed in the skilled sector are evaluated by firms. For simplicity, we suppose that in each period the probability that a type t worker passes the test is again p_t; the tests that firms use to screen workers are the same in each period, but conditional on the worker's type, test results are uncorrelated across periods. If a worker passes the test, he may work in the skilled sector. Otherwise he works in the unskilled sector. Again, test results are known to at least two firms, and firms compete for workers.

In period 2, a firm (in the skilled sector) simply offers a wage to workers who are qualified. Since test results in this period are known to firms, and firms compete with each other, in equilibrium, qualified workers must be paid their expected productivity. In period 3, a firm observes an applicant's employment history, which states whether he has been employed in the unskilled sector in period 2. Then firms offer wages to qualified workers based on their employment histories.[16]

In this game, we can distinguish four "classes" of workers: type G workers who pass the test in period 2, type G workers who fail the test in period 2, type B workers who pass the test in period 2, and type B workers who fail the test in period 2. Any worker who is still in the market in period 3

[16] Here we also assume that test results in period 2 are not observable to firms in period 3. Otherwise, in general, firms may want to condition period 3 wage offers on both employment histories and period 2 test results. This will be true if type G and type B workers who pass (in period 2) choose differently in period 2. For example, suppose a type G worker who passes chooses L, while a type B worker who passes chooses J. Then in period 3, a firm will infer that a worker who is still in the labor market and has passed the test in period 2 must be of type G.

will accept a job in the skilled sector whenever he is qualified; this is because B > M, i.e., the minimum wage this worker obtains in the skilled sector is higher than the wage in the unskilled sector. A worker who passes firms' evaluation tests in period 2 faces a somewhat more complicated decision. He can accept a job in the skilled sector immediately; this is optimal if (expected) future wages (possibly conditional on employment history in period 2) are not attractive. However, if by becoming unemployed (U) in period 2, he can successfully signal that he is a worker with a good message, it may be optimal for him to turn down the job offer in period 2.

Not surprisingly, the model can generate many sequential equilibria.[17] We will only consider symmetric equilibria in which workers in the same "class" choose the same action; there are eight of these.[18] Since any worker still in the market in period 3 accepts a job in the skilled sector if he passes the qualifying test, and works in the unskilled sector otherwise, all equilibria can be characterized by workers' choices in period 2. The strategy combination of workers will be represented by a four dimensional vector. The vector (W,X;Y,Z) means that type G workers who have passed (resp. failed) in period 2 choose W (resp X), and type B workers who have passed (resp. failed) in period 2 choose Y (resp. Z).

First, we observe that for all parameter values, (J,L;J,L) is a sequential equilibrium. This is a full employment equilibrium, and (if B > M) it maximizes (expected) output. It is never incentive compatible for a qualified type B worker to choose either U or L, when it is optimal for a qualified type G worker to choose J. Therefore, there are only two other possible equilibria in which workers who are qualified accept jobs in period 2: (J,U;J,L) and (J,U;J,U). Together with (J,L;J,L) these are the only

[17] Notice however that many of these equilibria may not exist simultaneously.

[18] We found these by listing all possible strategy combinations and determining which are sequential equilibria.

equilibria in which type G workers who are qualified in period 2 choose J. Note that in the (J,U;J,U) equilibrium, among otherwise observationally indistinguishable workers, those with previous spells of unemployment have lower wages.

Next, we list the possible (symmetric) equilibria in which only type B workers choose J. In these equilibria, qualified and unqualified type G workers must choose the same action in period 2. This is because when qualified type G workers refuse jobs in period 2, they have the same choice set as unqualified type G workers. These equilibria are (L,L;J,L), (U,U;J,M), and (U,U;J,U). Finally, there are two completely pooling equilibria: (U,U;U,U) and (L,L;L,L).

As in the simpler model, it is again always possible to impose a balanced budget tax-subsidy scheme that eliminates unemployment, and leaves (J,L;J,L) as the unique sequential equilibrium.[19] The reason for the existence of unemployment equilibria is that there may be some (future) gain for good workers when they signal their information by becoming unemployed. Hence, to eliminate unemployment, it is sufficient to eliminate the potential gain to signaling. Let

$$\omega_1 = \frac{\alpha\ (1 - p_G)\ p_G\ G + (1 - \alpha)\ (1 - p_B)\ p_B\ B}{\alpha\ (1 - p_G)\ p_G + (1 - \alpha)\ (1 - p_B)\ p_B} .$$

ω_1 is the expected productivity of a worker randomly chosen from among those who have failed the screening test in period 2 but have passed in period 3. Notice that in the (J,L;J,L) equilibrium, ω_1 is the wage that firms offer for skilled jobs in period 3. In this equilibrium, firms offer ω in period 2. Also, note that ω_1 is less than ω.

<u>Proposition 10</u>: Suppose the tax-subsidy scheme t(W) is given by

[19] As we have mentioned before, we only use tax-subsidy schemes that are functions of wage incomes; moreover, there is no tax imposed on unemployment.

$$t(W) = \begin{cases} W - (\sigma - \epsilon) & W < M \\ W - \sigma & M \le W < B \\ W - \tau & B \le W, \end{cases}$$

where σ and τ are chosen to satisfy $0 < \epsilon < \sigma < \tau$, $\sigma/(\tau - \sigma) > p_G$, and

$$\left[\alpha\ p_G + (1 - \alpha)\ p_B\right](\omega - \tau) + \left[\alpha\ (1 - p_G)\ p_G + (1 - \alpha)\ (1 - p_B)\ p_B\right](\omega_1 - \tau)$$
$$= \left[\alpha[1 - p_G + (1 - p_G)^2] + (1 - \alpha)[1 - p_B + (1 - p_B)^2]\right](\sigma - M)\ .$$

Then the unique sequential equilibrium is the (J,L;J,L) full employment equilibrium. Expected taxes are zero in the full employment equilibrium.

The last equation is the balanced budget constraint. It is straightforward to show that σ and τ can be chosen to satisfy all the conditions in Proposition 10. Under the tax scheme, workers' net incomes in the unskilled and skilled sectors are respectively σ and τ. Since we have chosen $\sigma < \tau$, a worker never turns down a job in the skilled sector. Now if in period 2 a worker is not qualified for the skilled sector, he chooses a job in the unskilled sector, because the maximum potential from signaling, $(\tau - \sigma)p_G$, is less than the wage he can obtain from the unskilled job, σ.

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Appendix A: $B < M$ $(< \omega)$

First, we indicate how the characterization results in Sections 3 and 4 will be modified when $B < M$ $(< \omega)$. Next, we construct tax-subsidy schemes to induce full employment.

When $B < M$, if a worker is ever offered a skilled job with a wage B, he will refuse it and accept employment in the unskilled sector. Hence, in the equilibria described by Propositions 2B, 3A and 3B, no type B worker will accept employment in the skilled sector in period 3, if they are offered less than M. For these three propositions to be valid, therefore, we need to change B to M in the necessary and sufficient conditions (resp. (6); (9) and (10); and (14)). In Proposition 2A, B is never offered by firms in equilibrium. Nevertheless, for the proposition to be valid, we also need to change B to M in the necessary and sufficient condition (3). The proofs of these propositions when $B < M$ are immediate. Proposition 1 in Section 3, and Propositions 4 and 5 in Section 4 remain true as stated if $B < M$.

Proposition 6+ characterizes the taxes and subsidies needed to eliminate unemployment when $B < M$. In general, these taxes and subsidies will be non-zero. We continue to assume that the government cannot tax unemployment, impose sector specific tax schedules or make taxes a function of past employment histories.

Proposition 6+: If $B < M$, let

$$t(W) = \begin{cases} W - (\sigma - \epsilon) & W < M \\ W - \sigma & M \le W < \omega \\ W - \tau & \omega \le W, \end{cases}$$

where σ and τ are chosen to satisfy $0 < \epsilon < \sigma < \tau$, $\sigma/(\tau - \sigma) > p_G$, and

(A.1) $\quad A\,(\omega - \tau) = (2 - A)(\sigma - M), \quad$ with $\quad A = \alpha\, p_G + (1 - \alpha)\, p_B$.

Then the unique sequential equilibrium is the full employment equilibrium. The equilibrium also satisfies the intuitive criterion. Expected taxes are zero in the full employment equilibrium. (See Figure 3 for an illustration of a worker's after-tax income.)

Proof: It is easy to show that there are σ and τ that satisfy the requirements stated in the proposition. (Figure 4 illustrates the possible σ's and τ's.) If the equilibrium is the full employment equilibrium, then budget balancing holds. To see this, note that the total taxes in the second period is $M - \sigma$. In the third period, a fraction A of all workers are employed by firms in the skilled sector. Hence, the total tax they pay is $A(\omega - \tau)$. The remaining workers are then hired by firms in the unskilled sector, and their taxes are $(1 - A)(M - \sigma)$. (A.1) then says total expected taxes are zero.

The proof that full employment is the unique sequential equilibrium, and that it satisfies the intuitive criterion closely follows that of Proposition 6 above.||

Note that in Proposition 6+, equilibrium taxes and subsidies are in general nonzero. But for some parameter values of the model, there are tax (subsidy) schedules that eliminate unemployment without taxes being actually collected. If $M > p_G(\omega - M)$, then the following tax schedule[20] will eliminate unemployment, and no taxes are collected in equilibrium:

$$t(W) = \begin{cases} 0 & W < \omega \\ W - \omega & \omega \le W. \end{cases}$$

The reason that this works is that when any income above ω is taxed away, and income below ω is neither taxed nor subsidized, the maximum gain for a type G for choosing unemployment is $(\omega - M)p_G$. But when $M > p_G(\omega - M)$, this potential gain is less than what he can obtain by choosing an unskilled job.

[20] This tax schedule may be regarded as a special case of the one used in Proposition 6+ with $\sigma = M$ and $\tau = \omega$.

When $B < M$, another issue must be considered: the full employment outcome may not be the one that maximizes total (expected) output. In period 3 of the full employment equilibrium, some type B workers are employed by firms in the skilled sector. But their productivity in that sector is lower than that in the unskilled sector. Consider the equilibrium in Proposition 2B, i.e., in the second period, all type G workers choose U and all type B workers choose L. Then none of the type B workers will ever work in the skilled sector in the third period: in the separating equilibrium, type B workers "reveal" themselves by choosing an unskilled job and will be offered a wage B in the third period if they pass the hiring test. But since their productivity is higher in the unskilled sector, they never accept such job offers. From a social point of view, this means that in the third period, all workers are allocated to jobs in which they have the highest productivities. Of course, in the separating equilibrium, type G workers are unemployed in the second period. When the social loss due to the unemployment of type G workers is smaller than the gain in efficient allocation in the third period, the total (expected) output in the separating equilibrium is bigger than in the full employment equilibrium. In other words, the allocation resulting from the separating equilibrium is a "constrained" optimum.

In particular if $\alpha M < (1 - \alpha)p_B(M - B)$ then the gain in total output from employing all type G workers in period 2 will be less than the loss in output from employing a fraction p_B of the type B workers in skilled jobs in period 3. From Propositions 1 and 5, we can see that a full employment equilibrium satisfying the intuitive criterion may exist even if output would be higher under a separating equilibrium in which some (or all) of the type G workers choose unemployment.

Appendix B: Proof of Propositions 8B and 9B

Proof of Proposition 8B:

We will show that there are tax schedules under which full employment is the unique equilibrium outcome, and each type of worker has a higher expected equilibrium life-time income than in the equilibrium where type G workers choose U and type B workers choose L in period 2 (i.e. the equilibrium in Proposition 2B).

Consider the following tax schedule:

$$t(W) = \begin{cases} W - (\sigma - \epsilon) & W < M \\ W - \sigma & M \le W < B \\ W - (\tau - \epsilon) & B \le W < \omega \\ W - \tau & \omega \le W. \end{cases}$$

where

$$\text{(B.1)} \qquad \tau > \sigma > 0 \ , \quad \text{and}$$

$$\text{(B.2)} \qquad \sigma > \epsilon \, p_G \ .$$

(See Figure 5 for the after tax income schedule.) (B.1) guarantees that a worker prefers an unskilled job to unemployment, and that a worker who has passed the evaluation test prefers the skilled job to an unskilled job. We shall prove that there are suitable choices for σ, τ, and ϵ such that the proposition is true.

Any tax schedule satisfying (B.1) and (B.2) generate full employment as the unique sequential equilibrium. The argument is identical to the one used in Proposition 6.

Second, if expected taxes are zero, then we must have

$$\text{(B.3)} \qquad A\,(\omega - \tau) = (2 - A)(\sigma - M), \qquad \text{with} \quad A = \alpha\, p_G + (1 - \alpha)\, p_B \ .$$

(B.3) is exactly the balanced budget condition that we have used in Proposition 6+ in Appendix A.

Under the proposed tax schedule, in the full employment equilibrium, a type t worker has an expected life-time income of $\sigma + p_t \tau + (1 - p_t) \sigma$. From Proposition 2B, $p_G G + (1 - p_G) M$ and $M + p_B B + (1 - p_B) M$ are the expected incomes of type G and type B workers respectively, in the separating equilibrium. For the tax scheme to induce a Pareto improvement, the differences between expected incomes for the two types of workers in the full employment equilibrium under the tax scheme and expected incomes in the separating equilibrium must be positive:

$$\text{(B.4)} \qquad \sigma + p_G \tau + (1 - p_G) \sigma - p_G G - (1 - p_G) M \geq 0$$

$$\text{(B.5)} \qquad \sigma + p_B \tau + (1 - p_B) \sigma - M - p_B B - (1 - p_B) M \geq 0.$$

The proposition is proved if we can find σ, τ and ϵ to satisfy (B.1) to (B.5). However, ϵ only appears in (B.2), therefore, for any σ, one can always find ϵ such that (B.2) is true. So our task reduces to finding σ and τ to satisfy (B.1) and (B.3) to (B.5).

Since (B.3) is a linear equation, any choice of σ also determines the value of τ. Hence, we only need to consider appropriate choices of σ. In σ,τ space, (B.3) is a downward sloping straight line intersecting the positive quadrant. It is easily verified that (B.3) intersects the 45° line (in σ,τ space) at $\sigma^* = [A\,\omega + (2 - A)\,M]/2$. Therefore the choice of σ that satisfies (B.1) and (B.3) must lie in the interval $[0,\sigma^*]$. See Figure 6.

From (B.3) we obtain

$$\text{(B.6)} \qquad \tau = \frac{A\,\omega + (2 - A)\,M}{A} - \frac{2 - A}{A}\,\sigma .$$

Substituting (B.6) to (B.4) and (B.5) and defining the LHS's of (B.4) and (B.5) by $G(\sigma)$ and $B(\sigma)$ respectively, we get (after simplification)

(B.7) $$G(\sigma) = \frac{2\sigma(1-\alpha)(p_B - p_G)}{A} + p_G(\omega - G) + \frac{M}{A}\,[p_G + (1-\alpha)(p_G - p_B)]$$

(B.8) $$B(\sigma) = \frac{2\sigma\alpha(p_G - p_B)}{A} + p_B(\omega - B) - \frac{2M\alpha(p_G - p_B)}{A} .$$

Notice that $G(\sigma)$ is decreasing in σ and $B(\sigma)$ is increasing in σ. We next find σ^G and σ^B such that $G(\sigma^G) = 0$ and $B(\sigma^B) = 0$. That is, we put (B.7) and (B.8) to zero and solve for the values of σ:

$$\sigma^G = \frac{p_G(\omega - G)A + M\,p_G}{2(1-\alpha)(p_G - p_B)} + \frac{M}{2} \qquad \sigma^B = M - \frac{p_B(\omega - B)A}{2\alpha(p_G - p_B)} .$$

Since $G(\cdot)$ is a decreasing function, from the definition of σ^G, it follows that $G(\sigma) > 0$ if and only if $\sigma < \sigma^G$. Similarly, since $B(\sigma)$ is an increasing function, from the definition of σ^B, it follows that $B(\sigma) > 0$ if and only if $\sigma > \sigma^B$. We now want to show that $\sigma^B < \sigma^*$, $\sigma^B < \sigma^G$, and $\sigma^G > 0$, which says that the intervals $[0,\sigma^*]$ and $[\sigma^B,\sigma^G]$ overlap. Since any σ in $[\sigma^B,\sigma^G]$ makes each type of workers better off, and any σ in $[0,\sigma^*]$ is sufficient to induce full employment as the unique equilibrium, any element common to both intervals will achieve the desired result.

We compute the difference between σ^* and σ^B. Using their definitions and after simplification, we get

$$\sigma^* - \sigma^B = \frac{A}{2}\,(\omega - M) + \frac{p_B(\omega - B)A}{2\alpha(p_G - p_B)} > 0 .$$

Then we compute $\sigma^G - \sigma^B$:

$$\sigma^G - \sigma^B = \frac{A\,M}{2(1-\alpha)(p_G - p_B)} > 0 .$$

Finally, we show that σ^G is positive. It is somewhat more convenient to consider

$$\frac{\sigma^G(1-\alpha)(p_G - p_B)2}{p_G\,M\,A} = K .$$

Obviously K has the same sign as σ^G. From the definition of σ^G, we obtain

$$K = \frac{\omega - G}{M} + \frac{1}{A} + \frac{(1-\alpha)(p_G - p_B)}{p_G \, A} \quad .$$

We use the definitions of A and ω to simplify the above to

$$K = \frac{2}{A} - \frac{1}{p_G} - \frac{(1-\alpha)(G-B)p_B}{A\,M} \quad .$$

Recall from Proposition 2B that the necessary and sufficient condition for the separating equilibrium is $p_B \le M/(G - B) \le p_G$, which implies that $p_B(G - B)/M \le 1$. Therefore

$$K \ge \frac{2}{A} - \frac{1}{p_G} - \frac{1-\alpha}{A} = \frac{1}{A} - \frac{1}{p_G} + \frac{\alpha}{A} > 0 \quad .$$

where the second inequality follows from $A < p_G$.||

Proof of Proposition 9B:

The proof of Proposition 9B closely follows that of Proposition 8B. First, we prove that (19) is necessary for there to exist a tax scheme to induce a Pareto improvement if the initial equilibrium is the one in Proposition 3A. In any tax scheme $t(\cdot)$ that induces such an outcome, let $M - t(M) = \Sigma$ and let $\omega - t(\omega) = T$. Recall that we assume that there is no tax imposed on the unemployed workers, and that the after tax income is non-decreasing. For full employment to be an equilibrium,

(B.9) $\quad T > \Sigma > 0$.

The balanced budget constraint is again

(B.10) $\quad A\,(\omega - T) = (2 - A)(\Sigma - M), \qquad \text{with} \quad A = \alpha\, p_G + (1-\alpha)\, p_B$.

(B.9) and (B.10) are identical to (B.1) and (B.3).

In the equilibrium described in Proposition 3A, a fraction $1 - \hat{\beta}$ of type B workers choose L and all type G workers choose U. Suppose the tax scheme induces a Pareto improvement, then

$$(B.11)\qquad \Sigma + p_G T + (1 - p_G)\Sigma - p_G \hat{\omega} - (1 - p_G) M \geq 0$$

$$(B.12)\qquad \Sigma + p_B T + (1 - p_B)\Sigma - M - p_B B - (1 - p_B) M \geq 0.$$

In (B.11), $\hat{\omega} = \hat{\omega}(\hat{\beta})$; this is the wage that a type G gets if he is hired by a firm in the skilled sector (see (10) in Section 3.) We show that (B.9)-(B.12) imply (19). As in the proof of Proposition 8B, we use (B.10) to express T in terms of Σ. (B.9) and (B.10) together imply that the feasible choices of Σ must be in the interval $[0,\Sigma^*]$, where $\Sigma^* = \sigma^*$ (see Figure 6). Then we substitute (B.10) into (B.11) and (B.12) to eliminate T, and obtain

$$(B.13)\qquad G(\Sigma) = \frac{2\Sigma(1 - \alpha)(p_B - p_G)}{A} + p_G(\omega - \hat{\omega}) + \frac{M}{A}[p_G + (1 - \alpha)(p_G - p_B)]$$

$$(B.14)\qquad B(\Sigma) = \frac{2\Sigma\alpha(p_G - p_B)}{A} + p_B(\omega - B) - \frac{2M\alpha(p_G - p_B)}{A} \quad .$$

We next find Σ^G and Σ^B such that $G(\Sigma^G) = 0$ and $B(\Sigma^B) = 0$.

$$\Sigma^G = \frac{p_G(\omega - \hat{\omega})A + M\, p_G}{2(1 - \alpha)(p_G - p_B)} + \frac{M}{2} \qquad\qquad \Sigma^B = M - \frac{p_B(\omega - B)A}{2\alpha(p_G - p_B)} \quad .$$

It is straightforward to show that $\Sigma^B < \Sigma^G$ and $\Sigma^B < \Sigma^*$. Since $G(\Sigma)$ is decreasing in Σ and $B(\Sigma)$ is increasing in Σ, if there is a tax scheme to induce a Pareto improvement, there must be a Σ that is in both $[0,\Sigma^*]$ and $[\Sigma^B,\Sigma^G]$, which implies that $\Sigma^G \geq 0$. We now show that $\Sigma^G \geq 0$ if and only if (19) is true. Again it is somewhat more convenient to consider

$$\frac{\Sigma^G(1 - \alpha)(p_G - p_B)2}{p_G\, M\, A} = Q \ ,$$

which has the same sign of Σ^G. We simplify Q to

$$Q = \frac{\omega - \hat{\omega}}{M} + \frac{1}{A} + \frac{(1 - \alpha)(p_G - p_B)}{p_G\, A} \quad .$$

Simplifying the last two terms of the above, we have

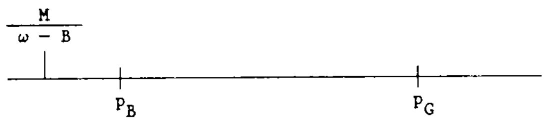
$$Q = -\frac{\hat{\omega} - \omega}{M} + \frac{2}{A} - \frac{1}{p_G} \quad .$$

Therefore Σ^G is positive if and only if the above expression is positive. Q is exactly the condition stated in the proposition.

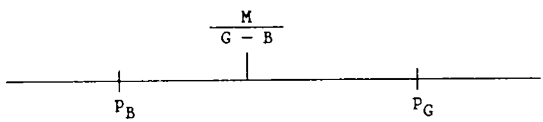
For the sufficiency part of the proposition, note that if Σ^G is positive, then one can use the tax scheme in the proof of Proposition 8B, replacing σ and τ by appropriate Σ and T. Then if the initial equilibrium is one described by Proposition 3A, the tax scheme will eliminate unemployment and induce a Pareto improvement.||

Figure 1

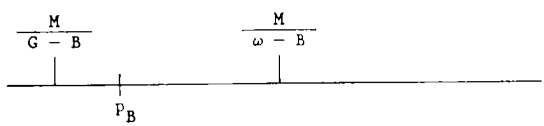
Proposition 2A: All workers choose U.



Proposition 2B: Type G workers choose U. Type B workers choose L.



Proposition 3A: Type G workers choose U. $\hat{\beta}$ of all type B workers choose U and $(1 - \hat{\beta})$ of all type B workers choose L.



Proposition 3B: $\tilde{\gamma}$ of all type G workers choose L and $(1 - \tilde{\gamma})$ of all type G workers choose U. Type B workers choose L.

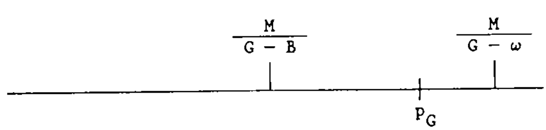


Figure 2

Workers' net income under tax scheme.

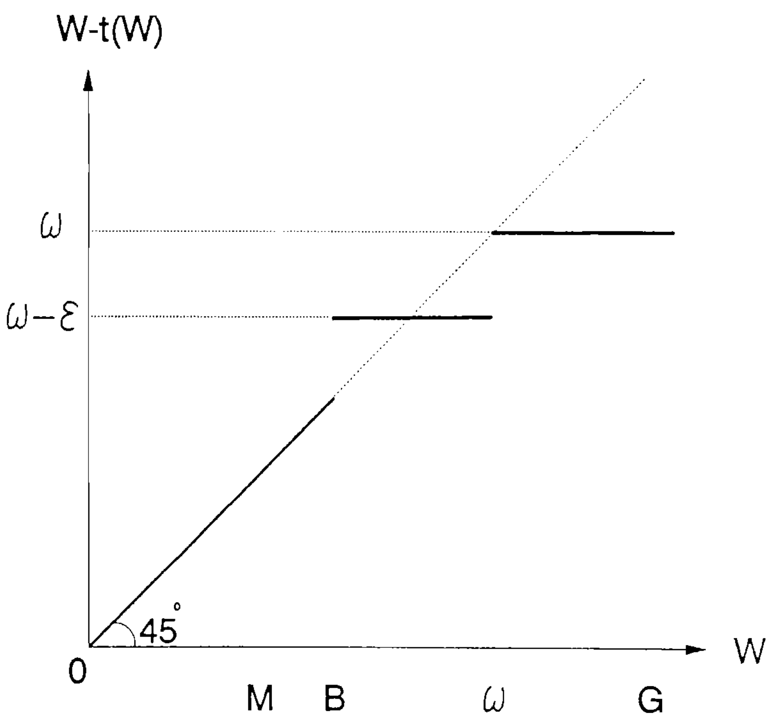


Figure 3

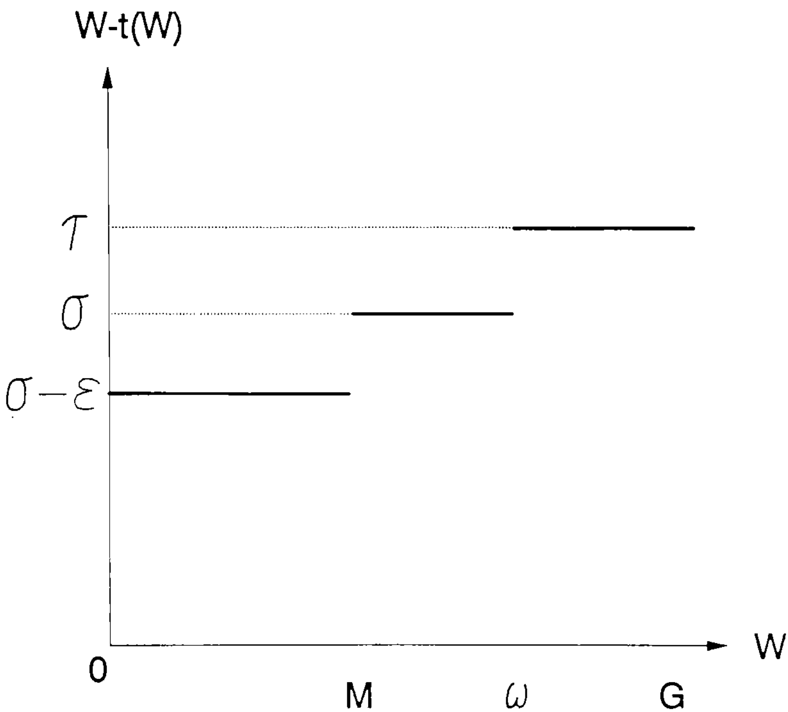


Figure 4

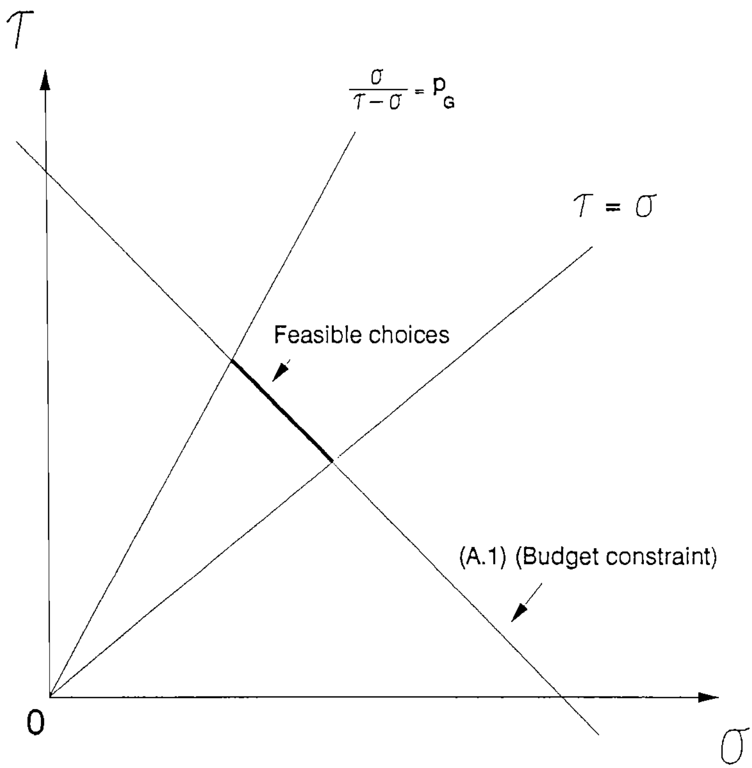

τ
$\frac{\sigma}{\tau - \sigma} = p_G$
$\tau = \sigma$
Feasible choices
(A.1) (Budget constraint)
0
σ

Figure 5

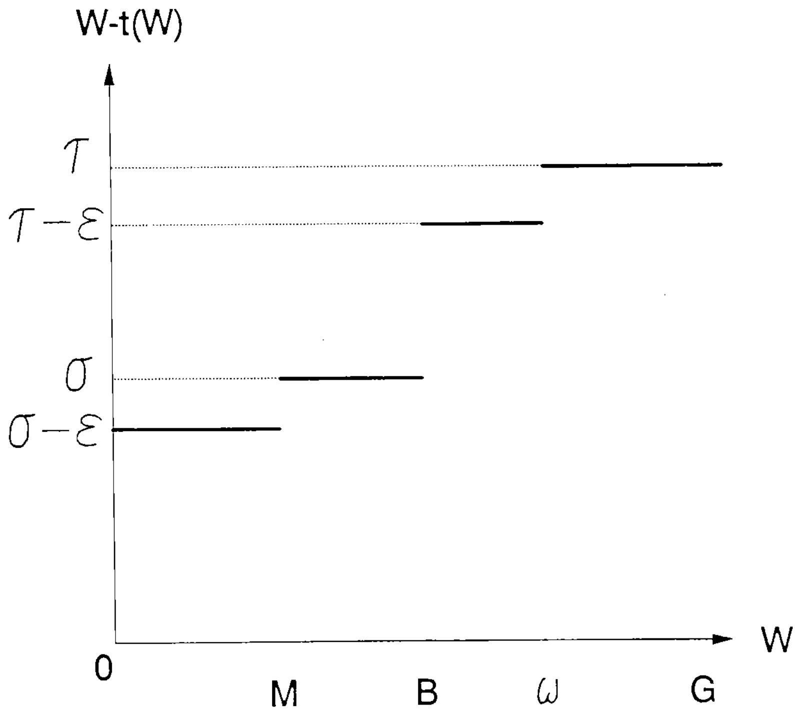
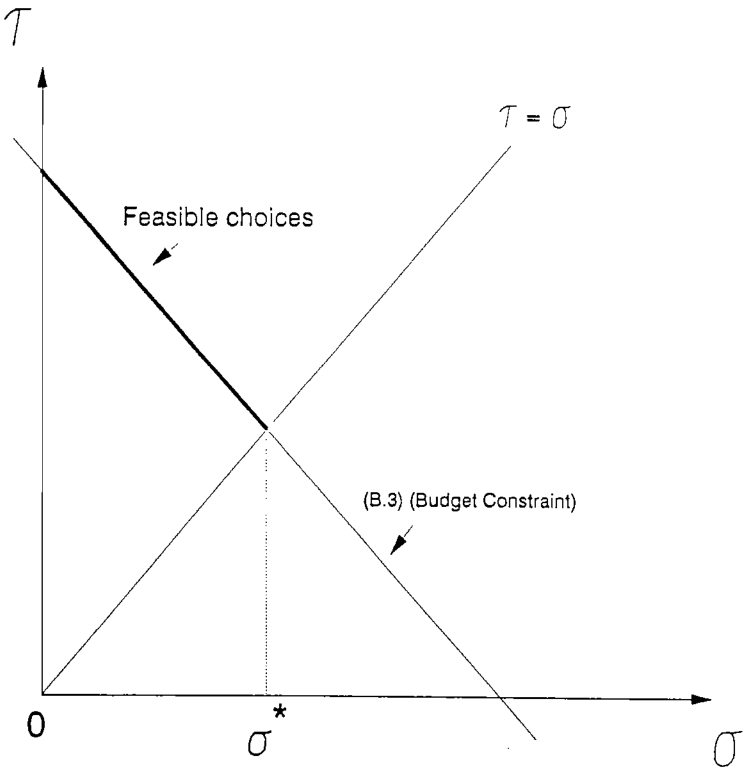


Figure 6



τ
$\tau = \sigma$
Feasible choices
(B.3) (Budget Constraint)
0
σ^*
σ