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Numerical Analysis of Combustion Instability in Annular Combustion Chamber with Spatially Distributed Heat Release Rate Model

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ABSTRACT

In order to analyze the thermo-acoustic instability in combustion chamber, an unsteady heat release rate model is established, which is based on the linear analysis of chemical reaction kinetics. In this model, the fluctuation of heat release rate is determined by the fluctuation of fuel mass fraction, density, temperature and pressure, and influenced by both the convection of flow and propagation of acoustic wave. To verify this model, the unsteady heat release rate of premixed swirling flame and staged flame are calculated respectively, and compared with the experimental data. It is demonstrated that the transport velocity is the key parameter affecting the heat release rate fluctuation. Subsequently, the heat release rate model is employed to analyse the thermo-acoustic instability of the annular combustor. The growth rate of the characteristic frequency of the combustor is obtained by analytically solving multi-dimensional acoustic wave equation in the frequency domain. The solution is used as the basis for determination of the self-excited thermo-acoustic instability of the combustor. The results predicted by this model are compared with the results obtained by high fidelity numerical method such as large-eddy simulation. The comparison show that three types of self-excited oscillation frequencies corresponding to the first-order circumferential, first-order longitudinal and first-order circumferential and longitudinal mixed modes of the combustor, can be properly predicted by this model.

INTRODUCTION

Combustion instability occurs in lean fuel combustion chambers such as civil aviation engines and gas turbines which can cause severe damage to combustor components (Lieuwen, 2005). The mechanism of its inception is due to the acoustic field fluctuation, induced by unsteady combustion, interacting with the combustion flame. If the acoustic fluctuation and heat release rate oscillation occur in-phase, the continuous increase of the amplitude of pressure and heat release fluctuation will be initiated, and more intense combustion fluctuations will be introduced, as well as

many other undesirable transient combustion phenomena such as flame blowout, flash back and increased emissions.

At present the widely used prediction method of combustion instability is based on a linearized small disturbance theory to analyze the growth rate of acoustic pressure in a combustion chamber. The method takes the whole combustion chamber as the research object and the unsteady heat release rate is coupled into the control equation of acoustic field as the source term. The growth rate of the corresponding frequency is obtained by calculating the eigenvalue of the control equation and it is possible to estimate the development trend of the acoustic pressure fluctuation amplitude at this frequency. If the amplitude of the pressure fluctuation increases continuously, it means that the combustion instability has been triggered in the combustion chamber.

Although the unsteady heat release in the combustor is influenced by multiple physical factors such as the geometry, boundary conditions, thermal dynamics, and the acoustic pressure modes, the perturbed heat release is usually correlated to the acoustic fluctuation of the combustor in the calculation for the purpose of introducing no new unknown variables. The unsteady heat release fluctuation source term is coupled with the pressure (or velocity) fluctuation, which is called the flame transfer function (Stow and Dowling, 2001), is the core of the model for combustion instability prediction and determines the accuracy of prediction results.

The flame transfer functions used currently can be roughly divided into two categories, one based on global or lumped heat release analysis (Crocco and Cheng, 1956; Dowling, 1995; Stow and Dowling, 2001; Dowling and Stow, 2003; Mongia et al., 2003; Evesque and Polifke, 2003; Ecksteom and Sattelmayer, 2006; Temme et al. 2014; Yoon, 2017), the other based on specific flame morphology (Park et al., 2002; Lieuwen, 2003; Schuller et al., 2003; You et al., 2005; Huang et al., 2006; Huang and Yang, 2009; Palies, 2011; Candel et al. 2014; Magina et al., 2015; Magina and Lieuwen, 2016; Rani V, and Rani S, 2018; Li et al., 2018; Yao and Zhu, 2012). A typical global heat release model is

so-called $n \sim \tau$ model, which was first proposed by Crocco in 1956 for the combustion instability study of rocket engines. In this model, it is considered that the heat release pulsation in the combustion chamber originates from a reference point in the upstream flow field and transmitted to the combustion zone with flow convection. Here, n describes the amplification of the fluctuation amplitude of the heat release in flame zone to the fluctuation amplitude of the reference point, and τ is the convection time from the reference point to the flame. The premise of applying this model is that the thickness of flame front is acoustically compact which can be deemed as a lumped point. Thus this model is basically applicable in the low-frequency, longitudinal thermo-acoustic oscillation of the slender combustor, and suitable for the analysis of one-dimensional, two-dimensional low-frequency thermo-acoustic problems. For example, Dowling (1995, 2003), Polifke (2002), Stow (2001), Temme and Driscoll (2014), Eckstein (2006) and others have already worked on the acoustic network analysis in single injector, single swirler, simplified model annular combustor, and some preliminary instability analysis focused on the engineering applications in such as GE (Mongia et al., 2003), Siemens (Yoon, 2017) and other lean burn combustors.

However, in fact, the flame shape in industrial combustor is very complicated, which involves the swirling, spray, mixing and evaporation. The flame heat release fluctuation is a nonlinear superposition of various physical factors in the combustion and fuel injection zone. The applicability of the simple global heat release model is limited. Therefore, more physical quantities that contribute to the combustion instability are required to take into account in flame transfer function. Lieuwen (2003) reviewed the development of unsteady heat release models considering the flame details and two main method can be used to calculate the heat release rate, one is to analytically solve the transport equation of the flame front, the other is to analyze the chemical reaction rate. For the former one, the key technical concept is to solve the flame surface equation and integrate the heat release rate along the flame shape. Schuller (2003), Palies (2011), Candel (2014), Huang (2009), You (2005), Rani (2018) and Li (2018) etc., have already studied the heat release model of premixed flame considering the interaction with turbulent fluctuation, equivalent ratio fluctuation and swirl vortex and flame staging by solving the G equation of premixed flame; Magina (2015, 2016) and Yao (2012) studied the heat release model of diffusion flame by solving the transport equation of mixture fraction. For the other approach, as reported in the work of Park (2002), the correlation between the heat release rate and the fuel mass fraction, the flow rate fluctuation is mainly determined by linearly solving the Arrhenius formula.

In practical application, the form of the flame transfer function needs to be consistent with the solution of acoustic equations. To analyze low-frequency combustion instability in one-dimensional or two-dimensional thermo-acoustic network models, simple $n \sim \tau$ model or other global heat release model are applicable. For the analysis of three-

dimensional combustion instability at medium and high frequency, it is necessary to describe the unsteady heat release of the flame with spatial distribution, from which more physical details and acoustic modal characteristics can be acquired. For instance, Prasad and Feng (2004a, 2004b) utilized the linearized Euler equation for the study of 3-D modes combustion instability in augmentor considering the effect of the transportation of the fuel component. To discretely solve the Helmholtz equation in frequency domain, which method was adopted by Nicoud (2007), Andreini (2013), Campa (2014), Silva (2013), Poinot (2017), etc., the location of the time-averaged flame zone can be determined by CFD calculation, and the spatial distribution of the discrete heat release pulsation in that area can be used in the prediction of the combustion instability. Mongia et al. (2003) proposed a method to correlate the upstream fluctuation source point with local unsteady heat release rate by tracing back along the streamline, therefore, the flame transfer function can be expressed as a set of $n \sim \tau$ models. All these flame transfer functions have the same characteristics of the spatial distribution of the stimulated flame and the thermodynamic quantities are related with the heat release model through these functions, which results a more universal flame transfer function for more accurate prediction. In this paper, a spatial distributed unsteady heat release model based on linearization analysis of chemical reaction kinetics is proposed and verified using the published experimental data, and finally this model was employed to predict the combustion instability of a model annular combustor.

UNSTEADY HEAT RELEASE MODEL

The acoustic wave equation including the heat release fluctuation is shown in equation (1):

$$\frac{1}{\bar{C}^2} \frac{\partial^2 p'}{\partial t^2} - \bar{\rho} \nabla \cdot \left(\frac{1}{\bar{\rho}} \nabla p' \right) = \frac{\gamma - 1}{\bar{C}^2} \frac{\partial Q'}{\partial t} \quad (1)$$

where p' is the fluctuated pressure, $\bar{C}$ is the time-averaged acoustic velocity, $\bar{\rho}$ is the time-averaged density, γ is the specific heat ratio, and Q' is the fluctuated heat release rate per unit volume. The flame transfer function is the correlation of Q' and the pressure fluctuation p' .

Establishment of the Flame Model

A premixed or partially premixed combustion organization is generally applied in industrial combustion chamber (Zhang et al., 2013), that is, the fuel and air flow into the front part of the combustion chamber through different flow paths, and a mixture of fuel and air is obtained and ignited after being sprayed into the combustion chamber through the nozzle, which is shown in Figure 1.

Figure 1 Schematic diagram of reactive flow in an industrial combustor

A simplified model is constructed based on two main physical features during the process mentioned above, as shown in Figure 2.

Figure 2 Schematic diagram of heat release model

The fuel is mixed with air in a mixing zone (expressed as MZ) and burned in a flame zone (FZ). In the mixing zone, the relevant physical quantities are considered to be transported with the flow, and the chemical reactions take effect in the flame zone with heat generation and fuel consumption. The transport equation for the mass fraction of fuel can be expressed as:

$$\frac{\partial}{\partial t}(\rho Y_f) + \nabla \cdot (\rho \bar{u} Y_f) = \begin{cases} 0 & x_i \in \Omega_{MZ} \\ S_f & x_i \in \Omega_{FZ} \end{cases} \quad (2)$$

where Y_f is the fuel mass fraction, S_f is the fuel consumption rate caused by combustion, which can be expressed by the Arrhenius formula:

$$S_f = -W_f A_f T^\beta \left(\frac{\rho Y_f}{W_f} \right)^{n_f} \left(\frac{\rho Y_o}{W_o} \right)^{n_o} \exp\left(-\frac{Ea}{RT}\right). \quad (3)$$

where W_f is the molar mass of the fuel, W_o is the molar mass of oxygen, Y_o is the mass fraction of oxygen, T is the temperature, ρ is the density, Ea is the activation energy, R is the ideal gas constant, A_f is the frequency coefficient, n_f and n_o are the reaction order of fuel and oxygen in the chemical reaction equation, respectively, β is temperature coefficient and always be zero.

On the other hand, from the perspective of energy conservation, we have:

$$Q = \frac{-S_f \Delta H}{\rho}. \quad (4)$$

If Eq. (4) is substituted into Eq. (2) and is performed linearization, the fluctuated quantities can be expressed as:

$$\frac{\partial Y'_f}{\partial t} + \bar{u}_i \cdot \nabla Y' = \begin{cases} 0 & x_i \in \Omega_{MZ} \\ -\frac{Q'}{\Delta H} - u'_i \cdot \nabla \bar{Y} & x_i \in \Omega_{FZ} \end{cases} \quad (5)$$

and $i = i, j, k$, which are the Cartesian coordinate directions.

Correlation of Unsteady Heat Release

Substituting Eq. (3) into Eq. (4) and linearizing it, the unsteady heat release fluctuation can be expressed as:

$$\begin{aligned} \frac{Q'}{\bar{Q}} = & n_f \frac{Y'_f}{\bar{Y}_f} + n_o s \frac{Y'_f}{Y_{o,0} - s Y_{f,0} + s \bar{Y}_f} \\ & + (n_f + n_o - 1) \frac{\rho'}{\bar{\rho}} + \frac{Ea}{RT} \frac{T'}{\bar{T}} \end{aligned} \quad (6)$$

where, $Y_{f,0}$ is the initial mass fraction of fuel in the calculation domain, $Y_{o,0}$ is the initial mass fraction of oxygen in the calculation domain. The mass fraction fluctuation of fuel is obtained by solving the Eq. (5) and the density fluctuation is isentropic in the mixing zone and affected by the increase of the entropy in the flame zone, which can be expressed as:

$$\frac{\rho'}{\bar{\rho}} = \begin{cases} \frac{1}{\gamma} \frac{p'}{\bar{p}} & x_i \in \Omega_{MZ} \\ \frac{1}{\gamma} \frac{p'}{\bar{p}} - \frac{s'}{Cp} & x_i \in \Omega_{FZ} \end{cases} \quad (7)$$

Regardless of the temperature fluctuation of the mixing zone, the temperature fluctuation of the flame zone is:

$$\frac{T'}{\bar{T}} = \begin{cases} 0 & x_i \in \Omega_{MZ} \\ \frac{\gamma-1}{\gamma} \frac{p'}{\bar{p}} + \frac{s'}{Cp} & x_i \in \Omega_{FZ} \end{cases} \quad (8)$$

The entropy increase caused by combustion can be represented by the entropy transport equation:

$$\frac{Ds}{Dt} = \frac{Q}{\rho T}. \quad (9)$$

Linearized expression of Eq. (9) is available as

$$\frac{\partial s'}{\partial t} + \bar{u}_i \cdot \nabla s' + u'_i \cdot \nabla \bar{s} = \frac{R\bar{Q}}{\bar{p}} \left(\frac{Q'}{\bar{Q}} - \frac{p'}{\bar{p}} \right). \quad (10)$$

and i is marked as the vector direction in the Cartesian coordinate system.

Boundary Conditions

In the case of diffusion flame, the inlet boundary is:

$$Y_f(x_0) = Y_{f,0}. \quad (11)$$

In the mixing zone, the fuel nozzle generally has choked end, and the pressure fluctuation in the combustion chamber hardly affects the flow change of the fuel, so the acoustic pressure fluctuation is mainly introduced by the inlet air flow fluctuation. The fuel mass fraction fluctuation at the boundary can be deduced as:

$$\frac{Y'_{x0}}{\bar{Y}_{x0}} = - \left(\frac{p'}{\gamma \bar{p}} + \frac{u'}{\bar{u}} \right)_{x_0}. \quad (12)$$

The remaining boundary conditions are $s'(x_0) = 0$, $T'(x_0) = 0$. In the calculation and analysis of the heat release rate fluctuation, Eq. (5), (7), (8), and (10) are substituted into Eq. (6) for simultaneous solution, and combined with the boundary conditions, the heat release rate fluctuation can be acquired.

VERIFICATION OF HEAT RELEASE MODEL

In the design of practical combustor, in order to achieve the working adaptability under a wide aerodynamic and thermal dynamic range, staged flame is usually carried out for combustion organization. Taking the low emission civil aero-engine combustor as an example (Foust et al., 2012), the flame structure is swirling and staged along the radial direction of the dome center line, where in the swirling core is a diffusion flame and in the outer area is a premixed flame, as shown in Figure 3. Since the equivalent ratio and the heat transfer characteristics vary in the region where the premixed flame and the diffusion flame intersect, the simulation of the concentric stratified flame is more complicated. For the purpose of verification of the applicability of the model, the two types of flame are studied respectively. First, a single premixed swirling flame is selected to calculate the heat release rate, and then the staged premixed flame with a pilot flame is calculated. Both calculations are compared with the published experimental study regarding the heat release rate.

Figure 3 Typical flame types in an industrial combustion chamber (Foust et al., 2012).

Model of Premixed Swirling Flame

The characteristics of heat release fluctuation of methane/air premixed swirling flame under acoustic excitation are systematically studied in literature (Palies, 2011). In the experiment, a mixture of methane and air is fed

into a relatively large volume plenum. The mixed gas is injected from the bottom of the plenum towards the top via the pressure difference and burned outside of the mixing chamber. An acoustic loud speaker is placed beneath the plenum, and the velocity fluctuation of the mixed air is driven by the drum vibration of the loud speaker. In each test, the vibration frequency of the loud speaker is a preset single frequency and the principle of the test equipment is shown in Figure 4.

Figure 4 Schematic diagram of forcing flame test rig

The heat release rate is calculated from the chemiluminescence imaging of the OH^* radical recorded by a CMOS camera with an intensifier. The colored image in Fig. 4 is a time-averaged 2D distribution of OH^* radical. The velocity fluctuation of a single point is measured utilizing a hot wire anemometer or a laser Doppler velocimeter. The overall heat release of the flame is acquired via a photomultiplier tube to obtain the instantaneous light intensity of the OH^* radical over the full flow field. Therefore, the relationship between the whole heat release rate fluctuation and the relative fluctuation of the flow at the observation point can be described approximately:

$$\frac{Q' / \bar{Q}}{u' / \bar{u}} \approx \frac{I'_{\text{OH}} / \bar{I}_{\text{OH}}}{u' / \bar{u}}. \quad (13)$$

Flame Response Study of Premixed Swirling Flame

According to the flame spatial distribution described in reference (Palies, 2011), the heat release calculation model is established, and the orthogonal grid is used for flame zone meshing, as shown in Figure 5. According to the calculation method of unsteady heat release rate in Eq.(6)~Eq.(11), the excitation velocity of different frequencies is added to the inlet of the flame zone shown in figure 5, and the fluctuation of heat release rate in this area is calculated. It can be seen from the above deduction, that in the computational domain the transport effect of time-averaged and fluctuating velocity is the prime factor which can bias the analysis of the characteristics of fluctuating transport. The time-averaged transport velocity can be obtained either from experimental measurement or from CFD calculation. Due to the limited

time-averaged velocity information can be reached in the literature, the flow field information is calculated by CFD which will not be described here in detail, and the calculated velocity results are used as a known input parameter for this simulation. The fluctuating velocity in the computational domain includes the velocity corresponding to the fluctuating pressure in Eq.(1) and the convective velocity of the vortex wave after the air swirler. The unsteady heat release rate fluctuation in the whole flame region is solved in the frequency domain. If the spatial distribution of the heat release rate at different disturbance frequencies is resolved, a volume integral of the heat release rate fluctuation along the flame zone is carried out to obtain a transfer function similar to that of Eq. (13), as shown in Figure 6. The transfer function is the relationship between the fluctuation of lumped heat release rate and the excitation frequency, including the changing tendency of the fluctuation amplitude (gain) and the characteristics of the phase of the fluctuation of heat release rate after excitation. Additionally, a comparison between the calculation method and the experimental results (Palies, 2011) has been displayed in Figure 6.

Figure 5 Spatial profile and computational grid of swirling flame.

Figure 6 Fluctuation of the swirling flame heat release rate vs. excitation frequency (top: fluctuation amplitude; bottom: fluctuation phase)

It can be seen from Fig.6 that the calculated fluctuation amplitude and phase of the heat release rate show good

agreement with the experimental results. There are two main factors affecting the heat release fluctuation of swirling flame, one is the lifting effect of longitudinal disturbance on the flame, and the other is the circumferential disturbance diffracted by the cyclone and transported to the flame front and detached with the flow. At different excitation frequencies, the two effects falls and rises and work together to determine the characteristics of the fluctuation of the flame front fluctuation and the global heat release fluctuation value after integration can be evaluated. Figure 7 shows the spatial distribution of the amplitude of the heat release fluctuation at four typical excitation frequencies, and the corresponding experimental measurements can be referred to the work of Palies (2011).

From the comparison shown in Fig. 7, it can be seen that under the excitation disturbance of 20 Hz, the heat release fluctuation of the flame exhibits a longitudinal distribution along the flow direction. Under the excitation disturbance of 60 Hz, the flame presents the characteristics of vortex shedding with circumferential swirling, thus the region with strong heat release rate fluctuation is concentrated on the edge of the swirl flow. Under the disturbance of 90 Hz, the heat release rate fluctuation still shows the characteristics with stretching and contracting along the extension direction of swirl angle, so the heat release fluctuation achieved in the simulation locates along the flame extension direction as well. Under the external excitation disturbance of 230 Hz, the heat release fluctuation characteristic is similar to that of 60 Hz, and the strong and weak intensity change alternately with the vortex shedding. The results of calculation and analysis show good agreements with the experimental results (Palies, 2011), which indicates that the calculation of the heat release fluctuation demonstrated in this paper has good applicability.

Figure 7 Spatial distribution of heat release rate fluctuation at four different excitation frequencies

Model of Staged Flame

The experimental study on the characteristics of the heat release fluctuation of a staged flame has been carried out (Li et al., 2018). In the study, the main stage is a premixed V-shaped methane/air flame with an equivalent ratio of 0.7, and a pilot conical diffusion flame is formed by direct injection of methane into air with an equivalent ratio of 0.9. At the exit of the fuel injector, the diffusion flame and premixed flame intersect with each other. The schematic diagram of study object is shown in Figure 8. According to the flame structure observed experimentally, the geometric model of the heat release zone and its grids resolution are shown in Figure 9.

Figure 8 Schematic diagram of staged flame(Li et al., 2018)

Figure 9 Spatial profile and computational grid of staged flame

Flame Response Study of Staged Flame

A volume integral of the heat release rate fluctuation along the flame zone under the excitation range from 10Hz to 80Hz is carried out to obtain a global transfer function, as shown in Figure 10. Additionally, a comparison between the calculation method and the experimental results (Li et al., 2018) has also displayed.

Figure 10 Fluctuation of the staged flame heat release rate vs. excitation frequency (top: fluctuation amplitude; bottom: fluctuation phase)

It can be seen from Fig.10 that the calculated fluctuation amplitude and phase of the heat release rate show agreement with the experimental results. Especially, the phase difference between 10Hz and 20Hz has been observed in the flame response calculation.

COMBUSTION INSTABILITY PREDICTION OF AN ANNULAR COMBUSTOR

Prediction Methods

For the solution of Eq.(1), it is convenient to transfer it into frequency domain. The study combustor is modeled as several discrete transfer acoustic units. The air swirlers are modeled as straight ducts, and connected to the annular chamber wall directly, thus there is abrupt area increase between these two parts. Radial dependence of the flow fluctuations is assumed negligible under the cut-off frequency, therefore the acoustic waves in the swirler ducts are supposed to be planar, and only longitudinal and azimuthal modes are calculated in the annular chamber. In this model, the general solutions of acoustic waves in each swirler are known as:

$$\begin{aligned} p'_{1,j} &= e^{i\omega t} \left(A_{1,N}^+ e^{-ik_N x} + A_{1,N}^- e^{ik_N x} \right) \\ u'_{1,j} &= \frac{1}{\bar{\rho}_{1,j} \bar{C}_{1,j}} e^{i\omega t} \left(A_{1,j}^+ e^{-ik_N x} - A_{1,j}^- e^{ik_N x} \right) \end{aligned} \quad (14)$$

where i is the imaginary unit, N is the N -th swirler, k is the longitudinal wave number, can be written as:

$$k_N = \frac{\omega}{\bar{C}_{1,N}} \quad (15)$$

In the annular chamber it can be written as the sum of multiple modes:

$$\begin{aligned} p'_2 &= \sum_{m=-M}^M e^{i\omega t + im\theta} \left(A_{2,m}^+ e^{-ik_m x} + A_{2,m}^- e^{ik_m x} \right) \\ u'_2 &= \sum_{m=-M}^M \frac{1}{\bar{\rho}_2 \bar{C}_2} e^{i\omega t + im\theta} \left(A_{2,m}^+ e^{-ik_m x} - A_{2,m}^- e^{ik_m x} \right) \end{aligned} \quad (16)$$

In these expressions, the subscripts 1 and 2 denote the swirler and the chamber region respectively, It is noted that the mean flow effects are ignored, hence the azimuthal wave

number of the m -th azimuthal mode in the chamber (k_m) are given by:

$$k_m = \sqrt{\left(\frac{\omega}{C_2}\right)^2 - \left(\frac{m}{r}\right)^2}. \quad (17)$$

where r is the mean radius of the annular chamber.

At zero Mach number, the matching condition for waves in the swirlers and the chamber is given by the conservation of pressure fluctuation (p') and acoustic flux (Su') across the dump plane. Due to the axisymmetric geometry, same relation can be found between one swirler and one collocated annular sector (see Fig.11). The jump condition for each swirler can be expressed in matrix form (see Eq.18) through a point-to-line matching procedure shown in Fig.11.

$$\begin{pmatrix} 1 & 0 \\ 0 & S_{1,j} \end{pmatrix} \begin{pmatrix} p'_{1,j} \\ u'_{1,j} \end{pmatrix} = \sum_{m=-N_m}^{N_m} \begin{pmatrix} 1 & 0 \\ 0 & S_{2,j} \text{sinc}\left(\frac{m\pi}{N}\right) \end{pmatrix} \begin{pmatrix} p'_{m,\theta_j} \\ u'_{m,\theta_j} \end{pmatrix}. \quad (18)$$

(a) Collocation of one swirler to one annular sector.

(b) Representation of line integral by point value.

Figure 11 Schema for the jump condition of a swirler and the point-to-line matching technique

To close the system, infinite acoustic impedance at the inlet boundary of the swirler ducts is applied and the choked outlet condition is equal to a velocity node when only acoustic modes are considered in the chamber. These treatments give both $u' = 0$ on the inlet and outlet boundary of the model. The system matrix of the linear acoustic model can be deduced from acoustic networks combined with the boundary conditions and the heat release model. The complex mode frequencies can be obtained by solving the dispersion equation of the numerical matrix. The growth rates of the frequencies of the combustor used as the basis for determination of the thermo-acoustic instability in the combustor.

Instability Frequencies and Modes of the Full Annular Combustor

The annular combustor studied in this paper is identical to the one studied with large-eddy simulation (LES) in the previous works. In this paper only brief introduction of the combustor will be present and for detail information you can find it in our publication (Zhang, 2016; Zhang, 2017). As illustrated in Fig.12, four air swirlers are evenly located along the azimuthal direction inside the annular chamber. For this combustor, pre-vaporized gaseous kerosene is injected from the central fuel nozzle of each swirler, and the hot air is guided into the combustor through the radial swirler around each nozzle. The combustion chamber is terminated with a choked annular nozzle at the top exit. Details of the swirler geometry and operating conditions are listed in Table 1.

Figure 12 Full-annular combustor in this study (left: sketch of the combustor; right: simplified model for analysis)

Table 1 Inflow conditions and geometry information of the model full-annular combustion.

Operating conditions	Value	Geometry information	Value
Pressure (MPa)	0.6	Swirler number	1.2
Fuel mass (g/s)	40	Chamber length (mm)	200
Fuel temperature (K)	673	Chamber diameter (mm)	200
Air mass (g/s)	650	Nozzle diameter (mm)	4
Air temperature (K)	623	Swirler diameter (mm)	28

Figure 13 shows the characteristics of the self-excited oscillation frequencies of the annular combustor calculated in this work. The horizontal direction is the real part of the frequency, which denotes the physical frequency, and the vertical direction is the imaginary part of the frequency, which means the frequency growth rate. When the frequency growth rate is negative, it is deemed that the amplitude at the corresponding frequency will continue to increase to trigger the combustion instability. At the same time, the self-excited oscillation frequency which is easy to occur when using the large eddy simulation (Zhang, 2017) is given in this figure. It can be seen from the comparison that the self-excited oscillation frequency of the combustion chamber calculated

using this method is close to the results acquired by the large eddy simulation.

Figure 13 Combustion instability frequencies of annular combustor (Top: predicted in this work; Bottom: predicted with LES)

(a) $f=1521$ Hz, mode (0,1)

(b) $f=2200$ Hz, mode (1,0)

(c) $f=2695$ Hz, mode (1,1)

Figure 14 Structure of the modes corresponding to unstable frequencies

Figure 14 shows the acoustic modes corresponding to the unstable frequencies. For comparison, the relative phase-averaged pressure fields with LES are also given in Fig.15. These unstable frequencies are the first-order circumferential, first-order longitudinal and first-order circumferential and longitudinal mixed instability modes of the combustor. It also confirms that the frequencies and acoustics modes predicted in this work agree very well with the LES results.

0

$\pi/2$

π

$3\pi/2$

(a) Phase-averaged pressure fields for one cycle of 1595Hz.

(b) Phase-averaged pressure fields for one cycle of 2251Hz.

(c) Phase-averaged pressure fields for one cycle of 2720Hz

Figure 15 Snapshots of the phase-averaged pressure fields from LES

CONCLUSIONS

In order to predict the thermo-acoustic instability of combustion chamber, an unsteady heat release rate model is developed in this paper, which is based on the linear analysis of chemical reaction kinetics. Through the comparison with experimental data of typical flames, it demonstrated that the global flame response and transient flame distribution can be properly predicted. Furthermore, the verification of the combustion instability prediction with this model shows that the frequencies and modes of annular combustor can be properly predicted with this heat release model.

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