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TOPOLOGY OPTIMIZATION OF GAS TURBINE BLADES

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ABSTRACT

Unprecedented levels of manufacturable complexity can be achieved by new additive manufacturing technologies. Traditional design methods are being superseded by automated design methods, such as Topology Optimization, that can fully exploit these new capabilities. The paper presents the structural optimization of two existing turbine blade designs and a performance assessment of an open-source topology optimization algorithm. The low-pressure T106C and high-pressure C3X (cooled) turbine blade models were re-designed to minimize the material volume. The Solid Isotropic Material with Penalization (SIMP) and Sequential Element Rejections and Admissions (SERA) methods were used and compared. The SIMP method was applied with the commercial optimization code in the Abaqus Topology Optimization Module and the SERA method with an open-source optimization code. The open-source code was modified for implementing geometric constraints, extending its utility to the structural optimization of structures with multi-disciplinary design considerations. The resulting optimal geometries from SIMP and SERA were predominantly similar, revealing hollow, thin-walled blade topologies with axially distributed material reinforcements at the leading edge and suction side, the locations of maximum bending and tensile stresses. Despite this similarity, considerable differences in the intricacy and complexity of the designs were observed. The SIMP-derived topologies were characterized by smooth boundaries and an even material distribution across the camber line. The SERA-derived topologies were composed of intricate features with unrealistic connectivity, reflecting the finite element discretization applied on the original geometry. The SERA method was demonstrated to be less computationally efficient, consistently requiring about 2.6 times longer than SIMP to achieve convergence but produced better-performing geometries, with 24.7% and 37.9% lower objective functions for the T106C and C3X geometries respectively. The effect of Laplacian mesh smoothing applied by the ATOM SIMP optimizer on the performance and optimal design is believed to play a major role and therefore commented in the paper.

INTRODUCTION

The rapid development of Additive Manufacturing (AM) technologies over the past 30 years enabled levels of complexity in the manufacture of structural components never achieved before. The gap between the increased manufacturing flexibility and traditional design methods highlights the necessity of considering a new design perspective. The potential of automated design methodologies makes them effective design tools, capable of designing optimally loaded, intricate components to manufacturable standard, while fully exploiting the current manufacturing capability.

Structural design techniques based on numerical optimization have been developed since the 1970s and have evolved from Size Optimization to Shape Optimization and, most recently, to Topology Optimization (TO). Topology Optimization methods are the current state-of-the-art of automated design methods, capable of determining the solid material distribution in space that optimally satisfies boundary conditions and design constraints without the need for an a priori definition of structural shape.

Such techniques are highly valuable in high-performance engineering applications where the minimization of structural mass is critical in the pursuit of efficiency and performance. Their successful utilization in the aerospace industry can drive significant weight-saving and reduced design times by producing optimal designs with high structural efficiency to manufacturable standards.

A high structural efficiency – the specific load-carrying capacity of a structure - is of paramount importance in the design of high-performance components, as all solid regions are effectively utilized in the process of load transmission. Such structures are

particularly important to the aerospace industry, where the minimization of weight – and consequently fuel consumption – is critical to ensure compliance with strict emissions regulations.

In this study, established topology optimization methodologies are used in the design of lightweight versions of existing gas turbine blade models. Two different blade designs were considered: the high-pressure cooled NASA C3X and the low-pressure T106C turbine rotor blades. Each blade model was optimized using the Abaqus Topology Optimization Module (ATOM) with the Solid Isotropic Material with Penalization (SIMP) method and with an open-source Sequential Element Rejections and Admissions (SERA) optimization algorithm. The results were used to determine the relative performance of the two methods and the validity of the open-source code, with ATOM considered as a benchmark algorithm due to its wide acceptance.

TOPOLOGY OPTIMIZATION TECHNIQUES

Structural Optimization is the application of constrained mathematical optimization methods to the design of structurally efficient solid structures. It involves the definition of an objective function to be maximized or minimized and a set of design constraints, typically restrictions on the maximum volume of available material.

In this study, two types of topology optimization methodologies were used: Solid Isotropic Material with Penalization (SIMP) and Sequential Element Rejections and Admissions (SERA).

The **Solid Isotropic Material with Penalization (SIMP)** method is a subset of the Density-based family of TO methods, which employ a continuous design function to quantify the stiffness tensor in space. Originally proposed by Bendsøe in 1989 [1] and later by Zhou and Rozvany [2] the SIMP method introduces a penalization factor p (where $p > 1$) and formulates the material stiffness tensor as shown in Equation 1.

$$E_{ijkl}(x) = [\rho(x)]^p \bar{E}_{ijkl} \quad (1)$$

$\bar{E}_{ijkl}$ is the fourth-order stiffness tensor of the solid material and $\rho(x)$ is the continuous design variable, known as artificial density ($0 < \rho(x) \leq 1$). The positive penalization factor steers the stiffness of intermediate density elements ($0 < \rho(x) < 1$) towards zero, making them structurally inefficient and consequently rejected by the optimization algorithm. The solution is thus directed into a predominantly discrete (0-1) topology with minimal ‘grey regions’ of intermediate density. Subsequent to the 1999 study of Bendsøe and Sigmund [3], a penalization value of 3 has been considered ideal for obtaining discrete topologies without premature convergence.

The **Sequential Element Rejections and Admissions (SERA)** method belongs to the Biologically inspired family of TO methods, where a structural performance threshold is used to determine the retention and rejection of solid elements. It was first proposed by Xie and Steven [4] and named ‘Evolutionary Structural Optimization’, a term which was criticized in literature because, in the field of computational optimization, the label ‘Evolutionary’ is used to denote population-based solvers such as Genetic Algorithms. In this paper, the authors considered appropriate to refer to the method as SERA, as originally suggested by Rozvany [5].

The method is based on the rejection and admission criteria, determined by the structural performance of each element relative to a pre-determined threshold. Elements not meeting the threshold are assigned a low-magnitude elasticity modulus and those that do are assigned the solid elasticity modulus. For example, using a Von-Mises stress criterion the rejection and admission criteria would be formulated as shown in Equations 2 and 3 respectively.

$$\sigma_i^{VM} < RR \cdot \sigma_{max}^{VM} \quad (2)$$

$$\sigma_i^{VM} > IR \cdot \sigma_{max}^{VM} \quad (3)$$

RR and IR denote the *Rejection Ratio* and *Inclusion Ratio* respectively. The thresholds are shifted by updating RR and IR and the process is repeated until a global structural performance target, such as elemental stress level or specific stiffness, is achieved. In this study, the open-source optimization algorithm proposed by Zuo and Xie [6] is used.

METHODOLOGY

In this study, *compliance* was used as the objective function. As it is a measure of the elastic strain energy stored in the material by its force-induced displacement, minimizing its value maximizes the stiffness of the structure. The complete formulation of the optimization problem is illustrated in Equation 4.

$$\begin{aligned}
\min: C(\mathbf{X}) &= \frac{1}{2} \mathbf{F}^T \mathbf{U} = \frac{1}{2} \mathbf{U}^T \mathbf{K} \mathbf{U} \\
s. t: \mathbf{X} &= x_i, \quad x_i = [x_{min}, 1] \quad i = 1, \dots, N \\
\mathbf{F} &= \mathbf{K} \mathbf{U} \\
V(\mathbf{X}) &= \sum x_i v_i = V^*
\end{aligned} \tag{4}$$

In the above equation, the compliance matrix is denoted by $C(\mathbf{X})$, where $\mathbf{X}$ is a vector of the element design variables that take the value $x_i=1$ for solid elements and $x_i=x_{min}$ for void elements. $\mathbf{F} = \mathbf{K} \mathbf{U}$ represents the equilibrium constraint for linear elasticity where $\mathbf{F}$, $\mathbf{U}$ are the force and displacement vectors respectively and $\mathbf{K}$ is the stiffness matrix. $V(\mathbf{X})$ is the total volume of the structure which is defined by the available material volume V^* , where $0 < V^* < 1$.

The optimization strategy in this project was based on the definition of a target volume fraction V^* as the stop criterion for the optimization algorithm. It was chosen as such to follow the structure used by Zuo and Xie [6] when designing the optimization algorithm.

The SERA optimization algorithm uses the results of the linear elastic FEM solver to extract the strain energy density (compliance) for each computational node, as shown in Equation 4. It then computes the rate of change of this function with respect to the design variable x_i known as the design sensitivity (α). Mesh independence is achieved with a smoothing operation on the design sensitivities which computes a weighted average of the values at a computational node with those of its direct neighbours. The sensitivities are then subjected to a historical averaging step to aid convergence.

Having defined all the parameters, the code proceeds to optimise the structure using a threshold sensitivity number (α_{th}), obtained using the bisection method. In elements where $\alpha_i > \alpha_{th}$, the design variable is set as $x_i = 1$ and in all remaining elements, it is set as $x_i = 0$ (no material). The elements are then assigned to two element-sets, according to their x_i values, where they are assigned the appropriate structural properties (solid for $x_i = 1$ and void for $x_i = 0$).

TEST CASES

The T106C low-pressure turbine blade and the cooled C3X high-pressure turbine blade used in this study were modelled using the XY coordinate data of their cross-section, as given by Hylton et al. [7] The cross-sections are shown in Figure 1.

The T106C profile is characterized by a high degree of camber, with sharp leading and trailing edges and a maximum blade thickness at the 35% chord length location. It is a highly-loaded blade, designed to generate maximum lift – hence thrust - from the exhaust gases at the latter stages of the gas turbine while minimizing stagnation pressure loss.

The C3X profile is characterized by a very thick leading edge with a maximum thickness at the 5% chord length location, a sharp trailing edge and 10 circular penetrating cooling channels. Due to its position at the exit of the combustor, it is subjected to extremely high thermal loads, and the design is hence optimized for maximizing heat transfer from the blade to the cooling air.

Figure 1 Turbine Blade Cross Sections

MODEL SETUP

Take-off conditions - the maximum loading conditions - were assumed throughout the analysis, with a 12400 rpm stage angular velocity. The total blade mass was assumed to be $m_0 = 300$ g and the mean radius from the centre of rotation was set at $r_m = 0.216$ m. For this study, static loading conditions were assumed, resulting in a constant centrifugal force $F_c = 109.3$ kN. The gas pressure on the turbine blades was accounted for using the experimental values of Chana and Jones [8] of stage pressure ratio and turbine inlet pressure as a reference, resulting in a $\Delta P_s = 316$ kPa pressure difference across the blade. Figure 2 illustrates the loads and boundary conditions on the two blades. The blades are fixed at the root using an “Encastre” boundary condition, illustrated by the blue and orange arrows, and subjected to the centrifugal load and bending pressures, shown by the purple arrows.

Grade 310 Stainless Steel was used as the blade material in this study, due to the requirement of a solid isotropic material by the optimization algorithm. The material properties used are shown in Table 1.

a) T106C b) C3X
Figure 2 Boundary Conditions

Property	Value	Unit
Density	8000	kg/m ³
Elasticity Modulus	200	GPa
Poisson's Ratio	0.30	-----

TABLE 1: MATERIAL PROPERTIES OF ASTM 310 STAINLESS STEEL

COMPUTATIONAL MODEL

A dense, structured finite element mesh is necessary for the stable operation and convergence of the TO code. Linear hexahedral (C3D8R) finite elements are used to discretize both geometries, due to the high computational efficiency achieved by their structured connectivity and a reduced number of integration points. A mesh dependency study was conducted to determine the effect of mesh density on the converged objective function using the ATOM SIMP optimizer. A compliance minimization objective function was defined, and the total volume was constrained to be less than half its original value ($V_f \leq 0.5 V_0$).

Mesh dependence is reduced as the number of elements are increased above 100,000, but due to restrictions in computational resource, 50,000 elements were used in this study, as shown in Figure 4. Increasing the mesh density resulted in fundamentally identical topologies, with the predominant difference being the detailed definition of the structural boundary. Analogous results are expected using the SERA method, but further investigation is necessary for this to be confirmed.

In the design of gas turbine blades, static structural performance is only one of numerous functions to be performed by the turbine blades. Structural dynamic performance, aerodynamic and thermal functionalities are of equally paramount importance to the overall engine performance. The aerofoil shape is optimized for minimizing stagnation pressure losses across the stage. Similarly, the cooling channel dimensions in the C3X blade are optimized for maximum heat transfer from the blade to the cooling air. These performance-critical features are defined as ‘frozen regions’, shown in red in Figure 4, which cannot be altered by the optimization algorithm.

Figure 3 Mesh Dependence

The optimization code of Zuo and Xie was originally intended for designing geometrically unconstrained structures. Although a technique for implementing geometric constraints involving the introduction of an additional solid part mated to the original one to act as a geometric constrained, it was not used in this study as it introduces significant flaws to the optimization process.

The original method involved the creation of an additional solid part mated to the original, that would act as a geometrically constrained region by being completely ignored by the optimization algorithm. Such an approach is fundamentally flawed as the ‘geometrically constrained’ regions and their load-carrying capability are not taken into consideration, resulting in sub-optimal, geometrically wrong structures. Besides, the quantitative properties (objective function, volume fraction) would be invalid, as they would only be representative of the designable region and not the whole structure.

For the correct implementation of geometric constraints, the BESO subroutine of the python source code was instead significantly modified. The new constraint definition strategy consists of a range of data-structure manipulations to isolate the operation of the optimization algorithm on only the unconstrained elements (the design domain).

RESULTS

The quantitative performance of each optimization method and qualitative analysis of the resulting optimal topologies are presented in this section.

T106C BLADE

Structural finite element analysis was conducted on the blade models to determine the strain energy distribution, which would serve as the input variable to the optimization, the results of which are shown in Figure 5. The highest strain energy with a magnitude of 1.237 MPa, is observed at the blade root and the suction side of the aerofoil, as it is subjected to a combined tensile and bending load. Lower induced strains are observed in the central part of the blade, around the camber line, while the minimum strains are observed at regions close to the trailing edge.

Figure 5 T106C Original Structural Analysis

The FEA results shown above were then fed into the SIMP and SERA algorithms to design an optimally loaded structure. The evolution of objective function, volume fraction and the final optimized topologies of each method are shown in Figure 6 below.

Figure 4 Constraints and meshes

Figure 6 T106C optimization

The SIMP optimization converged after 32 iterations, satisfying the 50% volume constraint while reducing compliance by 41.6% relative to the original 50% volume structure. The operation of the optimization algorithm is revealed by the volume fraction trace. Upon initiation, large volumes of inefficient material are removed by the optimization algorithm, achieving a 50% volume fraction at the first iteration. At the second iteration, further material is removed (≤ 0.05 Vf), reducing compliance by 29% in a single step. Between iterations 3 and 8, the volume is increased as solid material is re-introduced as a structural reinforcement for highly-stressed regions, further reducing compliance. The optimized topology, shown in Figure 5, is characterized by hollow rectangular slots along the camber line, at the 0.1 to 0.65 and the 0.85-0.95 camber line length positions, extending from the blade root to 0.9 blade height.

The SERA optimization converged after 57 iterations at a compliance value at 4.14×10^4 , which is 36.7% lower than the 6.54×10^4 value achieved by SIMP. The optimization strategy, as described in the methodology section, is evident by the quasi-linear shape of the Volume Fraction trace as the target volume is reduced in each iteration until the global target is achieved. The global target volume V^* for the SERA optimization was set at 0.60 V_0 as computational constraints prevented a sufficient mesh density to achieve a 50% designable area.

C3X BLADE

Figure 7 illustrates the results of the structural analysis on the C3X turbine. The highest strain energies are observed at the blade root and are concentrated on the leading edge of the aerofoil, peaking at a magnitude of 1.205 MPa. The qualitative and quantitative optimization data from both algorithms are shown in Figure 8.

Figure 7 C3X Original Structural Analysis

Figure 8 C3X optimization

The optimizer converged after 28 iterations, achieving a final design with 50% of the original volume and a 33.6% reduction in compliance, relative to the original 50% volume structure. Although the objective function is monotonically decreasing, as with the T106C, the blade undergoes a systematic removal/addition processes until convergence is reached at the target volume. The increased geometric complexity of the C3X blade is shown to harm the optimization performance, requiring more iterations for the same relative performance benefit than in the T106C case.

The optimized C3X blade, shown in Figure 6, is comprised of a hollow structure, locally reinforced at the cooling channel boundaries and the leading edge.

The SERA optimization converged after 91 iterations. As a volume-driven optimization process, material is steadily removed, and compliance is increased relative to the original solid structure. At convergence, the achieved compliance was set at 6.70×10^4 , a value 22.0% lower than the 8.58×10^4 value of the SIMP-produced design. Despite the conceptual similarities between the resultant topologies - hollow cavities with local structural reinforcements distributed throughout the design - the two methods are

fundamentally different in their optimization strategy. Lower structural performance is achieved by the SIMP method relative to the SERA method, but the resultant topology is substantially smoother, a characteristic of designs of manufacturable standard.

DISCUSSION

Predominantly similar topologies were produced with the two optimization methods, their main difference being the boundary smoothness and the distribution of structural reinforcements.

The optimal T106C geometry was comprised of a thin, highly stressed skin structure with rectangular cavities along the camber line, extending to from the blade root to a near-tip position. The SIMP-derived result was shown to have symmetric skin thickness across the camber line. The SERA-derived result was characterized by asymmetric material distribution across the camber line, biased towards the suction side of the blade, where the highest elastic strain energies are observed.

The optimized C3X topology is a hollow structure with a thin outer skin, which is reinforced at the channel boundaries and the highly stressed leading edge. The predominant difference between SIMP and SERA derived topologies is the axial and lateral distribution of material reinforcements. With the SIMP method, material is retained near the blade tip, while with the SERA method, solid material is distributed in a column-wise arrangement, spanning from the blade root to approximately 85% blade height.

Interpreting these results highlights the ATOM SIMP optimization algorithm as a more computationally efficient method, capable of producing smooth, highly manufacturable designs. In contrast, the SERA method is shown to produce designs with higher structural performance but higher complexity. The difference in optimization is believed to be caused by the integrated Laplacian mesh smoothing filter, built-in to the ATOM SIMP algorithm. Additional verification studies are required to test this hypothesis and further extend the utility of the open-source algorithm via incorporation of an analogous smoothing filter. The difference in structural performance between the designs produced by the two methods can also be justified by this hypothesis (if proved), as the process of mesh smoothing compromises structural performance for increased topology smoothness and thus manufacturability.

CONCLUSION

This research project aimed to structurally optimize the T106C and C3X gas turbine blades using the ATOM SIMP topology optimization algorithm and an open-source SERA optimization algorithm. The open-source algorithm was modified to allow the definition of non-designable regions, expanding its capability to geometrically constrained problems.

Both methods resulted in thin-walled, hollow blade designs, locally reinforced at high-stress regions. The optimal T106C design consisted of a thin cavity, centred around the camber line, extending from blade root to blade tip. The optimal C3X design is a hollow geometry, with material reinforcements around the channel boundaries and at high-stress locations along the blade skin.

The ATOM SIMP algorithm exhibited higher computational efficiency and resulted in smooth, highly manufacturable geometries. The SERA algorithm resulted in geometries with higher structural performance, with its T106C and C3X designs having 36.7% and 33.6% lower compliance than the SIMP-derived result. It is hypothesized that the differences are caused by the application of a mesh smoothing filter in the ATOM SIMP optimization, decreasing structural performance for increased manufacturability. Further investigations on this discrepancy and the effect of mesh density on the optimized topologies are necessary to verify the observations of this study.

NOMENCLATURE

C	Compliance	m	Mass
E_{ijkl}	Fourth-Order Stiffness Tensor	m₀	Initial Mass (g)
$\bar{E}_{ijkl}$	Fourth-Order Stiffness Tensor of solid material	N	Number of elements
F	Force	p	Penalization Factor
F	Force Vector	P	Pressure (kPa)
F_c	Centrifugal Force	r_m	Mean Radius (m)
K	Stiffness Matrix	U	Displacement Vector

v_i	Volume of element i	$x_{\min}$	Minimum design variable value
V	Volume	$\mathbf{X}$	Vector of design variables
V^*	Target Volume	α	Design Sensitivity
V_0	Initial Volume	ρ	Density
V_f	Final Volume	σ	Stress
VM	Von Mises		
x_i	Design Variable of Element 'i'		

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