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NUMERICAL VALIDATION OF AN ANALYTICAL SEAL FLUTTER MODEL

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ABSTRACT

An analytical model to describe the flutter onset of straight labyrinth seals has been numerically validated using a frequency domain linearized Navier-Stokes solver. The nondimensional work-per-cycle depends on three dimensionless parameters interrelating the effect of the pressure ratio, the geometry, the nodal diameter, the vibration frequency and the torsion centre location. Two of these parameters are new to the aeroelastic community. A comprehensive set of simulations has been conducted to assess the stability criterion derived from the analytical model. Special attention has been paid to increase the accuracy of the model by informing it with the CFD derived effective gaps. The numerical calculations show for the first time that the model can accurately predict the aeroelastic stability of labyrinth seals in a wide range of working conditions when the seal vibrating frequency does not coincide with the acoustic resonances of the adjacent cavities.

INTRODUCTION

Labyrinth seals are extensively used in aero-engines to control the leakage between regions with different pressures to prevent the ingestion of hot gas in the turbine disk cavities and for cooling operations (Chupp et al., 2006). They are comprised of rotating and stationary components. The rotating parts are usually equipped with a series of radial fins however the stationary elements have straight or stepped geometries. The flow is repeatedly forced to pass through small clearances generating kinetic energy that is dissipated in the inter-fin cavity. This process increases the resistance to flow compared to a smooth slot.

The first pioneering studies on the seal flutter were conducted during the 60' and 70' by Alford (1964; 1971; 1975). His works discuss the cause and prevention of fatigue failures in seals. He identified the importance of the support side of the seal in preventing self-excited vibration. Ehrich (1968) introduced the importance of the knife-edge clearance on the seal stability, but his findings were restricted to

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the 0th nodal diameter. Ehrich (1968) proposed an analytical model and identified a stability parameter which included the effect of the clearance, the support side of the seal and the geometry of the inter-fin cavity. The models described by Ehrich and Alford did not take into account the effect of the circumferential flow. Abbott (1981) found that a seal supported on the low-pressure side was unstable only if the mechanical frequency was lower than the acoustic frequency of the seal, and vice-versa. According to Abbott's criterion, the mechanical to acoustic frequency ratio and the support side of the seal are the only two parameters that control the stability of the seal.

All the previous studies were based on experimental analysis, failure evidence and often on bulk-flow analytical models. Nowadays, CFD analyses are a commonplace both in industry and academia. Phibel et al. (2009) conducted a parametric investigation on a multi-finned straight-through seal to investigate the influence of the frequency and the support location on the seal stability. They found that the aerodynamic damping depended mainly on the cavity shape, the mode-shape and the unsteady pressure phase distribution. Di Mare et al. (2010) describe the application of a time-marching Navier-Stokes solver to the stability analysis of an LP turbine labyrinth seal for two different configurations. The results are compared with Abbott's stability criterion suggesting that complex seal configurations are not appropriate to be treated with simple stability criteria based on the comparison of the structural and acoustic frequencies. Recently, Miura and Sakai (2019) have compared CFD results with experimental data obtained in an in-vacuum rotating rig. The numerical results show reasonably good agreement with the measurements.

Corral and Vega proposed a new comprehensive model for seal flutter (Corral and Vega, 2018; Vega and Corral, 2018a). The model provides an expression for the work-per-cycle involving two new dimensionless parameters to define the stability map of the seal. All the results of the classical analytical models (Abbot, 1981; Ehrich, 1968) are recovered and conciliated. The results were also extended by to stepped-seals Corral. et al. (2020). It has been identified in Corral. et al. (2019) that the labyrinth seal has an outstanding effect on the stability of shrouded turbine rotor blades. The numerical results are compared with the prediction of the Corral and Vega (CV) model corroborating the stabilizing effect predicted analytically. Recently, a new formulation of the Corral and Vega model (2018) accounting for non-isentropic perturbations has been proposed Corral et al. (2020). The model redefines the dimensionless parameters described in Corral and Vega (2018) and outlines a more general stability map of the seal.

This work presents a numerical validation of the CV model for labyrinth seal flutter, including non-isentropic perturbations. The paper is organized as follows: First, the analytical model and the test case are briefly introduced. Secondly, steady-state results are presented. Finally, the results of the numerical simulations have been compared with the prediction of the analytical model. The numerical results present a large degree matching with the predictions of the high-order analytical model in a broad range of operating conditions provided that some conditions are fulfilled.

GOVERNING EQUATIONS

The higher order formulation of the CV model is described in Corral et al. (2020) is an extension of the original model (Corral and Vega, 2018) to account for non-isentropic flow perturbations. In this formulation, the entropy conservation relationship is replaced by the integral energy equation of the seal inter-fin cavity. The main hypotheses used to derive the new formulation could be summarized as follow:

1. The flow through the seal fins is quasi-stationary.
2. The process is adiabatic ($RePr \gg 1$)
3. The mean total temperature, T_0 , in the inter-fin cavity is constant, which means that the work due to the seal rotation is negligible.

The expression of the dimensionless work-per-cycle, $\tilde{W}_{cyc}$, of the extended CV model can be written as:

$$\tilde{W}_{cyc} = \frac{W_{cyc}}{\pi p_{c,s} \delta^2 S L / |r| H h'} = \text{sign}(r) \hat{p}_c \quad (1)$$

where $\delta = r\Delta\theta$ is the seal torsion displacement, and $S = 2\pi RL$, is the surface of the seal land. All the parameters involved in the previous expression are described in the sketch 1 and 3. The dimensionless out-of-phase component of the unsteady pressure according to the CV model is:

$$\hat{p}_c = -\tilde{\Omega} \left[\tilde{e}'_{eff} + \left(1 - \frac{1}{St^2} \right) \right] / \left[\tilde{1} + \tilde{\Omega}^2 \left(1 - \frac{1}{St^2} \right)^2 \right]. \quad (2)$$

The dimensionless work-per-cycle depends on four nondimensional parameters

$$\tilde{W}_{cyc} = \tilde{W}_{cyc}(\tilde{\Omega}, St, \tilde{e}'_{eff}, h') \quad (3)$$

FIGURE 1. SKETCH OF THE LABYRINTH SEAL GEOMETRY

Cavity Radius (R)	Nominal Gap (H)	Length (L)	Height (s)
554 mm	0.2 mm	15.8 mm	10.1 mm

TABLE 1. PHYSICAL GEOMETRIC PARAMETERS OF THE SEAL INTER-FIN CAVITY

where the non dimensional parameter $\tilde{e} = \gamma r H / (s L)$ includes the effect of the vibration mode-shape, the seal clearance and the geometry of the seal. The expression of the out of phase component of the unsteady pressure is formally equal to that derived in the isentropic formulation (Corral and Vega, 2018) where the seal clearance non-dimensional parameter appears always multiplied by h' which is a function of the seal pressure ratio. Nevertheless in the higher order formulation the parameter $\tilde{e} h'$ is replaced by $\tilde{e} h'_{eff} = \tilde{e} h' F(\Omega)$ to account for non-isentropic perturbations. As described in Corral et al. (2020) the dependence of the $\tilde{W}_{cyc}$ on h' is due to the new barotropic relationship included in the model. Moreover, $\tilde{e} h'_{eff}$ introduces two limiting curves that bound the unstable region of the seal for each frequency, the boundary curves are obtained considering a seal with very low pressure ratio and for a choked seal. The dependence of the work-per-cycle with the vibration frequency is retained in the non-dimensional discharge time of the cavity, Ω , and in the non-dimensional acoustic frequency, St . An elaborated form of the former can be written as:

$$\bar{\Omega} = \omega \frac{p_{c,s} V_{c,s}}{\dot{m}_s a_0^2} \frac{1}{h'} \hat{\Omega} \quad (4)$$

where the auxiliary variable $\hat{\Omega}(\Omega)$ varies smoothly between $\hat{\Omega}(0) = \gamma$ and $\hat{\Omega}(\infty) = 1$ (the reader is referred to Corral et al. (2020) for further details. Finally,

$$St = \frac{\omega R}{ND a_0} \quad (5)$$

represents the ratio between the mechanical frequency and the acoustic frequency of the seal, and the function one tilde, $\tilde{I}(\omega)$, is a function of the frequency that for the cases of interest is very close to one. In general, $\omega = \omega(ND)$ and then St can be expressed either as a function of ω or ND . The physical interpretation of the parameters and their influence on the $\tilde{W}_{cyc}$ are described in detail in Corral et al. (2020) and Vega and Corral (2018a).

NUMERICAL SETUP

A simplified geometry consisting in a straight-through labyrinth seal with a single inter-fin cavity has been constructed to assess the accuracy of the CV model. This model only accounts for the work exerted by the seal in the inter-fin cavity land which frequently is the dominant one and therefore the actual shape of the adjacent cavities is irrelevant for this purpose. It is acknowledged that the neighboring cavities, especially in or close to the acoustic resonances, can exhibit a large level of unsteadiness that eventually can influence the inter-fin cavity. Especial care has been paid in this work to avoid such resonances to ease a clean comparison between the model and the CFD analyses. The schematic view of the model used in this study and the geometric details of the seal are shown respectively in Fig. 1 and Table 1. Special care has been taken sizing the upstream and downstream cavities. Both cavities have been designed to ensure a uniform pressure at the inlet and outlet of the seal and to avoid any eventual acoustic resonance in the frequency range of the simulations.

The CV model has been already partially assessed on the tip-shroud of an LP turbine (Corral et al., 2019) using a well-validated frequency domain linearized Navier-Stokes solver (Corral et al., 2003). (Vega and Corral, 2016) In this work, a two-dimensional hybrid

FIGURE 2. OUTLINE OF THE COMPUTATIONAL DOMAIN (A), MESH DETAIL AROUND THE FIN TIP CLEARANCE REGION (B), AND AZIMUTHAL OVERVIEW (C).

FIGURE 3. MODE-SHAPE DEFINITION. (A) THE SEAL MOVES AS A RIGID BODY AROUND AN ARBITRARY PIVOT POINT SUPPORTED IN THE HIGH-PRESSURE SIDE (RED LINES) OR IN THE LOW-PRESSURE SIDE (GREEN LINES). (B) RADIAL DISPLACEMENTS OF THE 3RD ND.

grid has been constructed in the meridional plane of the seal and extruded in the circumferential direction to form triangular prisms and hexahedra. A grid sensitivity study has been conducted by checking convergence of the mass flow-rate and the mean static pressure in the inter-fin cavity. Hereafter a model with approximately 350,000 points was chosen (see 2). The standard $k - \omega$ turbulence model is used in this study and the mesh is fine enough to ensure $y^+ \simeq 1$ in the whole domain. The model used in all the simulations is a sector of 10° with 10 layers in the circumferential direction. The seal geometry is considered axisymmetric, and cyclic symmetry with phase-shifted boundary conditions were applied in the azimuthal interfaces.

Mode-Shape Definition

As mentioned before, the $\tilde{W}_{cyc}$ of the seal flutter model (Corral et al., 2020) is a function of 3 independent parameters namely, $\bar{\Omega}$, $\tilde{e}h'_{eff}$ and St . To ease the set up of the simulations independently changing each of these three parameters, a synthetic mode-shape generator has been implemented. The seal in the meridional plane is considered as a rigid body moving around a pivot point meanwhile in the azimuthal direction each section is shifted of a proper angle according to the selected nodal diameter. Figure 3 (a) describes how mode-shapes are defined in the meridional plane. The mode-shape displacements are applied on the seal wall nodes of the fluid mesh, and the inner points are moved using a Laplacian smoother. The displacements are small enough not to generate large distortions in the mesh that could jeopardize mesh quality. Moreover, this requirement is needed to ensure that the unsteady perturbations are linear. For a given mode, only the seal surface is moved, as shown in Fig. 3 (b), while the rest of the solid walls are kept fixed. This mechanism allows easily changing the pivot point, the ND and the frequency for conceptual studies.

RESULTS

Steady State

The steady results obtained from the simulations are presented next. For all the simulations of this work we will assume that the seal is non rotating to reduce the uncertainties in comparing with the model. The high-pressure and low-pressure cavities are designed to act as a plenum to settle the flow that fills the seal. Both cavities are characterized by large re-circulations and very low Mach numbers (see

FIGURE 4. VELOCITY STREAMLINES FOR THE NOMINAL PRESSURE RATIO $\pi_T = 2.0$ COLORED BY THE STATIC PRESSURE CONTOUR (A) AND DETAIL OF THE INTER-FIN CAVITY (B).

FIGURE 5. INTER-FIN CAVITY PRESSURE RATIO(LEFT), $\pi_s = P_0/p_c$, AS A FUNCTION OF THE SEAL TOTAL PRESSURE RATIO, $\pi_T = P_0/p_e$, OBTAINED WITH THE 0D MODEL ($\square$) AND 2D AXISYMMETRIC RANS ANALYSIS ($\circ$). CONTRACTION OF THE STREAMLINES AT THE FIRST FIN TIP (A) AND MACH NUMBER CONTOURS AT THE OUTLET FIN FOR $\pi_T = 2.3$ (B)

4(a)). The streamlines displayed in the Fig. 4 (b) show that the wall jet created in the first fin impinges onto the second one separating the inter-fin cavity into a thin wall jet region and a large vortex zone. The flow recirculates in the inter-fin seal cavity dissipating his kinetic energy, while the wall jet reaches the downstream clearance carrying most of its kinetic energy out of the seal. The computed static pressure in the inter-fin cavity is nearly constant, and is compared with the prediction of the simple analytical steady model consistent with the CV model

$$\pi_s^{\frac{\gamma-1}{\gamma}} \left(\pi_s^{\frac{\gamma-1}{\gamma}} - 1 \right) = \left[\left(\frac{\pi_T}{\pi_s} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \left(\frac{\pi_T}{\pi_s} \right)^{-\frac{\gamma+1}{\gamma}} \quad (6)$$

in Fig. 5, where $\pi_s = P_0/p_c$ is the ratio between the inlet total pressure and the inter-fin cavity static pressure. This comparison is obtained by changing the inlet total pressure, P_0 , while the exit pressure, p_e , and inlet total temperature, T_0 , are constant and correspond to the ambient conditions, $p_e = 100$ kPa and $T_0 = 293$ K. Figure 5 displays the cavity pressure as a function of the total pressure ratio up to the choking pressure ratio of the exit fin ($\pi_T = 2.3$). The flow at this condition is shown in Fig. 5. The mass flow and the cavity pressure are constant for higher pressure ratios. The numerical results are in reasonable agreement with the analytical predictions considering that this is the simplest of all the possible models. The differences increase with the pressure ratio up to 7% for $\pi_T = 2.3$, which is acceptable considering that the analytical model does not even take into account the contraction of streamlines at the first fin. In fact the convergence of the streamlines (see fig 5 (a)), which increases with the pressure ratio, leads to an acceleration of the flow and therefore to a decrease of the static pressure in the cavity.

It was observed that the discharge coefficient of the baseline case changes from the first to the second fin. To properly compare the

FIGURE 6. DIMENSIONLESS WORK-PER-CYCLE CONTOUR PLOTS OF THE 0TH NODAL DIAMETER AS A FUNCTION OF THE DIMENSIONLESS FREQUENCY, $\bar{\Omega}$, AND TORSION CENTRE, $\tilde{e}h'$, FOR A TWO-FIN STRAIGHT SEAL. LEFT: CFD RESULTS. RIGHT: ANALYTICAL HIGH-ORDER MODEL (CORRAL ET AL., 2020). BLACK LINES MARK THE STABILITY LIMIT ($\tilde{W}_{cyc} = 0$) ON THE HPS.

numerical results with the model predictions we need to incorporate the effective gaps in the formulation. Since the discharge coefficients of the first and second fin are different ($C_{d,1} \neq C_{d,2}$) this automatically implies that if the geometric gaps are identical, the effective clearances are not ($H_{1,eff} \neq H_{2,eff}$). The discharge coefficients are consequently estimated as $C_{d,1} = 0.76$ for the inlet fin and $C_{d,2} = 0.9$ for the the outlet one. The CV model includes the effective gaps by using the effective non-dimensional seal clearance parameter, $\tilde{e}_{eff}$ (described in Vega and Corral (2018b)) instead of the nominal one.

Work-per-Cycle: 0th Nodal Diameter

In this section, the prediction of the analytical model is compared against the numerical results obtained with a 2D linearized Navier-Stokes solver for the 0th nodal diameter. The seal motion associated with this mode-shape does not create acoustic waves propagating in the azimuthal direction, and therefore there are no circumferential variations of the flow. The analytical expression of the work-per-cycle for the ND 0 is obtained from 2 for values of $St \rightarrow \infty$. The stability map in Fig. 6 compares the CFD simulations with the analytical model prediction. The non-dimensional work-per-cycle has been normalised by $|1 + \tilde{e}_{eff}h'|$. The unsteady simulations are obtained for a pressure ratio of $\pi = 2.0$, performing a frequency sweep up to a maximum dimensionless frequency of $\bar{\Omega} = 3.6$ for different torsion centers. The normalized work-per-cycle has been bounded $-1 < -\tilde{W}/|1 + \tilde{e}_{eff}h'| < 1$ to magnify the unstable zone. The matching is good and sound overall, and the critical work-per-cycle is very well predicted. This proves that most of the physics of the problem is retained by the analytical model.

The high-order model (Corral et al., 2020) predicts that for low-frequency of vibrations ($\bar{\Omega} \ll 1$) the perturbations are isothermic and the unstable zone expands up to $\tilde{e}_{eff}h' = -\gamma$ when $\bar{\Omega} \rightarrow 0$, this trend is retained by the CFD simulations. The non-isentropic nature of the perturbations at low-frequency is responsible for the bending of the stability limit on the HPS that otherwise would be a vertical line. As a consequence of the normalization adopted, the maximum and minimum of the $\tilde{W}_{cyc}$ occur when the seal is supported on the HPS, in a small interval around $\tilde{e}_{eff}h' = -1$. It is important to highlight that around this value the torsion centre is physically far away from the seal mid-point in the simulations ($r = +7L$) and the seal motion no longer corresponds to a torsion mode. This means that during the vibration, the difference between the two clearances is small and the seal is not able to generate high unsteady pressure in the inter-fin cavity. Due to that, the physical work-per-cycle is about two orders of magnitude less than that obtained when $|\tilde{e}_{eff}h'| \ll 1$. Moreover, one more effect can be observed from the simulations when $|r| \gg 1$. Figure 7 shows the out-of-phase component of the unsteady pressure for a pressure ratio of $\pi = 2.0$. It is clear that when the pivot point is far away from the inter-fin cavity ($|\tilde{e}_{eff}h'| \gg 1$) the unsteady pressure in the inter-fin cavity is not uniform anymore (see Fig. 7,B), and its magnitude is close to that of the downstream cavity. In this context, even a small fraction of the unsteady pressure generated in the outer cavities could be transmitted through the clearances to the inter-fin cavity, leading to misleading results when comparing with the model. Therefore, it is not surprising that simulations over-predict the magnitude of the work-per-cycle of the stable zone when the seal is supported on the HPS. On the contrary, when the pivot point is nearby the inter-fin cavity ($|\tilde{e}_{eff}h'| \ll 1$) the unsteady pressure generated in the inter-fin cavity (for the same amplitude of the seal displacement) is much higher and uniform in the whole volume (see Fig. 7,A).

FIGURE 7. INFLUENCE OF THE INLET/OUTLET CAVITIES: OUT-OF-PHASE COMPONENT OF THE UNSTEADY PRESSURE FOR $\pi_T = 2.0$. (A): Modes with pivot points nearby the inter-fin cavity ($|\tilde{e}_{eff}h'| \ll 1$). (B): Modes with pivot points far away from the inter-fin cavity $|\tilde{e}_{eff}h'| \gg 1$.

FIGURE 8. DIMENSIONLESS WORK-PER-CYCLE CONTOUR PLOTS AS A FUNCTION OF THE REDUCED FREQUENCY, St , AND THE DIMENSIONLESS TORSION CENTRE, $\tilde{e}h'$, FOR A TWO-FIN STRAIGHT SEAL. PRESSURE RATIO $\pi = 1.5$, NON-DIMENSIONAL FREQUENCY $\bar{\Omega} = 2.4$. LEFT: CFD RESULTS. RIGHT: ANALYTICAL HIGH-ORDER MODEL (CORRAL ET AL., 2020). BLACK LINES MARK THE STABILITY LIMIT ($\tilde{W}_{cyc} = 0$).

Work-per-cycle: Circumferential variations

The simulations of this section has been obtained by keeping the pressure ratio and the non-dimensional frequency, $\bar{\Omega}$, constant. This method is used to reduce the risk of crossing the acoustic resonances of the adjacent cavities of the seal. As a consequence, the circumferential variations has been accounted for in the St by changing the nodal diameter. In the high-order model, the critical reduced frequency, St_c , is obtained by imposing in Eq. 2 that $\tilde{W}_{cyc}(St_c) = 0$. It is readily obtain that this condition yields:

$$St_c^2 = \frac{1}{1 + \tilde{e}h'_{eff}(\bar{\Omega}, h')} \quad (7)$$

which is a generalisation of the expression derived in Corral and Vega (2018). The limiting value of $\tilde{e}h'$ below which seals supported on the HPS are always stable oscillates between $\tilde{e}_{eff}h' = -1$ for very low pressure ratios ($\tilde{h}' \rightarrow \infty$) and $\tilde{e}_{eff}h' \simeq -0.89$ for choked seals ($h' = 2.5$). The stability of seals whose torsion centres are located in this region is very sensitive to the pressure ratio.

Figure 8 represents the general stability map for a two-fin labyrinth seal operating at a pressure ratio of $\pi = 1.5$ for a fixed dimensionless frequency $\bar{\Omega} = 2.4$. The scale of the $\tilde{W}_{cyc}$ has been limited to the range $-0.5 < -\tilde{W}_{cyc} < 0.5$ to magnify the region about the stability limit. The non-dimensional torsion center has been evaluated by considering the effective clearance of the seal. The qualitative matching between the stability map obtained using linearized Navier-Stokes analyses and the one derived from the model is very good, especially when $\tilde{e}_{eff}h' < 0$. As predicted by the model, for a given torsion centre the resonant condition is always stable and is achieved at $St \simeq 1$. That condition corresponds to the maximum of the $\tilde{W}_{cyc}$ (see Eq. 1). The map of fig. 8 numerically proves for the first time that the seal can be stable independently of St when the support is located on the HPS. On the other hand, when the seal is supported on the LPS the CFD simulations show that the seal is unstable in a confined range of $\tilde{e}_{eff}h'$ unlike what is predicted by the analytical model. In order to clarify this point Fig. 9 displays the contribution to the work-per-cycle of the inner surface of the fins and the seal land. It is important

FIGURE 9. SEAL WORK-PER-CYCLE BREAKDOWN. LEFT: TOTAL $\tilde{W}_{cyc}$ OF THE SEAL. CENTRE: SEAL LAND CONTRIBUTION. RIGHT: CONTRIBUTION OF THE INNER SURFACE OF THE SEAL TEETH. BLACK LINES MARK THE STABILITY LIMIT ($\tilde{W}_{cyc} = 0$)

to recall that the analytical model assumes that the unsteady pressure is uniform on the inter-fin cavity and the contribution due to the surface of both fins cancels out. The numerical simulations confirm that this is actually the case except in the downstream fin which is impinged by the wall jet flow created in the inlet fin. Actually, the inner surface of both fins has a non-negligible net contribution to the stability of the seal. Figure 9 (right) shows that when the support is located on the LPS, the global effect of the fins is stabilising. This contribution reduces the unstable region observed on the LPS. On the other hand, the stability map of the land of the seal compares very well with the prediction of the analytical model in the whole range of the tested torsion centres. The shape of both contributions is alike which is an indication that the unsteady pressure on the fins is controlled by the same physical mechanism as the rest of the inter-fin cavity but with local effects in the downstream fin.

CONCLUDING REMARKS

The aeroelastic stability of labyrinth seals under different working conditions and vibration mode-shapes has been compared with the predictions of an analytical model. It has been found that it is essential to inform the model with the correct parameter values derived from the steady-state numerical simulations, especially the effective gaps which are the most critical parameters to allow a proper comparison with the model. The non-isentropic perturbations described in the high order model have been observed in the CFD simulations at low vibration frequencies as expected. The largest discrepancies between the numerical results and the model predictions have been observed when the discharge time of the seal is much smaller than the characteristic time of the vibration ($\Omega \ll 1$), especially when the seal is operating at high pressure ratios and supported on the HPS. However, this regime is of modest engineering interest since in practice these frequencies are extremely low. In this context, the effect of the non-uniform unsteady pressure field in the inter-fin cavity due to the outer cavities has been highlighted. However, the matching with the analytical model is excellent in a wide range of operative conditions, especially when the seal is operating at low-pressure ratios. It has been found that the range of pivot centres located on the LPS at which the seal is unstable is significantly reduced due to the stabilising contribution of the inner surface of the teeth. It is concluded that the model can predict not only the trends of the work-per-cycle of the seal but also quantitative results.

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NOMENCLATURE

a_0	Speed of sound in the cavity
CV	Corral and Vega (2018) Model
h'	Seal pressure function
H	Fin clearance
HPS	High-Pressure Side
L	Seal cavity length
LPS	Low-Pressure Side
$\dot{m}$	Mass flow per span-wise or circumferential length unit
ND	Nodal diameter
p_c	Cavity static pressure
p_e	Exit static pressure
P_0	Inlet total pressure
r	Torsion center position
R	Cavity radius
s	Cavity height
St	Vibration-to-acoustic frequency ratio
CC	Cavity center
t_d	$= \frac{p_{c,s} V_{c,s}}{\dot{m}_s a_0^2}$. Discharge time
W_{cyc}	Work per cycle

Greek symbols

γ	Heat capacity ratio
π	$= P_0/p_c$. Cavity pressure ratio
π_T	$= P_0/p_e$. Total pressure ratio
π_c	$= \pi_T/\pi$. Pressure ratios relationship
ω	Vibration angular frequency (rad/s)
Ω	$= \omega t_d$ Non-dimensional discharge time
$\Delta\theta$	Rotation angle
τ	Non-dimensional time

Super-scripts

$\sim$	Non-dimensional values
$'$	Time perturbation

Sub-scripts

e	Exit
c	Cavity
cyc	cycle
s	Steady state

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