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VALIDATION OF A SYNTHETIC-EDDY METHOD FOR MODELLING INCOMING WAKES IN SCALE-RESOLVING SIMULATIONS OF TURBOMACHINERY CASCADES

Mark Zieße*, Lars Wein

Junior Research Group Multiphysics of Turbulent Flows
Institute of Turbomachinery and Fluid Dynamics
Leibniz Universität Hannover
An der Universität 1
30823 Garbsen, Germany
Email: ziesse@tfd.uni-hannover.de

Christoph Müller-Schindewolf, Florian Herbst

MTU Aero Engines AG
Dachauer Str. 665
80995 München, Germany

Joerg R. Seume

Institute of Turbomachinery and Fluid Dynamics
Leibniz Universität Hannover
An der Universität 1
30823 Garbsen, Germany

ABSTRACT

An enhanced Synthetic-Eddy Method for the modelling of free stream turbulence and incoming wakes for scale-resolving simulations of turbomachinery cascades is validated. It is shown that a previously presented method produces valid and physically plausible wakes representative of turbomachinery, and correctly reproduces their influence on the downstream cascade. The validation test case is an experimentally extensively investigated low-pressure turbine cascade with bars moving in pitch direction upstream of the cascade. Validation takes place in two steps: First, only the bars are considered. Then the profile including the moving bars is evaluated. In addition to the simulations with the directly modelled incoming wakes via the new method, the upstream wakes of the above two configurations are also directly resolved. The validation is proven by comparing the experimental values with the results of simulations with directly modelled wakes and the simulations with meshed moving bars.

INTRODUCTION

The wakes caused by the rotation of a turbomachine and the high turbulence levels outside the wake flow typical of turbomachines have a significant influence on the transition behaviour and thus also the losses of a turbomachine. The type

of inflow therefore should be considered in the aerodynamic design of a turbomachine.

According to the current state of the technology, turbomachinery currently is aerodynamically designed using computer-aided simulation methods known as Unsteady Reynolds Average Navier-Stokes Equations (URANS). A major disadvantage of these methods, however, is their modelling of turbulence. While current RANS methods have high prediction accuracies for steady inflows, Müller et al. (2015) could show which deficits current URANS methods have with regard to transient inflows, hence the current focus on this topic.

Due to their modelling, URANS procedures require extensive calibrations. Testing and calibration of URANS methods is currently carried out using experimentally investigated cascades. However, the generation of engine-like wakes upstream of the cascade to be investigated is difficult in the experiment. Either the Strouhal numbers (Stadtmüller et al., 2000) of current experimental investigations are clearly below those in a real engine, or a significant influence of the measurement setup cannot be excluded (Berrino et al., 2014).

Due to the steady decrease of the cost of computational resources, however, it is now possible to carry out so-called scale-resolving simulations (SRS) such as Large Eddy Simulation (LES) for technically relevant flows. In order to achieve signifi-

Table 1 Geometric parameters of the validation test case

parameter	value	parameter	value
β_{in}	127.7°	l_{ax}	0.085 m
b_{wake}	0.040 m	Δx_1	0.070 m
b	0.105 m	Δx_2	0.060 m
d_{bar}	0.002 m	Δx_3	0.030 m
l	0.100 m		

cantly better results with these SRS compared to the URANS methods, the simple application of the methods is insufficient. As mentioned before, an inflow similar to that encountered in the engine must also be considered. While in URANS procedures, the turbulent inflow can be directly specified due to the complete modeling of turbulence, in SRS, additional methods have to be used to generate the turbulent inflow.

For the generation of turbulent inflows in SRS different methods have been developed and successfully applied (Dhamankar et al., 2015). One method for generating inflow turbulence is the so-called Synthetic-Eddy Method (SEM). A turbulent inflow can generally be described as a superposition of eddy structures of different sizes, shapes and time scales. The most accessible approach for the generation of synthetic turbulence is thus the introduction of eddies into the computational domain whose fluctuations on average produce the desired turbulent inflow. In a previous paper (Zieße et al., 2019), the authors presented an improved SEM for the generation of free-stream turbulence and incoming wakes, called Wakes and Compressible Divergence-Free Synthetic-Eddy Method (W-DFSEM), in SRS of cascades. The basic idea is to replace preceding blade rows and their wake directly by means of W-DFSEM. There are three main advantages of using W-DFSEM:

1. An upstream blade does not need to be meshed and simulated (computational resource reduction by approx. 50%)
2. no rotor-stator interface is required and
3. a higher flexibility with regards to different wake variations

The aim of this paper is to validate the W-DFSEM on the basis of the existing experimental data for low, non-engine-like wakes, in order to then be able to use the W-DFSEM to analyze new, still completely unknown profiles under the influence of engine-like boundary conditions, in particular wakes and free-stream turbulence.

These simulations with LES and W-DFSEM can then serve as a further database for the improvement of current URANS methods in addition to the experimental data.

Figure 1 Definition of geometric parameters of the validation test case

VALIDATION CONCEPT

This paper mainly aims at validating the following two hypotheses:

1. With the W-DFSEM boundary condition previously presented in (Zieße et al., 2019), it is possible to generate physically plausible and sensible wakes as an inflow boundary condition of SRS
2. The influence of these wakes is correctly reproduced throughout the modelling chain.

The validation test case is the T106D low-pressure turbine cascade with upstream bars moving in pitch direction for wake generation. The D in the name of the cascade stands for a changed load of the profile ($b = 0.105$ m) compared to the original T106A cascade ($b = 0.08$ m). This test case was chosen because of the good data base. The test case was extensively investigated experimentally by Stadtmüller et al. (2000), Stadtmüller and Fottner (2001), and Hilgenfeld et al. (2002), as well as numerically by Blaim and Niehuis (2013). The selected operating point has an isentropic outflow Mach number $Ma_{out,is}$ of 0.4, isentropic Reynolds number $Re_{out,is}$ of 200,000 and the wake movement velocity in pitch direction is 21.4 ms^{-1} . The definition of all relevant geometric quantities of the validation are shown in Fig. 1. The corresponding values are listed in Tab. 2.

The entire validation concept can be divided into two main steps:

- 1st Step: Validation based on wakes of moving bars, without cascade
- 2nd Step: Validation of the influence of upstream bar wakes on a cascade

For both steps, the experiment is directly compared with the

simulation with meshed and moving bars and with the simulation with incoming wakes generated directly with W-DFSEM. If this comparison shows reliable results for Step 1 and Step 2, the W-DFSEM is validated for the generation of incoming wakes and the prediction of the influence of these wakes on a cascade. The simulations with meshed and moved bars make it possible to eliminate general methodical errors during the analysis of the artificially-generated wakes using W-DFSEM and can be used to quantify the accuracy of the wakes generated by W-DFSEM. In addition, this allows a direct comparison with regards to the required computational resources.

In principle, a comparison of the simulation with W-DFSEM and the experimental values for the second step would be sufficient to prove the validation. However, this would have the disadvantage that in case of significant deviations between simulation and experiment, it would not be possible to classify more precisely whether these deviations result directly from the wrongly generated wakes by the W-DFSEM or from general method errors (e.g. mesh resolution). This uncertainty can be excluded by the first step, i.e. using the test case with only bars but without the downstream cascade. Due to the measurement setup and the instrumentation used, it was only possible to measure the bar wakes with non-installed blades, and due to the influence of the potential field of the downstream cascade, a direct comparison of the wake measured without installed blades with the wake generated by W-DFSEM with installed blades is not a consistent comparison.

In addition to the simulations with moving wakes, simulations of non-moving bar and only cascade were also initially performed to evaluate the mesh error. The simulations of the non-moving bar are also used to determine the input parameters of W-DFSEM. For this purpose, the wake is extracted at the corresponding position behind the non-moving bar (see Fig. 1). For a multiple integer ratio of t/t_{wake} the pitch of the bar was reduced to 0.035m. The domain size in span direction corresponds to 0.3l for all simulations and the inlet of the simulations with meshed bars is 0.04m upstream of the bars. The inlet of the simulations with W-DFSEM is Δx_2 upstream of the cascade. The outlet is located Δx_3 upstream of the cascade for the simulations with bar wake only and for the simulations including cascade 0.05m downstream of the cascade.

The simulation of the cascade without incoming wake also served to determine the inflow angle. Due to the measuring setup for the generation of the moving bar wakes, deviations in the inflow angle already occur without installed bars. For the simulations carried out, a good agreement with the experiment was achieved with $\beta_1 = 133.2^\circ$. For further details please refer to (Stadtmüller, 2001).

MODELLING DESCRIPTION

The modeling of the computational domain itself is the same for all test cases. A finite volume method of second-order in space and time is used. The LES approach is used to reproduce the turbulence. The modeling of the smaller, isotropic

structures necessary for this approach is done by the wall adapting local eddy-viscosity (WALE) subgrid-scale model (Franck and Ducros, 1999). The filter width is the mesh itself or a representative local cell size: the third root of the volume of the respective cell. The spatial discretization is done with the so-called LUST scheme. The LUST scheme is a combination of 25 % second order upwind and 75 % central differences schemes. The temporal discretization is done by means of the implicit second order backward schemata. For all simulations, it was ensured that the CFL number never exceeds 0.4.

The resulting equation system is solved using the solver *rhoPimpleFoam* for the test cases without moving mesh and for the test cases with moving mesh using *rhoPimpleDyMFoam* from the *OpenFOAM* software package. The *DyM* extension stands for dynamic mesh motion. The moving cells for the solution are taken into consideration by correcting the predominant fluxes in the affected cells. This correction is done for the simple case of continuous uniformly moving cells by replacing the fluid velocities U in all convection terms with a relative velocity $U - U_{\text{wake}}$. Then the same procedure for solving the equation system follows as for the solver *rhoPimpleFoam*. For more details, see (Jasak, 2009) and (Ferziger and Peric, 2001). Both solvers are based on an incompressible solver, which has been extended with a density correction.

For all test cases, the outer walls of the calculation domain are assumed to be periodic in pitch and span direction. In the case of adiabatic internal walls, like the walls of the bars and the blade, the boundary conditions of all the variables required for determination correspond to a Neumann boundary condition, except for velocity. The velocity is specified directly on the wall and thus corresponds to a Dirichlet boundary condition. For non-moving walls, the velocity at the wall is set to zero and for the moving bars the velocity at the wall corresponds to U_{wake} . As outlet boundary condition, the wave permeable *waveTransmissive* based on (Poinso and Lelef, 1992) was used. This was also used at the inlet, except in the simulations with W-DFSEM where the velocity is specified by W-DFSEM.

W-DFSEM

The W-DFSEM is based on the Synthetic-Eddy Method by Jarrin et al. (2009). This in turn was extended by the property of divergence-freedom by Poletto et al. (2013) to the Divergence-Free Synthetic-Eddy Method (DFSEM). The SEM and DFSEM were primarily developed for flows close to walls. Müller-Schindewolf and Herbst (2018) then optimized and recalibrated the DFSEM for turbomachinery in order, for example, to also consider periodic boundaries. In a further step, this Incompressible-DFSEM was coupled with the previously introduced *waveTransmissive* boundary condition to the Compressible-DFSEM. The C-DFSEM, on the other hand, was then extended by the specification of moving incoming wakes by Zieße et al. (2019) to the W-DFSEM and successfully verified.

The essential characteristics of moving wakes are a momentum

deficit, increased turbulence and the unsteadiness itself. Inside the W-DFSEM the momentum deficit is set by specifying a velocity deficit based on the density function of the Gaussian distribution. This velocity deficit is clearly defined by a parameter for the wake width ω , the wake depth σ , the velocity in free stream U_{FS} and the number of wakes N . A 2D field of all Reynolds stresses of a non-moving wake is given for the reproduction of the increased turbulence. This 2D field was taken Δx_2 upstream of the cascade from the simulation of the non-moving bar at midspan and the corresponding values for the velocity deficit are $\sigma = 2.40$, $\omega = -0.23$ and $U_{FS} = 97.98 \text{ ms}^{-1}$. For the unsteadiness itself, only the movement velocity of the wake in pitch direction U_{wake} must be specified. The velocity deficit and the synthetic eddies are convected in pitch direction according to this velocity. In combination with these input parameters, the W-DFSEM allows to specify incoming wakes upstream in a SRS of a cascade. A brief explanation of the underlying equations of W-DFSEM is given in the appendix. For detailed description of the W-DFSEM please refer to (Zieße et al., 2019).

RESULTS

All experimental data shown are taken from (Stadtmüller, 2001). The absolute accuracy of the experimental results is 22Pa for each pressure tapping of the profile pressure distributions and 6Pa for the total pressure for each traverse position in the wake. The associated error bars are within the ranges of the symbol sizes, which is why they are not shown. The ensemble averaging is based on the work of Lakshminarayana and Poncet (1974). Unless otherwise indicated, the number of averaged ensembles of the ensemble averaging is $M = 25$ for the simulations with moving wakes and $M = 300$ for the experimental results. The number of averaged ensembles in the experiment is thus 12 higher than in the simulations, which must be taken into account when interpreting the results (see Fig. 3). This lower number of ensembles in the simulations results from limited computational resources, but was chosen according to comparable numerical investigations (Michelassi et al., 2003; Michelassi et al., 2015; Koschichow et al., 2014). The number of samples of the temporal averaging for the simulations is about 700,000.

Non-moving Bar

The grid parameters of the grid study of the bar are listed in Tab. 2. y_{wall} stands for the wall distance of the first cell layer in wall normal direction, Δ_i for a mean cell width far away from the wall, RF for a refinement factor of the grids among themselves and N for the total number of cells. Furthermore, the non-dimensional cell sizes tangential to wall Δx^+ , normal to the wall Δy^+ and in spanwise direction Δz^+ are shown. The results of the grid study for the wake of the bar are shown in Fig. 2. In addition, an analytical velocity deficit based on the work of (Tennekes et al., 1972) is shown. For both grids G_3 and

Table 2 Grid parameters of the 4 grids used for the grid refinement study of the bar.

parameter	Grid			
	G_1	G_2	G_3	G_4
y_{wall}	14.9e-6 m	9.93e-6 m	7.45e-6 m	5.96e-6 m
Δ_i	10e-4 m	6.66e-4 m	5e-4 m	4e-4 m
RF	1.00	1.5	2.0	2.5
N	149,520	519,345	1,221,960	2,334,600
Δx^+	29	20	16	13
Δy^+	2,1	1,6	1,3	1,0
Δz^+	145	111	88	72

G_4 a good agreement with the analytical velocity deficit can be observed. Also the turbulent kinetic energy (TKE) distribution of these two grids largely matches. The grid G_3 was therefore used for all further calculations, since it represents a good compromise between grid resolution and accuracy. In addition, this grid resolution corresponds to the simulations with cascade.

Figure 2 Velocity deficit (left) and TKE (right) in the wake at midspan of the grid study of the non-moving bar.

Moving Bar

In Fig. 3, the results of the simulation with meshed and moving bar and the simulation of the moving bar wake directly modelled by W-DFSEM are compared with the results of the experiment. Additionally, the influence of the ensemble number of the ensemble-average on the results is visualised. All results shown were additionally shifted and averaged over the different phases. The wake width of the velocity deficit of the simulation with meshed bar agrees well with the experiment. The wake depth ($U_{FS} - U_{abs}$)/ U_{FS} is slightly too high, which is due to the different number of averaged ensembles between experiment and simulation. The wake depth of the W-DFSEM agrees well with the experiment. However, the wake width of W-DFSEM is slightly too large, but could be further reduced by fine-tuning σ , ω , and U_{FS} . A comparison of the TKE between moving bar, W-DFSEM and experiment shows a good agreement regarding the wake width. Differences between the simulations and the experiment can be observed in the height of the TKE peak. These differences can also be explained by the different number of ensembles.

In conclusion, there is a good agreement between the simulation of the moving bar, the simulation of W-DFSEM, and the experiment. The W-DFSEM thus correctly reproduces the moving bar wake and the first step of the validation concept is thus fulfilled.

Figure 3 Comparison of the ensemble-averaged velocity deficit (left) and the TKE (right) in the wake at midspan 0.03 m upstream of the cascade for the simulation with meshed and moving bar and with bar wake modelled by W-DFSEM.

Table 3 Grid parameters of the 3 grids used for the grid refinement study of the cascade.

parameter	Grid		
	G ₁	G ₂	G ₃
y_{wall}	2.48e-5 m	1.65e-5 m	1.23e-5 m
Δ_i	10e-4 m	6.66e-4 m	5e-4 m
RF	1.0	1.5	2.0
N	2,569,020	8,900,310	21,039,449
Δx^+	31	18	12
Δy^+	1,4	0,9	0,7
Δz^+	58	39	28

Cascade

The grid parameters of the grid study for the cascade are given in Tab. 3 and the corresponding results in Fig. 4. When the isentropic Mach number distribution is considered, a good agreement of all three grids can be observed, except for the front suction side area and the area of the separation bubble. In these areas the resolution of the grid G₁ is not enough. This untypical mesh dependency in the area of the suction-side leading edge results from a small separation bubble in this area, graphically shown in Fig. 5. Between the two grids G₂ and G₃ no significant differences can be observed for the Mach number distribution and also for the suction-side wall shear stress no differences can be found between the two grids. When considering the total pressure loss in the wake, however, differences can also be found between these two grids. The wake peak is underestimated by both G₁ and G₂, but G₃ reflects the experimental data well. For this reason, the mesh G₃ was used for all further simulations. Although no grid convergence of this grid with regard to the total pressure loss in the wake can be observed, the grid resolution is high enough so that significant errors due to the grid can be excluded and the grid resolution is sufficient for a method comparison.

Cascade with Incoming Wakes

As reference total pressure for the calculation of the isentropic Mach number distribution and the total pressure losses in the wake, the total pressure between the bars and the cascade was chosen. In the experiment, however, only the total pressure upstream of the cascade was measured. Therefore, the total pressure upstream of the cascade was corrected by the total pressure losses of the row of bars from the simulation with meshed bars $\Delta p_{t,bar} = 121$ Pa, which is in good agreement with the experimental investigations of Clinckemaulle and Arts (2019).

In Fig. 6 left, the time averaged pressure distributions for the experiment, the simulations with meshed bars and W-DFSEM

Figure 4 Wall shear stress at suction side (upper left), isentropic Mach number distribution at midspan (lower left) and total pressure losses in the wake at $x = 1.47l_{ax}$ (right) for the grid study of the cascade.

Figure 5 Velocity magnitude at midspan and detail view of the separation bubble at the leading edge

are compared. Both simulations can reproduce the pressure distribution of the experiment well. The deviations of both simulations from the experiment are in the same range as in the case without incoming wakes (see Fig. 4). A comparison of the total pressure loss in the wake of the cascade, graphically shown in Fig. 6 right, also shows a good agreement between the experiment and the simulations. The small differences between

the simulations can probably be attributed to the different grid resolution. For the simulation with meshed bars the grid G_2 was used. For the simulation with W-DFSEM the grid G_3 . The simulation with meshed bars was also initially started on the G_3 mesh, but the necessary computational effort is unexpectedly high due to the poor parallelizability of the rotor-stator interface, which is why the grid resolution was reduced. However, the grid resolution of G_2 is also high enough to reproduce the quantities of interest with sufficient accuracy (see Fig. 4).

In Fig. 7, the ensemble-averaged wall shear stresses of the simulations are additionally compared with the ensemble-averaged quasi wall shear stress from the experiment. In the experiment the wall shear stress is not measured directly, rather a quantity which is proportional to the wall shear stress. The wall shear stresses in 7 were therefore scaled with a reference value τ_{Ref} . This reference value is different for the experiment and simulations. For a detailed description of the determination of the quasi wall shear stress, please refer to (Stadtmüller, 2001). In addition to the wall shear stress, the separation and reattachment lines were drawn in white and the time offset between the earliest separation and the latest reattachment point $\Delta t/T$ is shown. Qualitatively, the simulations and the experiment are very similar. The results of the simulations are much more noisy due to the small number of averaged ensembles. A comparison of the axial positions shows a good agreement between experiment and W-DFSEM, whereas in the simulation with meshed bars, the separation and reattachment line are much further upstream, which is also due to the lower grid resolution. For the time offset $\Delta t/T$, the simulations deviate from the experimental value 0.32, but are in the range of the experiment with 0.30 for the simulation with meshed bars and 0.27 for the simulation with W-DFSEM.

Figure 6 Isentropic Mach number distribution at midspan (left) and total pressure losses in the wake at $x = 1.47l_{ax}$ (right) for the cascade testcase with incoming wakes

Figure 7 Comparison of the ensemble-averaged experimental quasi wall shear stress with the wall shear stress of the simulations with meshed bars and W-DFSEM for the suction side at midspan.

In summary, it can be said that the wakes generated by W-DFSEM are not only largely identical to the experiment and simulation with directly meshed bars, but the complete modelling chain also correctly predicts the influence of these wakes on the downstream cascade. Thus, the second step of the validation concept is also fulfilled and the W-DFSEM is successfully validated.

Table 4 Comparison of the required computational resources for the cascade with incoming wakes.

Ref	W-DFSEM	Meshed bar	Meshed bar without RSI
100 %	110 %	871 %	166 %

Comparison of Computational Cost

The simulations with W-DFSEM and with meshed bars are compared in Tab. 4 regarding the required computational

resources. The values shown refer to the test case of the cascade with incoming wakes, but for the test case only with bar, the behavior is very similar. The simulation of the cascade without incoming wakes and the mesh resolution G_3 served as a reference. The ratio of cells per processor was kept constant for all simulations and is about 15,000. The additional computational resources of W-DFSEM compared to the reference are 10 %. Although the additional number of cells between the simulation with meshed bars (23.4 million cells) and W-DFSEM (20.7 million cells) is only 13 %, the additional needed computational resources for the simulations with meshed bars is more than 771 % compared to the reference. This unexpectedly high computational effort can be explained by the poor parallelizability of the rotor-stator interface (RSI) and is also the reason for choosing a lower mesh resolution for the simulation with meshed bars. However, even without considering the rotor-stator interface, the additional computational effort of the simulations with meshed bars is significantly higher than with W-DFSEM (see Tab. 4). If, on the other hand, the influence of the wake of a profile and not of a bar were to be investigated, the number of cells would be about twice as high in the case of a meshed geometry upstream of the blade row and thus at least the additionally required computational resources. For these more realistic application cases, a significantly higher advantage of W-DFSEM with respect to the required computational resources can therefore be expected.

CONCLUSIONS

The aim of this study was the validation of the newly developed boundary condition Wakes and Compressible Divergence-Free Synthetic-Eddy Method (W-DFSEM) for the generation of realistic inflow conditions (including incoming wakes and free-stream turbulence) for scale-resolving simulations of turbomachinery cascades. The focus was especially on the synthetically generated incoming wakes. A validation concept was presented. The results of the simulations confirmed the following hypotheses...

1. The W-DFSEM allows the generation of realistic and physically plausible wakes.
 2. The influence of these wakes on a downstream cascade is correctly reproduced throughout the entire modelling chain.
- ... and also showed that...
3. the direct generation of wakes by W-DFSEM at the inlet of a cascade simulation allows a significant reduction of computational resources compared to a simulation in which a wake's generating geometry is meshed and moved upstream of the cascade, and
 4. these reduced computational costs, as well as the wide range of possible variations of the generated wakes of the W-DFSEM, allows the realization of extended parameter studies to investigate the influence of different wakes on a downstream cascade.

To summarize, the W-DFSEM has been successfully validated. To the author's knowledge, W-DFSEM can be considered as the first synthetic non-reflective boundary condition for scale-resolving simulations of compressible turbomachinery flows under various realistic inflow conditions. After successful validation, W-DFSEM can be used to investigate current blading and to gain new insights into turbulent wake-blade row interactions.

NOMENCLATURE

Latin Symbols

b	pitch
N	number of wakes, number of cells
T	bar period
U_{wake}	wake movement speed
U_{FS}	free-stream velocity
U_{abs}	velocity magnitude
Δx^+	tangential non-dimensional cell size
Δy^+	wall normal non-dimensional cell size
y_{rel}	relativ pitch
y_{wall}	wall normal distance first cell layer
Δz^+	spanwise non-dimensional cell size

Greek Symbols

Δ_i	average cell size far away from wall
ω	wake width
σ	wake depth
τ_w	wall shear stress

Characteristic numbers

Re	Reynolds number
Ma	Mach number

Subscripts

in, out	inlet, outlet
is	isentropic
ax	axial length

Acronyms

C-DFSEM	Compressible Divergence-Free Synthetic-Eddy Method
DFSEM	Divergence-Free Synthetic-Eddy Method
EXP	Experiment
I-DFSEM	Incompressible Divergence-Free Synthetic-Eddy Method
LES	Large-Eddy Simulation
RF	Refinement factor
RSI	Rotor-stator interface
SRS	Scale-Resolving Simulations
URANS	Unsteady Reynolds-averaged Navier–Stokes
WALE	Wall adapting local eddy-viscosity
W-DFSEM	Wakes and Compressible Divergence-Free Synthetic-Eddy Method

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Appendix: Modelling equations of W-DFSEM

The W-DFSEM only influences the velocity field at the inlet of the simulation. For reasons of simplification, first-order temporal discretization is assumed in the following. The velocity at any point of the inlet patch $u_{i,n}$ is then given in Einstein notation by:

$$u_{i,n} = \frac{u_{i,n-1} + k(U_{\text{Gauss}} + u'_i)}{1+k} + c_k \frac{1+\alpha+k}{1+k} \frac{du'_i}{dt} \quad (1)$$

The factor k is part of the property of wave permeability and is defined as follows:

$$k = \frac{w \Delta t}{l_\infty} \quad (2)$$

The variable w corresponds to the velocity of the structures that leave the domain through the patch considered:

$$w = \max(\vec{u} \cdot \vec{n} + a; 0) \quad (3)$$

formed with the patch normal velocity $\vec{u} \cdot \vec{n}$ and the speed of sound a . Thus $w \neq 0$ applies to the structures leaving the domain. The degree of reflection of the boundary condition can be adjusted by the characteristic relaxation length l_∞ . The variable α represents a dimensionless propagation velocity related to the local cell width Δx :

$$\alpha = \frac{w \Delta t}{\Delta x} \quad (4)$$

For a detailed description of the wave permeability property, please refer to the original paper (Poinsot and Lelef, 1992). The velocity deficit of the wake U_{Gauss} is based on a Gauss function and is given by

$$U_{\text{Gauss}}(t, y) = U_{FS} \cdot \left(1 - \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(f(t, y)\omega)^2}{2\sigma^2}\right) \right) \quad (5)$$

with an additional function

$$f(t, y) = \left(\frac{\left[(y - h(t, y) + g(y) - \min(y)) \bmod \left(\frac{b}{N} \right) \right]}{\frac{b}{N}} - 0.5 \right) \cdot \frac{10.0}{N} \quad (6)$$

The movement of the wake in pitch direction is controlled using the function

$$h(t, y) = U_{\text{wake}} t \bmod [\text{sign}(U_{\text{wake}})b] \quad (7)$$

The function

$$g(y) = \left[\frac{|y - (\min(y) + 0.5b)|}{0.5b} - 1 \right] 0.5b \cdot \int \left(\frac{y - (\min(y) + 0.5b)}{0.5b} \right) \quad (8)$$

ensures that the values are always within one pitch. For a detailed derivation of the respective function or velocity deficit, please refer to (Zieße et al., 2019).

To reproduce the turbulent fluctuations in the wake, eddies are continuously seeded by the boundary condition and convected through the inlet patch. The discretization of the eddy fluctuations u'_i only occurs when the eddies pass the inlet patch and result from

$$u'_\beta(x) = \sqrt{\frac{1}{N}} \sum_{k=1}^N \sigma_\beta^k \left[1 - (d^k)^2 \right] \epsilon_{\beta j l} r_j^k \alpha_l^k \quad (9)$$

with N eddies, the Levi-Civita symbol ϵ , length scale σ_β^k , intensity α_m^k of the k -th eddy and the overall distance from the patch cell to the eddy center d^k . The eddy intensity α_m^k follows directly from the prescribed Reynolds stress field R . To enable the specification of anisotropic turbulence, the eddies are additionally rotated from the principal axes system (P) to the global coordinate system (G) by using the Reynolds stress tensor:

$$u'_i{}^G(x) = C_1 R_{im}^{P \rightarrow G} u'_m{}^P \quad (10)$$

$u'_m{}^P$ and $u'_i{}^G$ are the velocity fluctuations along the principal axes and in the global system respectively, $R_{im}^{P \rightarrow G}$ the rotation transformation matrix and C_1 a normalization coefficient. For a more detailed description of the synthetic eddies please refer to (Poletto et al., 2013).

The movement of the turbulent field is realized by a repositioning and additional convection of the eddies in pitch direction. After the initial random positioning the eddies are repositioned in pitch direction by an offset

$$\Delta y_m = t \cdot U_{\text{wake}} \bmod (\text{sign}(U_{\text{wake}}) \cdot b) + b \cdot (\text{sign}(0.5b - t \cdot U_{\text{wake}} \bmod (\text{sign}(U_{\text{wake}}) \cdot b)) + 1) / 2 \quad (11)$$

The additional convection in pitch direction results directly from the shift velocity of the wake U_{wake} :

$$\Delta y = \Delta t U_{\text{wake}} \quad (12)$$

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