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NUMERICAL INVESTIGATION OF WALL COOLING EFFECT ON THE TRANSONIC COMPRESSOR PERFORMANCE

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ABSTRACT

In recent years, the effect of heat transfer in the compressors has been investigated. It shows that cooling via compressor surface can improve the overall performance of the axial compressor. In this paper, the mechanism of cooling effect on the transonic flow is studied. Casing cooling is applied in a transonic compressor rotor and a one and a half stage transonic compressor modelling the F class gas turbine first stage, respectively. Effect of cooling is numerically investigated at the near peak efficiency point. By changing the temperature near the casing, both the velocity and speed of sound in front of the rotor near the tip are affected. Cooling affects the shock wave in the flow path, which improves the total pressure rise capability in the tip region. Further, cooling in the inlet guide vane (IGV) is also applied together with casing cooling in the 1.5 stage compressor. It can also improve the aerodynamic performance of compressors.

INTRODUCTION

Multistage axial compressor is one of the major three components of gas turbines. The aerodynamic performance of compressor affects the efficiency of Brayton cycle. To improve the efficiency of cycle, total pressure ratio of the multistage compressor should be increased without an increase in the number of stages, and the efficiency and operating range of the compressor cannot decreased. Thus, complex flow phenomenon, such as tip leakage flow, shock wave, corner separation, etc., should be controlled.

In the compressors, surfaces including blades, hub and casing are usually considered as adiabatic walls. However, the effects of heat transfer between the structure and flow are investigated recently. Surfaces in the compressor are considered as non-adiabatic wall.

Since 2011, Ito and Nagasaki (Ito and Nagasaki, 2011; Ito et al., 2012; Ito et al., 2014; Ito et al., 2015) proposed a new intercooled recuperated (ICR) method for aero-engines and studied the effect on the Brayton cycle. They used the

stator vanes in the compressor as the air cooler and the vanes in the bypass duct as the radiator in the intercooling system. Five serial cooling channels are applied in the compressor vanes for the convective heat transfer and supercritical carbon dioxide is the coolant. This technique uses already equipped components in engines to increase the efficiency of Brayton cycle, without an increase in space and weight caused by additional heat exchangers.

In the precooled turbojet (PCTJ), precooling of the inlet air is widely applied. It can be achieved using a heat sink (such as a cryogenic fuel), or via mass injection (such as water or liquid oxygen). Shah and Tan (Shah and Tan, 2007) studied the overall performance of an axial compressor with heat extraction via blade surfaces. All the surfaces, including blades, hub and casing, are specified as a wall with constant temperature. Numerical results by mean-line method and CFD method indicate that cooling can increase the total pressure ratio and increase the choked mass flow rate with the same rotating speed, which increases the shaft power of compressors.

In the re-accelerated transient (also called as “Bodie” transients) of aero-engines, the rotating speed of compressor rapidly changes. Due to the thermal inertia, component temperatures lag behind the main stream temperature. Consequently, there are heat transfer on the surfaces of the components. The main stream temperature rapidly increases in the acceleration of compressor, and it is higher than the temperature of solid wall. The wall of components has a cooling effect on the main stream. First, the unsteady change in temperature leads to the change in geometry of the compressor casing and the tip clearance, which affects the aerodynamic performance. Schulte et al. (Schulte et al., 2008) studied the effect of unsteady heat transfer on the tip clearance of the rotor. Besides, the effect of unsteady heat transfer on the surge margin is evaluated by mean-line method (Kiss and Spakovsky, 2018). It is found that by changing the deviation angle and the load distribution in each

stage, cooling can alleviate the modal-type stall at the low rotating speed and increase the surge margin.

However, literatures mainly focus on the effect on the overall performance by the mean-line method, without consideration of the cooling effect on the detailed flow field such as boundary layers. For the reason that complex flow structures such as the separation may occur in the compressors, the cooling effect on the aerodynamic performance of the boundary layer is required to be investigated. In 2018, wall cooling is applied to reduce the separation in a two-dimensional compressor cascade and a three-dimensional compressor stator vane (Liu et al., 2018).

Since the total pressure ratio and efficiency of the axial compressor significantly affect the overall performance of the gas turbine, especially near the design point of the compressor, it is necessary to conduct an in-depth study on the detailed flow field near the design point. Because that transonic rotors are applied for the first stage in the modern compressors to achieve higher total pressure ratio, it is necessary to investigate the cooling effect in the transonic compressors.

The remainder of this paper is organized as follows. A brief description of the numerical method are presented. Next, the mechanism of wall cooling effect on the boundary layer is investigated. Finally, the effect of cooling in the casing and inlet guide vane (IGV) is studied in the transonic rotor NASA Rotor 37 and a 1.5 stage transonic compressor modelling the F class gas turbine first stage, respectively.

NUMERICAL METHOD

The single-passage, steady simulation is conducted using a cell-centered finite volume method.

The compressible form of the Reynolds-averaged Navier-Stokes (RANS) equations are expressed in the compact Cartesian notation.

The continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho u_j) = 0 \quad (1)$$

The momentum equation:

$$\frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_j}(\rho u_j u_i) = -\frac{\partial P}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} \quad (2)$$

The energy equation:

$$\frac{\partial}{\partial t}(\rho h) + \frac{\partial}{\partial x_j}(\rho u_j h) = \frac{\partial}{\partial x_j}[(\lambda + \lambda_{tur}) \frac{\partial T}{\partial x_j} + u_i \tau_{ij}] + S_E \quad (3)$$

where S_E is the volumetric heat source, and τ_{ij} is the stress tensor.

For the precise prediction of the boundary layer, S-A turbulence model is employed in the calculation of three-dimensional RANS equations. The compressor flow field is assumed to be fully turbulent.

The second-order upwind TVD scheme is applied for the estimation of convective fluxes. Local time stepping is selected as the time stepping technique.

The ideal gas equation of state, $\rho = (p_{abs} M)/(R_0 T)$, is used. The specific heat capacity at the constant pressure c_p

is specified as $1006.0 J / (kg \cdot K)$. The relationship between dynamic viscosity of air and temperature can be obtained using Sutherland's formula (Sutherland, 1893):

$$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0} \right)^n \left(\frac{T_0 + T_s}{T + T_s} \right) \quad (T > 120K) \quad (4)$$

where $\mu_0 = 1.7161 \times 10^{-5} \text{ (Pa} \cdot \text{s)}$, $T_0 = 273.0 \text{ (K)}$, $T_s = 124 \text{ (K)}$, and $n = 1.5$.

The relationship between thermal conductivity and temperature can be obtained:

$$k = -5.432 \times 10^{-4} + 1.039 \times 10^{-4} \cdot T - 5.0546 \times 10^{-8} \cdot T^2 \quad (5)$$

EFFECT OF COOLING IN THE TRANSONIC BOUNDARY LAYER

When wall cooling is applied, the decrease in temperature leads to a change in the density, dynamic viscosity and local speed of sound in the boundary layer, which affects the velocity profile.

In the turbulent boundary layer with an elevated freestream turbulence level, dynamic viscosity is of a lower order of magnitude than the turbulent eddy viscosity. As a result, the change in dynamic viscosity does not dominate. Temperature mainly affects the density to modify the distribution of turbulent eddy viscosity and velocity profile.

Due to the decrease in temperature in the boundary layer, density increases. The increase in density makes the stream tube near the wall contract, and results a thinner boundary layer due to mass conservation. Thinner boundary layer makes the velocity profile fuller in the boundary layer, as shown in Fig. 1. At the same distance from the wall, flow in the boundary layer has a higher velocity.

Figure 1 Variation of the velocity profile in the boundary layer.

In the viscous sublayer of the turbulent boundary layer, dynamic viscosity cannot be ignored because the turbulence is limited in this region. According to Equation 4, dynamic viscosity decreases with the decrease in temperature, and it leads to a higher velocity in the viscous sublayer.

For the transonic flow, the shock wave are affected by the relative Mach number in front of the shock. Local speed of sound is defined as:

$$c = \sqrt{\gamma RT} \quad (6)$$

which can be decreased by cooling. The decrease in speed of sound and the increase in velocity make the relative Mach number increase near the wall. Consequently, the shock can be strengthened.

EFFECT OF CASING COOLING ON A TRANSONIC COMPRESSOR ROTOR

NASA Rotor 37

A transonic compressor NASA Rotor 37 (Reid and Moore, 1978; Suder and Celestina, 1994) is applied, as shown in Fig. 2. Numerical method are verified and investigation of cooling effect are conducted in the single-row compressor. It has 36 blades, and the tip clearance is 0.356 mm. The design rotating speed and design total pressure ratio are 17,188 rpm and 2.106, respectively. According to the NASA's experiments, the hub with the axial coordinate from -2.64 mm to 45.21 mm are set to rotate with the rotor blades, and the rest of the hub is stationary.

Figure 2 Geometry of Rotor 37.

In the numerical simulation with the software Fluent, inlet total pressure (101,325 Pa), inlet total temperature (288.17 K), the flow direction (normal to the inlet) and turbulent viscosity ratio between turbulent viscosity and the dynamic viscosity (10) are specified on the inlet boundary. The static pressure at the hub is specified on the outlet boundary, and the radial distribution satisfies the radial equilibrium pressure distribution. Different operating conditions can be obtained by changing the outlet static pressure.

Because that shock wave exists at each blade height of Rotor 37, a large number of elements are required to accurately simulate the complex flow in the transonic compressor. The mesh is generated with structural elements, employing a HOH mesh topology, with O-grid covering the blade surface. Mesh independence study is conducted and the mesh with 6.25 million grids is applied. The mesh has 33 layers in the tip clearance of the rotor, and the distance between the solid wall and the first grid line is set to $y^+ < 2$.

Pressure ratio – mass flow characteristic and adiabatic efficiency – mass flow characteristic are shown in Figures 3 and 4, respectively. At the operating point with 80% choked mass flow rate, the distributions of the total pressure ratio

and the total temperature ratio along the blade height are shown in Figures 5 and 6, respectively.

Fig. 7 shows the contour of relative Mach number at the near peak efficiency point at 99.4% span. It can be notice that tip leakage flow injects from the suction surface near the front of the tip clearance due to the difference of the pressure between pressure surface and suction surface. A bow shock exists in the flow path and it interacts with the tip leakage flow in the tip region, as shown in Fig. 7. The shock also interacts with the boundary layer on the suction surface and leads to the separation on the suction surface, as shown in Fig. 8.

By comparing the numerical results with the experimental data, it can be obtained that the numerical method can simulate the complex flow phenomenon in the transonic compressor, and numerical results are in reasonably good agreement with experimental data by NASA.

Figure 3 Pressure ratio - mass flow characteristic.

Figure 4 Adiabatic efficiency - mass flow characteristic.

Figure 5 Distribution of total pressure ratio (98% choked mass flow rate).

Figure 6 Distribution of total temperature ratio (98% choked mass flow rate).

Figure 7 The contour of relative Mach number at the near peak efficiency point (99.4% span).

Figure 8 Limiting streamline on suction surface at the near peak efficiency point.

Casing Cooling in the Transonic Rotor

At the design rotating speed of Rotor 37, the mass flow rate is about 99.8% choked mass flow at the near peak efficiency point. For the near peak efficiency point, wall cooling is applied on the casing in the upstream of the leading edge, and the axial length of the cooling wall is 0.8 times the chord length of the rotor in the tip, as the grey region shown in Fig. 9. The temperature of the cooling casing is specified to 0.5 times the inlet total temperature, i.e., 144K.

Figure 9 Cooling on the casing.

With casing cooling, both the total pressure ratio and efficiency of the compressor are affected, as shown in Figs. 10 and 11. Because of the heat transfer via the surfaces of the compressor, efficiency is defined according to the recommendation by Shah and Tan (Shah and Tan, 2007) as:

$$\eta = \frac{\left(p_2^* / p_1^* \right)^{(\gamma-1)/\gamma} - 1}{T_2^* / T_1^* - 1 - \frac{Q}{\dot{m} c_p T_1^*}} \quad (7)$$

where Q is negative with the cooling wall.

It can be obtained from the numerical results that the pressure ratio can be increased by the casing cooling with the same inlet mass flow rate. The total pressure ratio increases by 0.52% at the near peak efficiency point. While, the mass

flow rate can be increased with the same total pressure ratio, which can also increase the shaft power of the compressor.

Figure 10 Effect of cooling on the total pressure ratio.

Figure 11 Effect of cooling on the efficiency.

It is the results of the casing cooling as the analysis mentioned above. Since the boundary layer on the casing upstream the leading edge is affected by the cooling, the velocity of the boundary layer in the absolute coordinate system is increased, as shown in Fig. 12. It can be noticed by the velocity triangle that the axial velocity of the flow increases and the angle of attack slightly decreases. Besides, the local speed of sound is significantly reduced by the cooling. As a result, the relative Mach number is significantly increased, which can be shown by the distribution of the relative Mach number in the radial direction at the upstream of the leading edge in Fig. 13. The same result can be also obtained by the distribution of the relative Mach number in the circumferential direction at 99.4% span in Fig. 14, where the area with circumferential angle greater than 5 degrees is the upstream of the shock wave, and that with circumferential angle less than 5 degree is the downstream of the shock wave. Since the relative Mach number upstream the shock increases, the shock is

strengthened, and the total pressure ratio of the tip region increases.

However, the efficiency is not only affected by the shock. It is also affected by the thickness of boundary layer, tip leakage flow and the interactions between them, which are all affected by cooling. In this numerical simulation, the efficiency increases. Although it is necessary to further investigate the effect of cooling on the efficiency of the compressor, conclusion can be obtained that the efficiency of the Brayton cycle can be increased by the increase in total pressure ratio.

Figure 12 Effect of casing cooling on the velocity triangle in the upstream of the leading edge.

Figure 13 Distribution of relative Mach number in the radial direction.

Figure 14 Distribution of relative Mach number in the circumferential direction (99.4% span).

EFFECT OF COOLING ON A 1.5 STAGE COMPRESSOR

A 1.5 Stage Transonic Compressor

In order to study the effect of wall cooling in multistage compressor, numerical investigation has been conducted in a one and a half stage transonic compressor modelling the F class gas turbine first stage in Tsinghua University, as shown in Fig. 15.

It includes an inlet guide vane (IGV), a rotor, a stator and an outlet guide vane (OGV). The number of blades in the rotor is 22, and that in the IGV, stator and OGV is 24. The tip clearance of the rotor is about 0.5% of the blade height. The rotating speed can be changed from 0 to 24,840 rpm. Tab. 1 shows the design parameters of the compressor.

(a)

(b)

Figure 15 Geometry of the one and a half stage transonic compressor.

Table 1 Design parameters

Design parameters	Value
Total pressure ratio	1.3
Tip Mach number	1.25
Design rotating speed (rpm)	24,840
Design mass flow rate (kg/s)	12
Number of blades in the rotor	22
Number of blades in the stator	24

Inlet total temperature is 288.15 K, and inlet total pressure is 101,325 Pa. Inlet flow direction is normal to the inlet. Different averaged static pressure is specified at the outlet, which also matches the radial equilibrium condition at each operating condition. Periodic boundary conditions are applied on the pitchwise boundaries of the flow domain. Nonslip conditions are applied on the blade surface, hub, and shroud. For the original compressor, the wall is adiabatic.

Mesh independence study is also conducted and the distance between the solid wall and the first grid line is set to $y^+ < 5$ for the S-A turbulence model. The overall performance in the numerical result is compared with the experimental data at the rotating speed of 16,000 rpm, 18,000 rpm and 24,840 rpm, as shown in Fig. 16.

Figure 16 Measured and predicted overall pressure ratio - mass flow characteristic.

Effect of Cooling on the Overall Performance

In this paper, three kinds of cooling method are employed in the one and a half stage transonic compressor at the near peak efficiency point with the design rotating speed of 24,840 rpm. The first case is the original compressor with the adiabatic wall. In the second case, a constant temperature of 140 K is specified on both the casing of IGV and the casing of the rotor. In the third case, only the temperature of the blade surface of IGV is specified as 140 K. Cooling is applied on both the IGV and casing in the fourth test case.

First, the comparison between the aerodynamic performance in case 1 and case 2 is carried out to study the effect of casing cooling in multistage compressor. Then, IGV cooling is applied in case 3 and case 4 to investigate the effect of IGV cooling.

Figs. 17 and 18 shows the predicted overall pressure ratio - mass flow characteristic and efficiency - mass flow characteristic of the four cases at the design rotating speed of 24,840 rpm. With the same inlet mass flow rate at a given rotating speed, the total pressure ratio increases and the maximum mass flow rate increases because of cooling, which is similar to the conclusion obtained by the study of Rotor 37. Cooling in the case 4 has a more significant effect than the cooling in case 2 and case 3 due to a larger area of heat transfer surface.

Figure 17 Effect of cooling on the total pressure ratio.

Figure 18 Effect of cooling on the efficiency.

Effect of Casing Cooling on the Aerodynamic Performance in 1.5 stage compressors

In the multistage compressor, cooling on the casing of IGV can affect the boundary layer upstream the tip of leading edge of the rotor in the first stage.

Because that the tip clearance of the rotor is about 0.5% of the rotor height, the tip leakage flow and the shock can be clearly shown by the flow filed at 99% span. Fig. 19 shows the contours of relative Mach number at 99% span. There is a bow shock in front of the leading edge. Behind the bow shock, the air reaccelerates at the leading edge of the rotor. Because of the pressure gradient between suction surface and pressure surface at the rotor blade tip, the tip leakage flow develops near the leading edge of the tip clearance on suction side.

Casing cooling in the IGV makes the boundary layer thinner and increases the relative velocity near the tip, and casing cooling in the rotor decreases the local speed of sound. Consequently, the relative Mach number upstream the bow shock increase, which can be shown by Figs. 19 and 20.

Fig. 21 shows the total pressure and static pressure in the streamwise direction at 99% span. It can be observed that pressure significantly increases due to the shock at the position of 0.7. With the decrease in the casing temperature in case 2, it is noticed that the stronger shock near the tip makes the total pressure and static pressure behind the shock higher than those of the case with adiabatic casing. It indicates that casing cooling can increase the total pressure rise capability in the tip region of the rotor.

Figure 19 The contour of relative Mach number at 99.4% span.

Figure 20 Distribution of relative Mach number in the radial direction.

Figure 21 Distribution of pressure in the streamwise direction (99% span)

Effect of IGV Cooling on the Aerodynamic Performance

Together with the casing cooling, IGV cooling is also applied to investigate the effect of wall cooling in the 1.5 stage compressor.

Cooling can be applied in the IGV as a compact inlet air-cooling method. The effect of cooling IGV is similar to the heat exchanger in the traditional inlet air-cooling. With the same total temperature at the inlet of the IGV, IGV cooling decreases the total temperature at the inlet of the rotor in each spanwise location, as shown in Fig. 22. In the near-peak efficiency operating condition where the mass-averaged outlet static pressure is 115 kPa, total temperature of the main stream decreases by 1.3 K, which is substantial for the compact heat transfer method. The decrease in the total temperature results in a higher total pressure at the inlet of rotor, as shown in Fig. 23.

According to the definition of corrected rotating speed and corrected mass flow rate:

$$n_{corrected} = n \sqrt{\frac{288.15}{T_{in,R1}^*}}, \quad (8)$$

$$G_{corrected} = G \frac{101325}{P_{in,R1}^*} \sqrt{\frac{T_{in,R1}^*}{288.15}}, \quad (9)$$

because of the increase in the inlet total temperature $T_{in,R1}^*$ and the decrease in the total pressure $P_{in,R1}^*$ of the rotor by IGV cooling, the corrected rotating speed increases with the same actual rotating speed. At the same time, corrected mass flow rate decreases with the same actual mass flow rate. Consequently, the rotor has a lower corrected mass flow rate and a higher corrected rotating speed with the same actual mass flow rate and rotating speed, and it leads to a higher total pressure ratio. The numerical results in Fig. 17 indicate that.

In the analysis of the detailed flow field, it can be also noticed that IGV cooling increases the velocity in the boundary layer on the IGV surface. As a result, the relative Mach number upstream the rotor slightly increases, as shown in Fig. 20. It indicates that IGV cooling can also increase the total pressure ratio by increasing the relative Mach number upstream the shock.

Figure 22 Distribution of total temperature at the inlet of the rotor.

Figure 23 Distribution of total pressure at the inlet of the rotor.

CONCLUSIONS

In this paper, effect of cooling on the detailed flow field in the transonic compressor has been investigated. The mechanism is analysed in the boundary layer, and the wall cooling is applied in the transonic rotor NASA Rotor 37 and a 1.5 stage transonic compressor respectively. Wall cooling makes the boundary layer thinner and increases the velocity near the wall. At the same time, wall cooling decreases the local speed of sound, which leads to a higher relative Mach number. With cooling on the casing upstream the leading edge of the transonic rotor, the increase in relative Mach number strengthens the shock and increases the total pressure rise capacity of the compressor at the near peak efficiency point. In the 1.5 stage transonic compressor, the casing cooling in IGV increases the velocity and casing cooling in the rotors decreases the local speed of sound, which also result in a higher relative Mach number and higher total pressure ratio in the tip region. Besides, IGV cooling can be also applied together with casing cooling to further achieve a higher pressure ratio of the compressor. The increase in total pressure ratio by wall cooling in the transonic compressor at the near peak efficiency point can improve the overall efficiency of the gas turbine.

NOMENCLATURE

G	mass flow rate
n	rotating speed
P	static pressure
P^*	total pressure
T	temperature
T^*	total temperature
u_j	components of velocity
u	peripheral velocity
v	absolute velocity
w	relative velocity
y^+	non-dimensional wall distance

Subscripts

1	adiabatic
2	casing cooling
in, R1	inlet of the rotor

Acronyms

CFD	computational fluid dynamics
IGV	inlet guide vane
LE	leading edge
OGV	outlet guide vane
PS	pressure surface
RANS	Reynolds-Averaged Navier-Stokes
SS	suction surface
TE	trailing edge
TVD	total variation diminishing

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