A Review on Examination of Structural Behaviour of a Bi-directionally Functionally Graded Beam

Milan Motta^{*1}, Amit Sarda²

^{*1}Associate Professor, Mechanical Engineering, Sreenidhi Institute of Science and Technology, Hyderabad, India
(Ph.D. Scholar OPJU, Raigarh, Chhattisgarh)

²Associate Professor, Mechanical Engineering, Christian College of Engineering & Technology, Bhilai, India
(Ph.D. Scholar CSVTU, Bhilai, Chhattisgarh)

ABSTRACT

Article Info

Volume 7, Issue 6

Page Number: 448-455

Publication Issue :

November-December-2020

A composite beam that exhibits variation of properties of a material which is created artificially as per a predefined function is a special category popularly termed as functionally graded beam. The notable part in such a variation in properties of material is that it can be in any of the three possibilities in any single, two dimensional direction or in all the three dimensional direction. It is quite possible to use the governing function in its standard or customized form as any researcher derives it. The nature of governing function of the properties of the graded beam is the reason behind the structural behaviour and the modal characteristic of a functionally graded beam as these two depends upon it. This research review focuses on a detailed examination of different governing equation which controls the variation of mechanical properties of a beam in different dimensional directions and their authority on structural and modal distinctiveness of the beam.

Article History

Accepted : 15 Dec 2020

Published : 30 Dec 2020

Keywords: Structural Behaviour, Functionally Graded Beam, Process Parameters, Response Parameters, Taguchi Method, Optimization techniques

I. INTRODUCTION

In recent times, a new division of composite materials identified as functionally graded materials (FGMs) has drawn extensive attention. These functionally graded materials (FGMs) are a type of material whose composition considered to be altering constantly within the solid. The gradient compositional variation of the constituents from one surface to the other provides a well-designed solution to the problem of

high transverse shear stresses that are stimulated when two dissimilar materials with large difference in material properties are bonded together. This concept of FGM instigated in Japan in the year 1984 at some stage in the space-plane project, in the form of a projected thermal barrier material competent enough to withstand a surface temperature of 2000 K and a temperature gradient of 1000 K that too across a cross-section in the limit of <10 mm. The proper understanding of dynamics behaviour of FG beams is

quite important since these are used in automotive industries, machine essentials and spacecrafts. In comparison to FG plates and shells, studies for FG beams are reasonably less.

In this recent year numerous works have been done on the examination of Functionally Graded Beam regarding the mechanical properties behaviour and variation in accordance to governing equation controlling the properties spatially and temporally.

L.W.Zhang, W.C.Cui and K.M.Liew [1] did a noteworthy work on Vibration analysis of functionally graded carbon nanotube reinforced composite thick plates with elastically restrained edges in the year 2015. In their work they performed the analysis of the problem of the free vibration of functionally graded carbon nanotube (FG-CNT) reinforced composite moderately thick rectangular plates with edges elastically restrained against transverse displacements and rotation of the plate cross section with the aid of the element-free improved moving least-squares Ritz (IMLS-Ritz) method for the examination. In their research they used the first-order shear deformation theory (FSDT), to report for transverse shear strains and rotary inertia. The mathematical results which they got were validated through comparison and convergence studies. The cause of elastically restrained edges on the vibration behavior of the FG-CNT reinforced composite plates was considered by taking into account the CNT volume fraction ratio, CNT distribution, plate thickness-to-width ratio and plate aspect ratio.

In very same year Jia Lou and Liwen He [2] did a work based on the modified couple stress theory on Closed-form solutions for nonlinear bending and free vibration of functionally graded micro-plates. In their effort the author studied the nonlinear bending and free vibration responses of a simply supported

functionally graded (FG) micro plate lying on an elastic foundation within the framework of the modified couple stress theory and the Kirchhoff/Mindlin plate theory together with the von Karman's geometric nonlinearity. The equations of motion and boundary conditions for the FG micro plate were derived from the Hamilton's principle. By solving the algebraic equations, closed-form solutions for the nonlinear bending deflection of the micro plate were derived. Closed-form solutions for the nonlinear vibration frequency were also obtained by applying He's variational method to the ODEs. Based on the acquired closed-form solutions, numerical examples were further presented to examine the effects of the material length scale parameter to thickness ratio, the length to thickness ratio, the power law index and the elastic foundation on the nonlinear bending and free vibration responses of the micro plates.

An effort on large amplitude nonlinear vibration analysis of functionally graded Timoshenko beams with porosities was done by FarzadEbrahimi and Majid Zia [3]. In their study they focused on functionally graded (FG) Timoshenko beams made of porous material and examined large-amplitude nonlinear vibration distinctiveness in it. The work of this type was very unique at that time and was done for the first time. The governing equations were derived based on Timoshenko beam theory through Hamilton's principle and they solved it utilizing both Galerkin's method and the method of multiple scales. In relation to the numerical results obtained, it was discovered and revealed that the proposed modeling method could provide precise frequency results of the FG porous material beams as compared to the studied and available literature. The detailed mathematical derivations were presented and numerical examinations were presented while the weightage is given to examination of the effect of the several parameters such as aspect ratio, material distribution profile, porosity volume fraction, and mode number

on the normalized natural frequencies of the FG porous beams in detail. It was unambiguously shown that the vibration behavior of a FG beams is significantly influenced by these effects. Numerical results were presented to serve as benchmarks for future analyses of FG porous beams.

Chunhua Jin and Xinwei Wang [4] applied the weak form quadrature element method on an accurate free vibration analysis of Euler functionally graded beams. In this research using the classical beam theory (CBT) and differential quadrature (DQ) rule, N-node novel weak form quadrature functionally graded (FG) beam element was established. They considered and varied both Young's modulus and mass density of the beam materials exponentially through the thickness.

An imperative work using He's variational method on size dependent nonlinear free vibration analysis of an axially functionally graded (AFG) micro-beam was done by Mesut Simsek [5]. In this work the author explained a nonlinear governing partial differential equation for the analysis of nonlinear free vibration of axially functionally graded (AFG) Euler-Bernoulli micro beams with immovable ends. He's variational method as mentioned in reference [4] was employed to obtain the ballpark closed form solution of the nonlinear ordinary governing equation. The nonlinear governing partial differential equation and the associated boundary conditions were derived by Hamilton's principle. By using Galerkin's approach, the nonlinear governing partial differential equation was reduced to a nonlinear ordinary differential equation.

Hasan Kurtaran [6] here author worked on large displacement static and transient analysis of functionally graded deep curved beams with generalized differential quadrature method. In his work he examined large displacement static and transient behavior of moderately thick deep functionally graded curved beams with constant curvature using generalized differential quadrature

method. Equilibrium equations for static and dynamic responses were attained using the implicit work principle. Spatial derivatives in the equilibrium equations were expressed with the generalized differential quadrature method. Large displacements were taken into consideration using Green-Lagrange nonlinear strain-displacement relations that are derived from elasticity theory equations. Transverse shear effect was considered through the first-order shear deformation theory. Static and dynamic equilibrium equations were solved using Newton and Newmark methods respectively. Several curved beam problems with different level of deepness were solved with the proposed method. Effect of functionally graded material properties on the behavior of curved beam was investigated using various ceramic and metal combinations such as Zirconium/Aluminum, Alumina/Aluminum, Zirconia/Monel, Silicon Nitride/Steel and Alumina/Steel materials in analysis.

A work on geometrically nonlinear finite element analysis of functionally graded 3D beams considering warping effects was done by Kyungho Yoon, Phill-Seung Lee and Do-Nyun Kim [7]. In their study they offered a geometrically nonlinear finite element formulation for analysis of functionally graded 3D beams. The proposed formulation employed the continuum mechanics based beam element with the warping displacement to model complex modes of deformation. The innovation of the developed method was that the warping function can be accurately calculated for any beam with arbitrary cross-sections and material grading patterns. Superb performances of the proposed beam element were demonstrated through several representative examples.

C.M.C. Roque and P.A.L.S. Martins [8] did a work on Differential progression for optimization of functionally graded beams. Differential evolution optimization was used to find the volume fraction that maximizes the first natural frequency for a

functionally graded beam. A formulation using three parameters was used to describe volume fraction. Beams with different ratios of material properties were taken into account. Two different methods namely analytical and mesh less numerical method were used to compute the natural frequencies. Results obtained confirmed that differential evolution is capable to find distributions for volume fraction that increase the natural frequency of beams. It was also bring into establishment that the RBF numerical method can be used with differential evolution to solve problems related to maximization of natural frequencies in functionally graded beams.

Authors, X.L. Jia, L.L. Ke, C.B. Feng, J. Yang and S. Kitipornchai [9] proposed work on the size effect on the free vibration of geometrically nonlinear functionally graded micro-beams under electrical actuation and temperature change. In this study the authors expressed the size effect on the free vibration of functionally graded micro-beams under the combined electrostatic force, temperature change and Casimir force based on Euler–Bernoulli beam theory and von Karman geometric nonlinearity. Taking into consideration the temperature-dependency of the effective material properties, material properties of the functionally graded materials (FGMs) were assumed to be graded in the thickness direction according to the Voigt model and exponential distribution model. The principle of minimum total potential energy was used to derive the nonlinear governing differential equation which is then solved using the differential quadrature method (DQM). A parametric study was conducted to show the significant combined effects of the size effect, material gradient, temperature change, geometric parameters and Casimir force.

Recently a remarkable and very important work has been done by Zhi-hai Wang, Xiao-hong Wang, Guo-dong Xu, Su Cheng and Tao Zeng [11] on bi-directionally functionally graded beam and their

work has been published a year later in 2016. In the work titled “Free vibration of two-directional functionally graded beams” they have presented a theoretical examination in free vibration of a functionally graded beam which has variable material properties along the beam length and thickness. It is assumed that material properties vary through the length according to a simple power law distribution with an arbitrary power index and have an exponential gradation along the beam thickness. The characteristic equations are derived in closed form. The governing equation can analytically reduce to the classical forms of Euler–Bernoulli beams if the gradient index disappears. Analytical solutions of the natural frequencies are obtained for graded beams with clamped-free and hinged–hinged end supports. Results show that the variations of material properties in the beam length and thickness have a strong control on the natural frequencies. It is also shown that there exists a critical frequency depending on the gradient parameter. The natural frequencies have an abrupt jump when across its critical frequencies. The derived results can be useful for designing non-homogeneous beams which may be required to vibrate with a particular frequency.

II. MATHEMATICAL MODEL FOR BI-DIRECTIONALLY FG BEAM

As per Zhi-hai Wang, Xiao-hong Wang, Guo-dong Xu, Su Cheng and Tao Zeng[12] a straight and uniform cross-section FG beam with the length l , the width b , the depth h , and a rectangular cross section has been considered (Figure 1). It is assumed that the material properties vary continuously along the length and thickness direction. Young's modulus E and mass density ρ are assumed to vary in the x - and z -direction according to the following expressions

$$E = E_0 \exp(\beta x) f(z) \quad \rho = \rho_0 \exp(\beta x) g(z) \quad (1)$$

Figure 1: Schematic of FG beam

Where, E_0 and q_0 are constants of Young's modulus and mass density respectively. β is a constant characterizing the gradual variation of the material properties along x -direction. $f(z)$ and $g(z)$ are the functions of variable z .

According to Euler–Bernoulli beam theory, the following assumptions made:

- All the cross-section of FG beams remains plane after deformation. However, they can undergo a rigid body displacement in x - z plane and also a rotation about y -axis.
- The effects of rotary inertia and shear deformation in x - z plane can be ignored.
- The angle of rotation is small so that the small angle assumption can be used.

Based on these assumptions, the axial and the bending rotation displacement fields are respectively given as

$$u = -\theta z \quad (2)$$

$$\theta = \frac{\partial w(x,t)}{\partial x} \quad (3)$$

Considering the small deformations and assuming the material of FG beams obeys Hooke's law, the strains and stresses in the beam are respectively obtained using Eqs. (1) and (2) as,

$$\epsilon_x = \frac{\partial u}{\partial x} = -z \frac{\partial^2 w(x,t)}{\partial^2 x} \quad (4)$$

$$\sigma_x = E\epsilon_x = -Ez \frac{\partial^2 w(x,t)}{\partial^2 x} = -E_0 \exp(\beta x) f(z) z \frac{\partial^2 w(x,t)}{\partial^2 x} \quad (5)$$

According to Eq. (5), the bending moment M is given

by

$$M = b \int_{z_0-h}^{z_0} \sigma_x z dz = -DE_0 \exp(\beta x) \frac{\partial^2 w(x,t)}{\partial^2 x} \quad (6)$$

Where z_0 is the distance between the bending line (when imposing a pure bending on the section, no normal stress appears on it) and the bottom surface.

$$D = b \int_{z_0-h}^{z_0} z^2 f(z) dz \quad (7)$$

When $f(z) = 1$, D is the second moment of area of the beam's cross-section.

Without loss of generality, the forces and moments on a FG beam element is shown in Fig. 1. dF and dM denote shear force and bending moment on a beam element, and $q(x, t)$ denotes transverse load.

The balance of forces on the beam element in z -direction is given by

$$F + dF - F + q(x, t) = adm \quad (8)$$

Where, dm is mass of the beam element, a is acceleration of the beam element in z -direction. They are respectively given by

$$a = \frac{\partial^2 w(x,t)}{\partial^2 t} \quad (9)$$

$$dm = \rho_0 B \exp(\beta x) dx \quad (10)$$

Where,

$$B = b \int_{z_0-h}^{z_0} g(z) dz.$$

The balance of moment on the beam element is written as

$$M + dM - M + Fdx + q(x, t) \frac{(dx)^2}{2} = 0 \quad (11)$$

Considering $dx \rightarrow 0$, the quadratic term $q(x, t) (dx)^2/2$ can be ignored. Then, Eq. (11) can be written as

$$F = \frac{\partial M}{\partial x} \quad (12)$$

Combining Eq. (6), (9), (10) and (12), Eq. (8) is reduced to

$$\frac{\partial^2}{\partial x^2} \left[DE_0 \exp(\beta x) \frac{\partial^2 w(x,t)}{\partial^2 t} \right] + B\rho_0 \exp(\beta x) \frac{\partial^2 w(x,t)}{\partial^2 t} = q(x,t) \quad (13)$$

Now the above equation can be solved to find out natural frequencies of a FG Beam for different mode shape.

In order to obtain analytical solution of Eq. (13), it is expedient to suppose that the transverse displacement takes the form

$$w(x,t) = A(x) \sin(\omega t) \quad (14)$$

For free vibration problem of FG beams, the following expression can be obtained

$$q(x,t) = 0 \quad (15)$$

Substituting Eq. (14) and (15) into Eq. (13), the governing differential Eq. (13) reduce to

$$E_0 D \frac{d^4 A(x)}{dx^4} + 2\beta E_0 D \frac{d^3 A(x)}{dx^3} + 2\beta^2 E_0 D \frac{d^2 A(x)}{dx^2} - \omega^2 B\rho_0 A(x) = 0 \quad (16)$$

According to the governing differential Eq. (16), a characteristic equation is written

$$E_0 D \lambda^4 + 2\beta E_0 D \lambda^3 + 2\beta^2 E_0 D \lambda^2 - \omega^2 B\rho_0 \lambda = 0 \quad (17)$$

By solving the above characteristic equation, the characteristic roots can be easily obtained.

$$\text{If } 4\omega \sqrt{\frac{B\rho_0}{DE_0}} - \beta^2 > 0$$

$$\begin{aligned} \lambda_{1,2} &= \frac{1}{2} \left(-\beta \pm i \sqrt{4\omega \sqrt{\frac{B\rho_0}{DE_0}} - \beta^2} \right), \\ \lambda_{3,4} &= \frac{1}{2} \left(-\beta \pm \sqrt{4\omega \sqrt{\frac{B\rho_0}{DE_0}} + \beta^2} \right) \end{aligned} \quad (18)$$

$$\text{If } 4\omega \sqrt{\frac{B\rho_0}{DE_0}} - \beta^2 < 0$$

$$\begin{aligned} \lambda_{1,2} &= \frac{1}{2} \left(-\beta \pm \sqrt{\beta^2 - 4\omega \sqrt{\frac{B\rho_0}{DE_0}}} \right), \\ \lambda_{3,4} &= \frac{1}{2} \left(-\beta \pm \sqrt{4\omega \sqrt{\frac{B\rho_0}{DE_0}} + \beta^2} \right) \end{aligned} \quad (19)$$

The possible expressions of analytical solution are given by

$$\begin{aligned} A(x) &= \exp\left(-\frac{\beta x}{2}\right) [C_1 \sin(\alpha_1, x) + C_2 \cos(\alpha_1, x) \\ &\quad + C_3 \sinh(\alpha_2, x) \\ &\quad + C_4 \cosh(\alpha_2, x) \quad \text{if } 4\omega \sqrt{\frac{B\rho_0}{DE_0}} - \beta^2 \\ &\quad > 0 \end{aligned}$$

$$\begin{aligned} A(x) &= \exp\left[(-\beta x/2)[C_1 + C_2] + C_3 \sinh(\alpha_2, x) + C_4 \cosh(\alpha_2, x)\right] \\ &\quad \text{if } 4\omega \sqrt{((\beta \rho_0)/(DE_0))} - \beta^2 < 0 \end{aligned} \quad (20)$$

Where,

$$\alpha_1 = \begin{cases} 4\omega \sqrt{\frac{B\rho_0}{DE_0}} - \beta^2, & \text{if } 4\omega \sqrt{\frac{B\rho_0}{DE_0}} - \beta^2 > 0 \\ \beta^2 - 4\omega \sqrt{\frac{B\rho_0}{DE_0}}, & \text{if } 4\omega \sqrt{\frac{B\rho_0}{DE_0}} - \beta^2 < 0 \\ 0, & \text{if } 4\omega \sqrt{\frac{B\rho_0}{DE_0}} - \beta^2 = 0 \end{cases}$$

And

$$\alpha_2 = 4\omega \sqrt{\frac{B\rho_0}{DE_0}} + \beta^2$$

III. PROPOSED MODIFICATIONS

In the above formulation the equation has been derived considering the free vibration analysis. The behavior of a FG Beam under forced vibration can be investigated as an extension of this work.

The equation (16) describes the governing equation for free vibration but to make it governing equation for forced vibration we have to incorporate the force variation along with this governing equation.

The force variation is

$$F_x = F \cos \omega t$$

where

F = Static force, and

ω = Angular velocity of
the periodic disturbing
force.

After this modification the job will be to solve for Complementary Function (CF) and Particular Integral (PI).

This extended work will give a in depth understanding about the behavior of the FG Beam under the condition of resonance and also with the development of a governing equation for generated stresses the effect of resonance on the structural stability of the beam can be analyzed.

IV. CONCLUSION

The aforementioned and proposed modification will help to examine a Functionally Graded Beam to predict its structural and modal behavior completely under external periodic excitations.

A mathematical model will help the engineer or designer to select perfectly the cross section of a beam for any predefined purpose under subjection of static or dynamic load.

V. REFERENCES

- [1] L.W.Zhang, W.C.Cui and K.M.Liew, "Vibration analysis of functionally graded carbon nanotube reinforced composite thick plates with elastically restrained edges", International Journal of Mechanical Sciences 103 (2015) 9–21
- [2] Jia Lou and Liwen He, "Closed-form solutions for nonlinear bending and free vibration of functionally graded micro plates based on the modified couple stress theory", Composite Structures 131 (2015) 810–820
- [3] FarzadEbrahimi and Majid Zia, "Large amplitude nonlinear vibration analysis of functionally graded Timoshenko beams with porosities", ActaAstronautica 116 (2015) 117–125
- [4] Chunhua Jin and Xinwei Wang, "Accurate free vibration analysis of Euler functionally graded beams by the weak form quadrature element method", Composite Structures 125 (2015) 41–50
- [5] MesutSimsek, "Size dependent nonlinear free vibration of an axially functionally graded (AFG) micro beam using He's variational method", Composite Structures 131 (2015) 207–214
- [6] HasanKurtaran, "Large displacement static and transient analysis of functionally graded deep curved beams with generalized differential quadrature method", Composite Structures 131 (2015) 821–831
- [7] Kyungho Yoon, Phill-Seung Lee and Do-Nyun Kim, "Geometrically nonlinear finite element analysis of functionally graded 3D beams considering warping effects", Composite Structures 132 (2015) 1231–1247
- [8] C.M.C. Roque and P.A.L.S. Martins, "Differential evolution for optimization of functionally graded beams", Composite Structures 133 (2015) 1191–1197
- [9] X.L. Jia, L.L. Ke, C.B. Feng, J. Yang and S. Kitipornchai, "Size effect on the free vibration of geometrically nonlinear functionally graded micro-beams under electrical actuation and temperature change", Composite Structures 133 (2015) 1137–1148
- [10] W. Sumelka, T. Blaszczyk and C. Liebold, "Fractional EulereBernoulli beams: Theory, numerical study and experimental validation",

European Journal of Mechanics A/Solids 54
(2015) 243e251

- [11] Zhi-hai Wang, Xiao-hong Wang, Guo-dong Xu, Su Cheng and Tao Zeng, "Free vibration of two-directional functionally graded beams", Composite Structures 135 (2016) 191–198

Acknowledgement

This research was supported by Sreenidhi institute of science and technology (Hyderabad), Christian college of engineering and technology (Bhilai), Bhilai Institute of Technology (Durg) and OP Jindal University (Raigarh). We are thankful to our supervisors and management who provided expertise that greatly assisted the research, to bring out the interpretations provided in this paper.

Cite this article as :

Milan Motta, Amit Sarda, "A Review on Examination of Structural Behaviour of a Bi-directionally Functionally Graded Beam", International Journal of Scientific Research in Science and Technology (IJSRST), Online ISSN : 2395-602X, Print ISSN : 2395-6011, Volume 7 Issue 6, pp. 448-455, November-December 2020. Available at doi : <https://doi.org/10.32628/IJSRST207679>
Journal URL : <http://ijsrst.com/IJSRST207679>