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Non-equilibrium turbulent phenomena in transitional flat plate boundary-layer flows

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ABSTRACT

Many recent laboratory experiments and numerical simulations supported a non-equilibrium dissipation scaling in decaying turbulence before it reaches an equilibrium state. However, these studies were mostly focus on canonical flows, such as homogeneous isotropic turbulence and grid-generated turbulence. Beyond that, is it possible to observe the non-equilibrium phenomena in wall turbulence? In the present contribution, by analyzing a direct numerical simulation (DNS) database of a transitional boundary-layer flow, we show that the transition region and the non-equilibrium turbulence region, which are divided in the streamwise direction, presents different non-equilibrium scalings. Moreover, in the wall-normal direction, buffer layer, log layer and outer layer show different non-equilibrium phenomena which are very different from that in transitional channel flows. Thus, the turbulence models which consider the non-equilibrium phenomena should be modelled according to the dissipation scalings both in the streamwise direction and in the wall-normal direction. These findings are expected to shed light on the future modelling of various types of non-equilibrium turbulent flows.

Keywords: non-equilibrium turbulence; flat plate boundary layer; turbulence model

1 INTRODUCTION

The traditional Richardson-Kolmogorov theory^[1-2] is the origin of equilibrium dissipation scaling

$$C_\varepsilon = \varepsilon L / \mathcal{U}^3 \quad (1)$$

in which the dissipation coefficient C_ε is constant, with $\mathcal{U}$ root-mean-square (rms) of the velocity fluctuations, L integral length scale and ε turbulent energy dissipation rate. This scaling allows the formulation of turbulence models which establish a relationship between the small scales and the large scales. However, evidence based on numerical observations reveals that non-equilibrium turbulent flows exist obviously in engineering applications with complex boundaries and strong adverse pressure gradient^[3-4]. Thus, traditional turbulence models without considering the non-equilibrium properties may give unreasonable predictive results on these engineering projects.

Many recent laboratory experiments and numerical simulations supported a non-equilibrium dissipation scaling in decaying turbulence before it reaches an equilibrium state^[5]. This very different dissipation scaling breaks the Richardson-Kolmogorov cascade and has a non-constant C_ε , which scales as $C_\varepsilon \sim Re_l^{m/2} / Re_\lambda^n$ with $m \approx n \approx 1$, where Re_l denotes the inlet Reynolds number, Re_λ denotes the Taylor-based Reynolds number which is a local quantity. This scaling is also verified by a systematic theory research, in which $m \approx n \approx 15/14$ can be derived analytically^[6]. Turbulence of such character has been termed non-equilibrium turbulence. Recent studies in grid-generated turbulence^[7-8] and isotropic turbulence^[9] have shown that the skewness of longitudinal velocity derivative, $S_k = \langle (\partial u / \partial x)^3 \rangle / \langle (\partial u / \partial x)^2 \rangle^{3/2}$, is also a reasonable candidate for describing non-equilibrium turbulence. Significant effort

has been devoted to the non-equilibrium phenomena of turbulence in different configurations, such as homogeneous isotropic turbulence^[10-11], grid-generated turbulence^[12-14], turbulent wakes^[15-16], planar turbulent plume^[17], and turbulent planar jet^[18]. Apart from these simple flows, is it possible to observe the non-equilibrium phenomena in wall-bounded turbulence?

Wall-bounded turbulence can be classified into two main groups, one is the turbulence around the wall, and the other is the turbulence within a space bounded by the wall. Generally, the former can be considered as external flow and the latter as internal flow. In a previous study we investigated non-equilibrium turbulent phenomena in transitional channel flows^[19], that is, the non-equilibrium phenomena in the latter group (although only the two-dimensional channel flows belong to the former strictly). In the present contribution, we focus on the non-equilibrium turbulent properties in a transitional flat plate boundary-layer flow which belong to the former group. Note that the non-equilibrium phenomena are studied not only in the streamwise direction but also for different sublayers in the wall-normal direction, which is different from previous study in channel flows. The results are shown by analysis of a direct-numerical simulation (DNS) database of a transitional flat plate boundary layer. We will describe the numerical details in Section 2. In Section 3 the observations are presented, discussed and compared to previous studies. Conclusion remarks are in Section 4.

2 METHODOLOGY

An in-house code ASTR, which has been applied to DNS^[20-21] and large-eddy simulation(LES)^[22] of boundary layer and channel flows, is used as the flow solver. The Navier-Stokes equations are projected to the Cartesian coordinate system of the computational domain and solved by the high-order finite difference method. All the spatial derivatives are approximated with a sixth-order pade-type compact central scheme^[23]. The compact scheme is decoupled at the subdomain interface by using a recently proposed method^[24] to handle the domain-decomposition based parallel computation. To remove the small-scale wiggles due to aliasing errors resulting from discrete evaluation of the non-linear convective terms, a tenth-order compact filter is adopted, which limits the filter only at high wavenumbers^[25]. Once the spatial terms are solved, a third-order total variation diminishing Runge-Kutta method^[26] is used for the time integration. See Ref. [20] for more numerical details.

A DNS of a spatially developing transitional boundary layer on a flat plate is performed, as shown in Figure 1. The size of the computational domain is $L_x \times L_y \times L_z = (1 - 10) \times 0.9 \times 0.3$ in the streamwise x , wall-normal y and spanwise z directions, respectively, corresponding to the grid resolutions of $4020 \times 180 \times 256$. The mesh is refined towards the wall in the wall-normal direction and uniformly distributed in the streamwise and spanwise directions. The inlet Reynolds number based on the distance from the leading edge is $Re = 2.1 \times 10^6$, and the coming Mach

number is 0.3, defining a weakly compressible flow. A laminar boundary layer profile is prescribed at the inlet plane, combined with the characteristic soft inflow boundary condition proposed by Kim and Lee^[27]. At the outlet plane, the characteristic nonreflecting outflow conditions is used with a prescribed far-field pressure. To minimize the numerical error at the outlet plane, a sponge layer with a stretched mesh and a low-order filter is incorporated, as sketched in Figure 1. At the upper surface, a characteristic far-field boundary condition is used. The no-slip wall condition is imposed at wall, and a wall-normal periodic suction and blowing (PSB) is implemented in $1.36 \leq x \leq 1.56$ to trigger a boundary layer transition. This means that in the PSB region the non-slip wall conditions should be modified as

$$u = 0, w = 0, \quad (2)$$

$$v = A_1 f(x) \sin(\omega_1 t) + A_2 f(x) g(z) \sin(\omega_2 t),$$

where $u_i = (u, v, w)$ corresponds to the streamwise, wall-normal, and spanwise velocity components, respectively. ω_1 and ω_2 are the frequencies of the two-dimensional Tollmien-Schlichting and oblique waves, and A_1 and A_2 are the amplitudes of the waves. By referencing Fasel and Konzelmann^[28] and Sayadi et al.^[29] $f(x)$ is given as,

$$f(x) = 15.1875\xi^4 - 35.4375\xi^2 + 20.25\xi^3, \quad (3)$$

where ξ is defined as,

$$\xi = \begin{cases} \frac{x-x_1}{x_m-x_1}, & \text{if } x_1 \leq x \leq x_m \\ \frac{x_2-x}{x_2-x_m}, & \text{if } x_m \leq x \leq x_2 \end{cases}, \quad (4)$$

and $x_1 = 1.36$ and $x_2 = 1.56$ are the start and end location of the PSB region, with $x_m = \frac{x_2+x_1}{2}$. To trigger a K-type boundary layer transition, the parameters in Eq. (2) are given as, $A_1 = 0.0001$, $A_2 = 1.5 \times 10^{-5}$, $\omega_2 = \omega_1 = 1.1 \times 10^{-4} Re$.

Figure 1 Sketch of the computational domain and reduced numerical mesh. The mesh is plotted every 5th grid lines in both x and y directions.

3 RESULTS AND DISCUSSIONS

3.1 The division of the flows

3.1.1 The division of the flows in the streamwise direction

The purpose of this subsection is to divide the transitional boundary-layer flows in the streamwise direction. Figure 2(a-d) shows the streamwise evolution of the skin-friction coefficient C_f , the displacement thickness δ^* , the momentum thickness θ and the shape factor H , respectively. The C_f profile indicates that the main transition process is

complete at $x \approx 3.5$ since the turbulent drag coefficient exhibits an overshoot in the later stage of transition. This is supported by the variation of the displacement thickness and momentum thickness, especially by the decreasing displacement thickness during the transition. In fact, the shape factor $H = \delta^*/\theta$, as suggested by earlier studies, is an excellent indicator to evaluate the beginning and ending locations of a boundary-layer transition process. In a laminar boundary-layer flow, H is 2.6 according to the Blasius solution. Figure 2(d) reveals that in the interval $x < 2.2$ there is good agreement between the computed shape factor and its theoretical value. By $x = 2.2$ the H has moved substantially away from the laminar solution, and this factor changes rapidly in the interval $2.2 < x < 3.5$. After $x = 3.5$, the H settles down to a value of around 1.45, implying that the transition process is complete and the flow turn into turbulent. The shape factor presented in the turbulence region is consistent with the results obtained by Spalart^[30]. Thus, we can conclude that the transition region is approximately in the interval $2.2 < x < 3.5$. In a recent study we observed that there exists a non-equilibrium turbulence region before the beginning of the full-developed turbulence in a transitional channel flow^[19]. It is then natural to expect that such region appears also in present transitional boundary-layer flows. Indeed, this region locates around $x = 3.5$, which will be shown in Section 3.2. Note that in the present contribution we will not clarify a global non-equilibrium turbulence region as this region locates in different intervals which are intimately related to different wall-normal locations.

3.1.2 The division of the flows in the wall-normal direction

As is well known, in the turbulent flow past a rigid boundary we can roughly distinguish two regions in the wall-normal direction. One is the inner region which is directly affected by wall condition, and the other is the outer region which is only indirectly affected by the wall through wall shear stress^[31]. It is commonly accepted that most of the turbulent activities occur in the inner region and turbulence is mainly supplied from there to the outer region by turbulent diffusion. Therefore, in this subsection we will focus on the division of the inner region in the wall-normal direction. Figure 3 plots the mean velocity profile U^+ as a function of distance from the wall y^+ in the full-developed turbulence region, U^+ and y^+ are normalised by the local friction velocity u_τ . It is obvious that within $y^+ < 5$ the mean velocity follows the linear law of the wall, and it follows the log law in the range $45 < y^+ < 140$. We can therefore roughly divide the boundary-layer flow into four layers in the wall-normal direction: (a) the viscous sublayer in the range $y^+ < 5$; (b) the buffer layer in the range $5 < y^+ < 45$; (c)

Figure 2 Streamwise evolution of (a) skin-friction coefficient C_f , (b) displacement thickness δ^* , (c) momentum thickness θ and (d) shape factor H . δ^* and θ are normalized by the inlet-boundary-layer momentum thickness θ_0 .

the log layer in the range $45 < y^+ < 140$; and (d) the outer layer (or outer region) in the range $y^+ > 140$. The division is perhaps not appropriate before the turbulence is full-developed, however, we will not consider too much about this problem as it has no effect on our research.

Figure 3 Normalized mean velocity U^+ versus normalized distance from the wall y^+ in the fulldeveloped turbulence region. Two dash-dot lines represent the classic law of the wall.

3.2 Non-equilibrium phenomena in transitional boundary-layer flows

In this subsection we will analyse the evolution of C_ε and S_k along three different statistical streamlines, as shown in Figure 4. Note that Streamline *I* locates in the buffer layer but close to the viscous sublayer; Streamline *II* locates in the log layer and Streamline *III* locates in the the outer layer. In addition, the energy spectra and transfer spectra at some specific locations are performed to illustrate the multi-scale interactions of turbulence.

Figure 4 Three different streamlines in flat plate boundary-layer flows.

3.2.1 Dissipation coefficient

It follows from this paper's introduction that dissipation coefficient C_ε is a traditional quantity for evaluating the non-equilibrium phenomena in homogeneous isotropic turbulence, grid-generated turbulence, turbulent wakes and transitional channel flows. Here, we concern the evolution of C_ε along streamline, as well as the relationship between C_ε and Re_λ . In homogeneous isotropic turbulence, the quantities ε , L and $\mathcal{U}$, which define the C_ε , are calculated by ensemble average method. However in the present paper we employ the spanwise- and time-averaged method since the boundary-layer case is homogeneous only in spanwise direction and in time. The rms of velocity fluctuations in Eq. (1) should be replaced by the rms of spanwise fluctuating

velocity $\mathcal{W}$, and the integral scale L has to be defined in the spanwise direction

$$L_w = \int_0^\infty R_{ww}(r) dr, \quad (5)$$

where $R_{ww}(r) = \langle w(z)w(z+r) \rangle / \langle w^2 \rangle$ is the autocorrelation of the spanwise velocity fluctuation.

Figure 5 shows the evolution of C_ε along three different statistical streamlines. For Streamline *I*, C_ε decreases at first and then increases, and finally tends towards a constant value; for Streamline *II*, the value of C_ε decreases rapidly to constant. However, C_ε decreases gradually along streamline *III*. According to the region division in grid-generated turbulence and either the understandings of non-equilibrium turbulence, we can therefore conclude that C_ε decreases in the transition region and increases in the non-equilibrium region, and tends to an approximately constant in the equilibrium region. The rough region division of each streamline is shown in Table 1. It is interesting that three streamwise regions exist in the case of Streamline *I*. In Streamline *II*, however, there are only transition and equilibrium regions observed. The transition region exists alone in Streamline *III*. This demonstrates that the evolution of the boundary-layer flows along the streamwise direction is related to the wall-normal distance from the wall. Different sublayers embrace different number of streamwise regions. Thus, for the study of the non-equilibrium phenomena in external flows, not only the different regions should be distinguished along the streamwise direction, but also the flow characteristics in different sublayers should be investigated along the wall-normal direction.

Figure 5 Evolution of C_ε along three different streamlines.

Table 1. The region division of three different streamlines.

Streamline	Transition region	Non-equilibrium region	Equilibrium region
Streamline <i>I</i>	$2.5 < x < 3.4$	$3.4 < x < 4.0$	$4.0 < x < 9.0$
Streamline <i>II</i>	$2.8 < x < 3.6$	-	$3.6 < x < 9.0$
Streamline <i>III</i>	$3.6 < x < 9.0$	-	-

The relation between dissipation coefficient C_ε and Reynolds number Re_λ in the case of Streamline *I* is plotted in Figure 6(a) and (b). For reference, the evolution of Re_λ along Streamline *I* is provided in Figure 6(c). It is shown that

in the transition region, Re_λ increases and C_ε decreases, and C_ε satisfies the Re_λ^{-2} or $Re_\lambda^{-1.5}$ non-equilibrium law, as shown in Figure 6(a). This is similar to observations in a recent study of time-reversed turbulence, in which there are clearly dissipation scaling $C_\varepsilon \sim Re_\lambda^{-2}$ in the non-equilibrium process of backward energy transfer^[32]. In time-reversed turbulence, backward energy transfer is induced by extremes disturbance, leading an anomalous energy cascade from small to large scales. The large-scale energy is also increase in the transition region, although not exactly the same as the dynamics in time-reversed turbulence. In the non-equilibrium region, Re_λ decreases and C_ε increases, and Figure 6(b) shows a scaling $Re_\lambda^{-1.5}$, which is qualitatively in agreement with existing observations in grid-generated turbulence but with a different exponent. C_ε tends to a constant, independent of Re_λ , in the equilibrium region.

Figure 6 Relation between C_ε and Re_λ in the (a) transition region, and the (b) non-equilibrium region (blue open triangle) and equilibrium region (black open circle) of Streamline I. (c) Evolution of Re_λ along Streamline I.

Figure 7 (a) Relation between C_ε and Re_λ in the transition region (red open square) and equilibrium region (black open circle) of Streamline II. (b) Evolution of Re_λ along Streamline II.

The relation between C_ε and Re_λ in the cases of Streamline II and Streamline III is shown in Figure 7(a) and Figure 8(a), respectively. It is obvious that C_ε scales as $C_\varepsilon \sim Re_\lambda^{-2}$ or $Re_\lambda^{-1.5}$ in the transition region, which agree with the observations in Streamline I. Therefore, the transition region exists in all sublayers of boundary-layer flows, and the characteristics of such region are similar to the process of backward energy transfer. The spectral results in Section 3.2.3 also illustrate this well. However, the non-equilibrium region is observed only in the log layer.

Figure 8 (a) Relation between C_ϵ and Re_λ in the transition region of Streamline III. (b) Evolution of Re_λ along Streamline III.

3.2.2 Skewness of longitudinal velocity derivative

As mentioned in Section 1, the skewness of longitudinal velocity derivative S_k , which is defined along the streamwise direction as

$$S_k = \frac{\langle (\partial u / \partial x)^3 \rangle}{\langle (\partial u / \partial x)^2 \rangle^{3/2}}, \quad (6)$$

is a reasonable candidate for evaluating the non-equilibrium properties of turbulence. S_k describes the global balance of energy transfer over all scales. Figure 9 shows the evolution of S_k along three different statistical streamlines. S_k is approximately zero at the inlet plane since all fluctuations can be considered as Gaussian noise. For Streamline I, S_k varies complexly in the transition region and decreases in the non-equilibrium region, and finally settles down to an approximately constant in the equilibrium region. This is consistent with the conclusions in transitional channel flows^[19]. There is no obvious non-equilibrium region in the case of Streamline II, and S_k tends to a constant after experiencing complex changes in the transition region. In the cases of Streamline I and Streamline II, the changes in S_k are in agreement with that of in C_ϵ .

For Streamline III, C_ϵ decreases along the streamline, however, S_k tends to a constant at $x \approx 5.0$. The inconsistency between C_ϵ and S_k can be explained by the definitions of both. C_ϵ establishes a relationship between the small-scale quantity ϵ and the large-scale quantities $\mathcal{U}$ and L , while S_k is a small scale quantity. It is understandable that their evolution in the flow is different. Streamline I and Streamline II locate in the near-wall region where large-scale structures and small-scale structures are directly constrained by the wall. Therefore, the time-scales of these two structures are close in the process from non-equilibrium to equilibrium, corresponding to the simultaneous evolution of C_ϵ and S_k along streamlines. However, Streamline III locates in the outer layer, and the flow is little affected by the wall, both large scales and small scales can be considered as freely decaying turbulent structures. Since the time-scale of large-scale structures from non-equilibrium to equilibrium is longer in freely decaying turbulence, C_ϵ changes continuously along the streamline, while S_k settles down to a constant earlier. This then calls for future studies on the relationship between C_ϵ and S_k .

Figure 9 Evolution of S_k along three different streamlines.

3.2.3 Spectral statistical analysis

In order to investigate more details on non-equilibrium turbulence, in this subsection we present results for spectral statistical analysis. In the present case homogeneity is only valid in the spanwise direction, hence the results are presented as one-dimensional spectra in z direction. In each streamline, we select different typical streamwise locations to represent different streamwise regions, as shown in Table 2.

Table 2. Selection of streamwise locations in three different streamlines.

Streamline	Transition region	Non-equilibrium region	Equilibrium region
Streamline I	$x = 2.8$	$x = 3.5$	$x = 5.0$
Streamline II	$x = 3.0$	-	$x = 5.0$
Streamline III	$x = 7.0$	-	-

Figure 10, 11 and 12 show the spectral results of Streamline I, Streamline II and Streamline III, respectively. $E(k)$ is normalized with the kinematic viscosity ν and the dissipation rate ϵ , $T(k)$ is normalized with the wavenumber k and the dissipation rate ϵ , and the wavenumber k is normalized by the dissipation scale η . We observe that in the transition region of each streamline, the energy spectra is somewhat distorted, but the energy spectra of non-equilibrium region and the equilibrium region satisfy the power-law of $-5/3$.

Figure 10 (a) Energy spectrum $E(k)$ and (b) transfer spectrum $T(k)$ of Streamline *I*.

Figure 11 (a) Energy spectrum $E(k)$ and (b) transfer spectrum $T(k)$ of Streamline *II*.

Figure 12 (a) Energy spectrum $E(k)$ and (b) transfer spectrum $T(k)$ of Streamline *III*.

The transfer spectrum in the transition region of Streamline *I* has a “valley” at the middle wavenumbers, as shown in Figure 10(b). This indicates that the energy of such wavenumbers is transferred to both the large scales and the small scales, which is a typical picture of backward energy transfer. In the non-equilibrium and equilibrium regions of Streamline *I*, the transfer spectra indicate a forward energy cascade from large to small scales. In the cases of Streamline *II* and *III*, the transfer spectra in the transition region are expressed as energy input at all scales, which means that the flow is still far from the completion of the transition. Therefore, we can conclude that in the buffer layer or the viscous sublayer which close to the buffer layer, the dynamics in the transition region is similar to the process of backward energy transfer.

4 CONCLUSIONS

In the present contribution, the non-equilibrium turbulent phenomena in external flows are studied by using a transitional flat plate boundary layer. The boundary-layer flows can be divided into three regions along the streamwise direction, including the transition, the non-equilibrium, and the equilibrium regions. The flows in non-equilibrium region satisfies the dissipation law $C_\epsilon \sim Re_\lambda^{-1.5}$, which is qualitatively in agreement with the observations in grid-generated turbulence. However, in different sublayers along the wall-normal direction, the above three streamwise regions may not always appear. Numerical results reveal that three regions exist in the buffer layer. In the log layer, however, there are only transition and equilibrium regions observed. The transition region exists alone in the outer layer. This shows that for the study of the non-equilibrium phenomena in external flows, not only the different regions should be distinguished along the streamwise direction, but also the flow characteristics in different sublayers should be investigated along the wall-normal direction. These results have certain guiding significance for the modelling of the non-equilibrium turbulence models which are applicable in external flows. In addition, C_ϵ scales as $C_\epsilon \sim Re_\lambda^{-2}$ or $C_\epsilon \sim Re_\lambda^{-1.5}$ in the transition region, which is similar to the scaling in backward energy transfer process of time-reversed turbulence.

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