V. ECOLOGICAL AND SUSTAINABLE AGRICULTURE

NEW DEVELOPMENTS IN CROP ROTATION

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Abstract: Climate change is already affecting agriculture, with effects unevenly distributed across the world. These changes will undoubtedly lead to a reconsideration of the question of the necessary crop rotations on the territory of the individual continents and sub-regions. Many researchers have conducted field experiments with different crop rotations on several soil types and have received determined results, but still lack concrete unified numerical estimates of the influence of different crops in them. This article presents several new developments of the problem in crop rotation. Different approaches to assessing the influences of predecessors are illustrated. Therefore, it is necessary to apply advanced mathematical-statistical methods and models for obtaining more precise estimates. The application of asymmetry analysis and Procrustes analysis in the evaluation of different crop rotations are given in details. Some results about favorable and unfavorable rotation pairs are given.

Keywords: crop rotation; sustainable agriculture; asymmetry analysis, Procrustes analysis

INTRODUCTION

S. Vorobyov [25] believes that modern agricultural practice of crop rotation with cereals accounts for around 57% of the increase in yields, 34% falls on fertilization, and 9% – the interaction of both factors. In row crops influence of fertilization increased to 80%, of the rotation falls to 22%, and interaction – 13-22%.

Agronomists and soil scientists see a clear relationship between crop rotations and the sustainability of agricultural production systems. Several interactions lead to positive crop-rotation effects. An overview of the advantages of crop rotations [5] is presented here:

- Reduced use of agrochemicals and synthetic fertilizers;
- Facilitate the timing of farming activities;
- Lower erosion due to longer period of land cover;
- Improved soil structure;
- Improved soil texture;
- Improved soil fertility and higher yields;
- Maintenance of long-term productivity and organic matter;
- Improved population of microorganisms;
- Reduced number of weed seeds;
- Increased biodiversity;
- Greater market opportunities and lower economic and climatic risks due to diversified production.

Climate change is already affecting agriculture, with effects unevenly distributed across the world. Future climate change will likely negatively affect crop production in low latitude countries, while positive trends are evident in some northern regions [19]. Studies on Projected Impacts for crops and

livestock in global regions and sub-regions under future scenarios show that yield impact of wheat, maize and soybean for Europe will be positive in Boreal and Alpine sub-regions [13, p. 509-512).

Ouda [17] discussed the role of crop rotation in Egypt: It increases land productivity, maintains soil sustainability, defeats pests and weeds, and suggested crop rotations to increase food security and reduce water scarcity.

Rübensam and Rauhe [21] present a classification of cultivated crops and divide them into three main groups:

- a. Stable or self-tolerated: rye, maize, yellow lupine, soy, hemp, tobacco, millet, and potatoes /in absence of nematodes/;
- b. Unstable or negatively reactive to repetition: wheat, oats, sugar and fodder beet, clover, alfalfa, peas, flax, sunflower, cabbage, and poppy;
- c. Crop rotation labile or mutually exclusive in rotation: wheat after barley, oats after barley and vice versa, flax after peas and vice versa, clover after alfalfa and vice versa, broad beans after clover.

Diversifying cropping systems improves environmental health and has the potential to reduce risk from climate change-related threats, but empirical evidence remains sparse. In the study, was found that maize yields were higher during adverse weather, including droughts when maize was grown as part of a more diverse rotation. Rotation diversification also increased maize yields over time and under better growing conditions [4]. Suitable crop rotation with optimum fertilizer application can balance the need for high crop yields with minimal environmental risk [26]. If a technology works to improve productivity for farmers and does not cause undue harm to the environment, then it is likely to have some sustainability benefits. Agricultural systems emphasizing these

principles also tend to be multi-functional within landscapes and economies [20].

In his overview Kolbe [15], the preceding effects of crop pairs were classified by establishing schematic diagrams for use in crop rotation planning in low external input or organic agricultural systems.

Results from previous crop rotation trials and earlier diagrams cited in the literature were evaluated to classify the succession effects for more than 25 main crop species into four ranks: very favorable, practicable, not favorable and very unfavorable. These are linguistic variables.

Fig. 1. Pre-crop suitability of crops in ecological farming by Kolbe [15].

The CropRota model integrates agronomic criteria and historical crop mixes at the field, farm, or regional scales to generate optimal crop rotations for a particular scale [24]. The comparison of model results against seven years of field survey data from 579 farms in the Mostviertel region of Austria indicates that CropRota is suitable and reliable in modeling typical crop rotations. Experts valued the crop sequences that were converted into a point-value matrix ranging from zero points (0 – agronomically impossible sequence) to ten points (10 – agronomically highly desirable sequence).

Several crops are grown in the countries from Boreal region, namely: wheat, barley, rye, oats, flax, potatoes and others. They can be found in Figures 1 and 2, which are useful for selection of appropriate crop sequences.

Many Bulgarian researchers have conducted field experiments with different crop rotations on the main soil types in the country and have received determining results. The significant contributions of Dzhumalieva [6] and Georgiev [8] should be emphasized. Georgiev introduces a coefficient for the role of the predecessor on the next crop. When the predecessor does not have a significant effect on the yield of the next crop, it receives a coefficient equal to 1. In the case of a favorable effect, the coefficient is greater than 1 and corresponds to the percentage increase in yield due to the influence of the predecessor. When the tolerability is negative, the coefficient is less than 1. But still lack unified numerical estimates of the influence of different crops in them. Therefore, it is necessary to apply advanced mathematical-statistical methods and models for obtaining such evaluations.

	main crop																																												
	winter wheat	summer wheat	winter rye	mixed summer grain	winter barley	summer barley	oats	winter triticale	mixed winter grain	spelt wheat	other grain	grain corn	CCM	silage corn	green corn	peas	field beans	winter vetch	summer vetch	lupine	other legumes	saradella	red clover	red clover grass	alfalfa	temporary grass	other grass	rapeseed	mustard	flax	poppy seed	sweed	sugar beet	fodder beet	potatoes early	potatoes med	potatoes late	sun flower	soy bean	vegetables	pumpkin	fallow			
pre-crop	winter wheat	4	4	6	6	6	6	8	6	6	4	5	10	10	10	10	10	10	10	10	10	10	10	6	6	6	6	8	10	8	10	10	10	10	10	10	10	10	10	10	10	10	10	6	
	summer wheat	4	4	8	4	8	6	8	6	5	4	5	10	10	10	10	10	10	10	10	10	10	8	6	6	4	4	6	8	8	10	8	10	10	10	10	10	10	10	10	10	10	8	8	
	winter rye	6	6	4	8	8	8	8	6	6	6	6	10	10	10	10	10	10	10	10	10	10	8	8	8	8	8	10	10	8	10	10	10	10	10	10	10	10	10	10	10	10	10	8	
	mixed summer grain	4	4	6	4	6	6	4	5	5	4	5	10	10	10	10	10	10	10	10	10	10	8	8	8	8	8	10	10	8	10	8	8	10	10	10	10	10	10	10	10	10	10	8	8
	winter barley	4	6	6	5	4	6	8	6	5	4	5	10	10	10	10	10	10	10	10	10	10	8	8	8	8	8	10	10	8	10	10	10	10	10	10	10	10	10	10	10	10	10	8	
	summer barley	4	4	6	5	6	4	8	6	5	4	5	10	10	10	10	10	10	10	10	10	10	8	8	8	8	8	10	10	8	10	8	8	10	10	10	10	10	10	10	10	10	10	8	
	oats	8	4	8	7	8	6	2	8	8	8	7	10	10	10	10	10	10	10	10	10	10	8	8	8	8	8	8	10	8	8	10	8	8	10	10	10	10	10	10	10	10	10	8	
	winter triticale	4	6	6	5	6	8	8	4	5	4	5	10	10	10	10	10	10	10	10	10	10	8	8	8	8	8	10	8	8	10	8	8	10	8	8	10	10	10	10	10	10	10	8	6
	mixed winter grain	6	4	6	4	6	6	8	5	6	6	5	10	10	10	10	10	10	10	10	10	10	8	8	8	8	8	10	10	8	10	8	8	10	8	8	10	10	10	10	10	10	10	8	6
	spelt wheat	4	4	6	6	6	6	8	6	6	4	5	10	10	10	10	10	10	10	10	10	10	6	6	6	6	6	8	10	8	10	10	10	10	10	10	10	10	10	10	10	10	10	6	
	other grain	5	4	6	4	6	6	6	6	5	5	5	10	10	10	10	10	10	10	10	10	10	8	8	8	8	8	8	10	7	8	10	8	8	10	10	10	10	10	10	10	10	10	8	6
	grain corn	8	8	0	8	0	8	8	0	4	8	5	8	8	8	8	8	8	0	10	10	10	8	8	8	8	8	8	6	0	8	8	8	8	6	6	6	6	6	8	10	6	8	6	
	CCM	8	10	0	10	0	10	8	0	6	8	6	8	8	8	8	8	10	10	0	10	10	8	8	8	8	8	10	0	8	6	8	8	4	4	8	10	10	10	10	4	8	6		
	silage corn	8	10	6	10	6	10	8	6	6	8	6	8	8	8	8	8	10	10	0	10	10	8	8	8	8	8	10	0	8	6	8	8	8	4	4	8	10	10	10	10	4	8	6	
	green corn	8	10	6	10	6	10	8	6	6	8	6	8	8	8	8	8	10	10	0	10	10	8	8	8	8	8	10	0	8	6	8	8	4	4	8	10	10	10	10	4	8	6		
	peas	10	6	10	6	10	6	4	10	10	10	6	10	10	10	10	2	2	2	2	2	2	4	2	4	2	2	2	4	6	6	6	4	4	6	6	10	10	10	10	4	4	4	4	
	field beans	10	6	8	8	6	6	8	10	10	10	6	10	10	10	10	2	2	2	2	2	2	4	2	2	2	2	2	4	6	6	2	2	2	6	10	10	10	10	2	2	4	4	2	
	winter vetch	10	6	8	6	8	6	6	8	10	6	8	8	8	8	2	2	2	2	2	2	2	4	2	4	0	0	0	2	6	0	4	0	6	6	6	8	6	6	6	6	4	4	2	
	summer vetch	10	6	8	6	8	6	6	8	10	6	8	8	8	8	2	2	2	2	2	2	2	0	0	0	0	0	2	6	0	4	0	6	6	6	8	6	6	6	6	4	4	2		
	lupine	10	6	6	8	6	6	10	10	10	10	10	10	10	10	2	2	4	4	2	2	0	0	0	0	0	2	6	0	6	0	6	0	6	10	10	10	10	6	2	4	4	2		
	other legumes	10	6	6	6	8	6	6	8	10	10	8	8	8	8	4	4	4	3	2	4	0	0	0	0	0	2	6	0	6	0	6	0	6	8	10	10	10	6	2	5	4	2		
	saradella	8	4	8	4	6	4	6	8	8	8	4	8	8	8	2	4	4	4	2	2	2	2	2	2	2	2	6	4	6	8	8	8	8	8	8	8	8	2	2	6	4	2		
	red clover	8	6	6	6	6	4	6	6	8	4	6	6	6	6	0	0	0	0	0	0	0	10	0	0	0	0	4	4	2	2	6	4	6	6	6	6	6	0	0	2	2	0		
	red clover grass	6	6	6	0	6	8	6	6	6	6	6	6	6	6	4	4	2	0	4	4	4	0	10	0	2	4	4	6	2	2	6	4	4	6	4	4	4	4	4	4	2	2	0	
	alfalfa	8	6	6	4	4	4	6	6	8	4	6	6	6	6	0	0	0	0	0	0	0	0	0	0	10	0	4	2	2	2	6	4	4	6	6	6	0	2	2	2	0			
	temporary grass	6	6	6	4	6	4	4	6	6	6	4	8	8	8	0	0	0	0	0	0	0	2	0	0	0	10	4	4	6	4	4	6	4	4	6	6	6	2	2	2	4	0		
	other grass	10	10	8	4	6	4	4	8	10	6	8	8	8	8	4	2	2	2	2	4	2	4	2	2	2	2	2	4	6	8	4	4	8	8	8	8	8	6	6	4	4	4		
	rapeseed	10	8	8	8	8	6	8	8	10	8	10	10	10	10	4	2	4	4	4	4	4	4	4	4	4	6	6	6	10	0	2	4	4	0	4	4	8	8	8	4	2	4	0	4
	mustard	8	6	8	4	8	6	8	8	8	8	6	10	10	10	4	4	4	4	4	4	4	4	4	4	4	6	6	4	4	4	4	4	4	6	6	8	8	8	8	8	2	0	4	
	flax	8	8	8	8	8	10	10	8	8	8	10	10	10	10	4	4	4	4	4	4	4	4	2	2	2	2	4	6	2	4	0	0	6	8	8	8	8	8	8	8	0	4		
	poppy seed	8	6	8	6	8	6	8	8	6	6	6	6	6	6	4	4	4	4	4	4	2	2	2	2	2	2	4	6	4	4	4	4	6	6	6	6	6	4	4	4	4	4		
	sweed	8	8	2	8	6	2	8	4	4	8	6	8	8	8	8	6	6	6	6	6	6	4	4	4	4	4	4	6	2	2	4	4	2	4	4	6	6	6	4	4	2	2	4	
sugar beet	8	10	0	10	10	10	4	4	8	8	8	8	8	8	6	6	4	4	4	4	4	4	4	4	4	4	4	8	0	8	4	6	0	2	2	8	8	8	6	6	4	4	4		
fodder beet	8	10	4	10	6	10	10	4	4	8	8	8	8	8	6	6	4	4	4	4	4	4	4	4	4	4	4	4	8	0	8	4	6	0	2	2	8	8	8	6	6	4	4	4	
potatoes early	10	6	8	10	8	6	10	10	10	10	10	8	8	8	8	8	10	8	4	4	4	4	4	4	4	4	4	6	10	4	4	4	0	10	10	2	2	2	10	10	4	4	4		
potatoes med	10	6	8	10	8	6	10	10	10	10	8	8	8	8	8	8	8	4	4	4	4	4	4	4	4	4	4	6	10	4	4	4	0	10	10	2	2	2	10	10	4	4	4		
potatoes late	10	6	8	10	8	6	10	8	8	10	8	8	8	8	6	6	4	4	4	4	4	4	4	4	4	4	4	6	10	4	4	4	0	10	10	2	2	2	10	10	4	4	4		
sun flower	10	8	8	8	8	8	8	8	10	8	10	10	10	10	6	4	2	4	4	4	4	4	4	4	4	4	4	6	6	8	0	0	4	4	4	2	2	2	2	4	4	4	8		
soy bean	10	8	6	8	8	10	10	8	10	10	8	6	6	6	2	2	2	2	2	2	2	2	2	2	2	2	2	4	2	2	4	4	4	8	8	8	8	8	4	4	4	4			
vegetables	8	10	10	10	10	10	10	8	8	10	10	10	10	10	4	4	4	4	4	4	4	4	4	4	4	6	6	8	8	4	2	4	4</												

Table 1. Influence of the previous crop on the yield by Dzhumalieva [6].

Previous Crop	Wheat	Barley	Rye	Oats	Maize
Wheat	412	476	378	325	472
Barley	461	450	390	323	469
Rye	523	441	359	311	462
Oats	504	502	402	279	473
Maize	536	547	427	352	455

In Table 2 Baeumer [1] classify the pairs of preceding and succeeding crops in five categories: 1= very unfavorable, 2 = unfavorable, 3 = possible, 4 = favorable, 5 = very favorable.

Table 2. Favorable and unfavorable rotational pairs by Baeumer [1].

Preceding crops	Succeeding crops												
	Grass-clover	Winter rape	Sugar beet	Potatoes	Sunflower	Faba beans	Field peas	Maize	Winter wheat	Winter barley	Winter rye	Spring barley	Oats
Grass-clover	1	5	1	4	1	1	1	5	5	4	4	2	2
Winter rape	3	1	1	4	2	3	3	4	5	4	4	1	1
Sugar beet	3	1	1	4	2	2	2	5	5	1	1	4	3
Potatoes	3	5	5	2	2	2	2	5	5	5	5	4	3
Sunflower	3	1	4	4	1	4	4	4	5	5	5	4	4
Faba beans	2	1	2	5	1	1	1	5	5	4	4	3	3
Field peas	2	4	4	4	1	1	1	4	5	5	5	3	3
Maize	3	1	5	5	5	5	5	3	4	2	2	4	4
Winter wheat	4	2	5	5	5	5	5	5	1	3	3	3	4
Winter barley	5	5	5	5	5	4	4	4	1	2	2	2	3
Winter rye	5	5	5	5	5	4	4	4	1	1	2	2	3
Spring barley	5	4	4	4	4	4	4	4	3	1	2	2	1
Oats	5	4	5	4	4	4	4	4	4	4	4	2	1

In his article Kubinec [16] provides Table 1 with the potential yield (percent of 1998-2007 average) of a crop on stubble combination, as compared to the average yield for the planted crops on all stubble types in Manitoba. Many crops are presented, but some crops are better suited to specific areas, and regardless of rotation will not perform in other areas. The table and all of the combinations are to provide interested producers with an additional tool to determine crop sequences and their yield risks. The

main disadvantage of this table, which makes it unsuitable for numerical analysis, is the presence of many missing values denoted by NSD.

It is the reason to apply the method of K-Nearest Neighbors to make imputations of missing values. The algorithm of cross-validation [3] is applied with STATISTICA software package. Table 3 gives the full combinations of previous crops with crops planted.

Table 3. Relative yield response of Manitoba crops [16] (with imputed missing values)

Previous Crop	Crop Planted											
	Winter Wheat	Spring Wheat	Barley	Oat	Canola	Flax	Field Pea	Soybean	Navy Bean	Potato	Sunflower	Corn
Winter Wheat	76	102	100	102	94	105	98	102	105	101	98	79
Spring Wheat	91	90	101	101	103	102	102	107	100	107	99	96
Barley	91	92	88	91	100	99	97	92	89	102	103	82
Oat	97	93	91	85	95	97	93	104	94	98	97	101
Canola	104	103	105	104	83	90	93	99	99	99	82	98
Flax	102	98	104	102	98	98	79	58	62	101	102	93
Field Pea	90	100	104	105	101	101	94	90	94	99	88	95
Soybean	95	109	98	89	84	99	94	72	98	100	46	93
Navy Bean	91	124	97	98	96	98	93	90	100	94	89	120
Potato	92	114	99	97	95	97	93	93	116	99	92	95
Sunflower	93	95	97	91	94	97	95	95	96	100	89	90
Corn	92	101	99	107	97	100	92	84	104	90	106	92

From the presented Tables 1 – 3, it is seen that this are asymmetric matrices in which the elements $d_{ij} \neq d_{ji}$. It follows the need to apply a method for the analysis of square non-symmetric matrices, whose rows and columns are classified by the same elements /crops/. We will use the method of analysis of asymmetry and orthogonality [11]; and [12] and the method for metric multidimensional unfolding [22].

The decomposition of a square matrix into its symmetrical and skew-symmetrical parts can be used to analyze proximity data in agricultural studies. The analysis of a square asymmetric matrix A is based on the well-known decomposition

$$A = S + K, \text{ where}$$

$$S = \frac{1}{2}(A + A') \text{ is symmetric and}$$

$$K = \frac{1}{2}(A - A') \text{ is skew-symmetric.}$$

Expressing $A = U'\Sigma V$ in its singular value decomposition form, where U and V are orthogonal and Σ are diagonal. Thus the orthogonal matrix $H = V'U$ may be regarded as a measure of asymmetry. The graphic presentation of results is possible by the method of correspondence analysis performed with NCSS software package.

In order to perform a joint analysis of two tables with data about preceding and succeeding crops, which are given in different units, it is necessary to make standardization of the values. To get the normalized score (Z-score) of the Value in the data set having mean (Average) and Standard deviation of the data, we use the below formula from statistics

$$Z\text{-score} = (\text{Value} - \text{Average}) / (\text{Standard deviation})$$

In several environmental, agricultural, and social studies questions arise to compare results of multivariate analyses derived from different data on the same domain or derived from different analyses of the same data. An appropriate method for such comparative studies is the so-called “Procrustes analysis”. The theory and analytical procedures of Procrustes analysis are presented in works of Eckart and Young [7], Schonemann, and Carroll [23], and Gower [9], [10]. With this name are indicated least-squares problems where independent and dependent variables are matrices.

The problem is defined mathematically as follow:

$$(1) \mathbf{B} = \mathbf{T}'\mathbf{A}\mathbf{S} + \mathbf{E}$$

when given matrices $\mathbf{A}_{p \times p}$ и $\mathbf{B}_{p \times p}$,

$$(2) \mathbf{T}\mathbf{T}' = \mathbf{T}'\mathbf{T} = \mathbf{I} \text{ и } \mathbf{S}\mathbf{S}' = \mathbf{S}'\mathbf{S} = \mathbf{I},$$

$$(3) \text{trace}(\mathbf{E}'\mathbf{E}) = \min.$$

The solution is performed by the help of Eckart-Young decomposition (still called SVD - decomposition by singular values) of the two given square matrices

$$\mathbf{A} = \mathbf{P}\mathbf{D}_a\mathbf{Q}', \mathbf{B} = \mathbf{X}\mathbf{D}_b\mathbf{Y}',$$

where $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{X}$ и $\mathbf{Y}$ are square and orthogonal, but $\mathbf{D}_a$ and $\mathbf{D}_b$ are diagonal from the same order p .

The orthogonal transformation matrices are

$$\mathbf{T} = \mathbf{P}\mathbf{X}' \text{ and } \mathbf{S} = \mathbf{Q}\mathbf{Y}',$$

Which give the solution

$$\mathbf{B}^* = \mathbf{T}'\mathbf{A}\mathbf{S} = (\mathbf{P}\mathbf{X}')'\mathbf{A}\mathbf{S} = \mathbf{X}\mathbf{P}'\mathbf{A}\mathbf{Q}\mathbf{Y}' = \mathbf{X}\mathbf{P}'\mathbf{D}_a\mathbf{Q}'\mathbf{Q}\mathbf{Y}' = \mathbf{X}\mathbf{D}_a\mathbf{Y}'$$

The problem is solved numerically with a computer program for singular value decomposition and the method is realized easily on a personal computer with the known package MATLAB.

RESULTS AND DISCUSSION

Here are the results from the analysis of Table 1.
The orthogonal matrix H may be regarded as

measuring departures from symmetry in a multiplicative sense.

Table 4. Orthogonal matrix H from Table 1.

Previous crop	Wheat	Barley	Rye	Oats	Maize
1-Wheat	-0.3629	0.5780	0.4619	0.5578	-0.0993
2-Barley	0.6551	0.1328	0.3581	0.1066	0.6431
3-Rye	0.2051	0.6424	-0.7318	0.0830	0.0522
4-Oats	-0.3612	0.4057	0.1905	-0.7591	0.3039
5-Maize	0.5163	0.2664	0.2942	-0.3073	-0.6939

The correspondence analysis was used to visualize relationships on a plain with the main factors. This alternative presentation on the map facilitates its interpretation since it concentrates the analysis on the distances between rows (previous

crops) and columns (next crops). Here are abbreviations used in Fig. 3:

C1-Row1=Wheat, C2-Row2=Barley, C3-Row3=Rye,
C4-Row4=Oats, C5-Row5=Maize.

Fig. 3. Proximity of previous and next crops.

Fig. 3 confirms the very unfavorable rotation pair R1-Wheat and C1-Wheat as the distance between the points is quite large while the pair R2-Barley and C5-Maize are very closed (favorable

in rotation).

Here starts the Procrustes analysis. We were matching Baeumer with Kubinec data in the following steps:

1. Starting with Baeumer matrix A (Table 5)

Table 5. Matrix A - extract from Baeumer [1]

Preceding crops	Succeeding crops					
	Winter wheat	Winter barley	Sunflower	Oats	Maize	Potatoes
Winter wheat	1	3	5	4	5	5
Winter barley	1	2	5	3	4	5
Sunflower	5	5	1	4	4	4
Oats	4	4	4	1	4	4
Maize	4	2	5	4	3	5
Potatoes	5	5	2	3	5	2

Table 6. Matrix B – extract from Kubinec [16]

Preceding crops	Succeeding crops					
	Winter Wheat	Winter barley	Sunflower	Oats	Maize	Potatoes
Winter Wheat	76	100	98	102	79	101
Winter barley	91	88	103	91	82	102
Sunflower	93	97	89	91	90	100
Oats	97	91	97	85	101	98
Maize	92	99	106	107	92	90
Potatoes	92	99	92	97	95	99

2. Get the normalized score (Z-score) of the Value in the data set A calculating $S_A = (\text{Data}_A - 3.67)/1.3522$, and receive Table 7.

Table 7. Standardized Table 5 - matrix A

Preceding crops	Succeeding crops					
	Winter wheat	Winter barley	Sunflower	Oats	Maize	Potatoes
Winter wheat	-1.679	-0.200	1.279	0.540	1.279	1.279
Winter barley	-1.679	-0.939	1.279	-0.200	0.540	1.279
Sunflower	1.279	1.279	-1.679	0.540	0.540	0.540
Oats	0.540	0.540	0.540	-1.679	0.540	0.540
Maize	0.540	-0.939	1.279	0.540	-0.200	1.279
Potatoes	1.279	1.279	-0.939	-0.200	1.279	-0.939

The same is made with data set B calculating $S_B = (\text{Data}_B - 94.50)/7.0737$, and receive Table 8.

Table 8. Standardized Table 6 - matrix B

Preceding crops	Succeeding crops					
	Winter Wheat	Winter barley	Sunflower	Oats	Maize	Potatoes
Winter Wheat	-2.615	0.778	0.495	1.060	-2.191	0.919
Winter barley	-0.495	-0.919	1.202	-0.495	-1.767	1.060
Sunflower	-0.196	0.353	-0.847	-0.495	-0.636	0.713
Oats	0.353	-0.495	0.353	-1.343	0.919	0.495
Maize	-0.350	0.689	1.626	1.767	-0.353	-0.636
Potatoes	-0.300	0.601	-0.358	0.364	0.071	0.693

3. Perform SVD-decomposition of the two given square matrices A and B.

Then find the orthogonal transformation matrices T and S (see Tables 9 and 10)

Table 9. Transformation matrix T

0.5719	-0.5383	0.2167	-0.0505	0.5668	-0.1114
0.3343	0.2522	-0.5031	-0.3608	0.1878	0.6372
-0.5451	-0.4693	0.2607	-0.5255	0.0307	0.3709
-0.4360	0.3229	-0.1837	-0.1156	0.7421	-0.3283
0.2298	0.1214	-0.0271	-0.7538	-0.2785	-0.5347
-0.1455	-0.5544	-0.7730	0.0973	-0.1196	-0.2242

Table 10. Transformation matrix S

0.4065	-0.5564	-0.3493	-0.1297	-0.0414	-0.6203
0.4093	0.6899	0.1928	-0.2562	0.2722	-0.4237
-0.4794	0.0201	0.4280	-0.1451	-0.5567	-0.5056
-0.4425	0.3554	-0.7881	-0.1995	-0.0513	-0.1197
-0.3644	-0.0373	0.0554	0.6215	0.5598	-0.4039
0.3299	0.2940	-0.1829	0.6859	-0.5461	-0.0515

4. Find the solution - transformed matrix B^* and residual matrix E (see Tables 11 and 12).

Table 11. Transformed matrix B^*

-2.9056	0.8311	0.6682	1.1027	-2.1764	0.9453
-0.4149	-0.8200	1.1236	-0.2534	-1.8998	0.9603
-0.0333	0.3786	-0.8027	-0.3464	-0.7172	0.6756
0.1081	-0.2472	0.5800	-1.3950	1.4512	0.6875
-0.4276	0.7343	1.6151	1.6352	-0.4030	-0.3412
-0.2946	0.7674	-0.1541	0.3705	0.2526	0.9832

Table 12. Residual matrix E

0.2906	-0.0531	-0.1732	-0.0427	-0.0146	-0.0263
-0.0801	-0.0990	0.0784	-0.2416	0.1328	0.0997
-0.1627	-0.0256	-0.0443	-0.1486	0.0812	0.0374
0.2449	-0.2478	-0.2270	0.0520	-0.5322	-0.1925
0.0776	-0.0453	0.0109	0.1318	0.0500	-0.2948
-0.0054	-0.1664	-0.2039	-0.0065	-0.1816	-0.2902

5. The interpretation follows from the values of diagonal elements (trace) of the residual matrix E.

Numerical values of the criterion for accuracy of solution's approximation B^* is

$$\text{eps} = \text{trace}(E' * E) / (6 * 6) = 0.03014,$$

which testifies for a very good concordance of the results.

To check the agreement between the original matrix B (Table 6) and the transformed matrix B^* is necessary to go back from normalized score (Z-score) to original data calculating $\text{Data}_B = S_B^* \cdot \text{Std.dev.}_B + \text{Ave}_B$. So we receive the recovered matrix in Table 13, which is very close to the original.

Table 13. Recovered matrix from Table 6

Preceding crops	Succeeding crops					
	Winter Wheat	Winter barley	Sunflower	Oats	Maize	Potatoes
Winter Wheat	74	100	99	102	79	101
Winter barley	92	89	102	93	81	101
Sunflower	94	97	89	92	89	99
Oats	95	93	99	85	105	99
Maize	91	100	106	106	92	92
Potatoes	92	100	93	97	96	101

CONCLUSION

Crop rotation is one of the most important agricultural practices, existing for thousands of years, and often used in organic farming but with less and less use in conventional agriculture.

Integrated Crop Management requires recognition of the fact that diverse crop rotation is the most effective indirect means of maintaining soil fertility and controlling pests. The aim should be to grow healthy, vigorous crops producing economically viable yields and quality by making optimum use of natural resources. In this way, the reliance on technological inputs such as fertilizers and pesticides is reduced, but the need for them is by no means eliminated.

Future climate change will likely negatively affect crop production in southern countries, while positive trends are evident in some northern regions. Finland, Sweden, Norway, and Iceland will reconsider the list of crops to grow in the future decades.

This article shows the application of symmetry analysis and Procrustes analysis in the evaluation of different crop rotations. The examples given confirm that such types of analyses can reveal favorable and unfavorable rotation pairs such as:

- Favorable: Wheat – Maize; Barley – Maize; Oats – Rye; Maize – Wheat and
- Unfavorable: Wheat – Wheat; Wheat – Barley; Maize – Maize; Oats – Barley.

Considering Figures 1 and 2, and Tables 2 and 3 we can refute the existing opinion that maize is not a suitable predecessor of sunflower, but only to wheat and oats.

The advanced mathematical-statistical analyses can help farmers to select good crop rotations if more reliable data from field experiments are available. Such methods may be used successfully for evaluation of the environmental quality and study of other analogous problems.

In the vast bibliography of the European project [2] the authors of this project emphasize the need for financing further research on crop rotations at the European level. Scientific data are missing on the multiple technical aspects linked to crop rotations (technical as well as economic aspects). Consistent statistics and databases are currently needed to help inventory crop rotations.

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