ON THE EXTREMUM-SEEKING CONTROL DESIGN AND APPLICATION FOR ANAEROBIC DIGESTION PROCESSES

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Abstract: The paper deals with the optimization of nonlinear systems by using Extremum Seeking Control (ESC) without any prior knowledge of the system model. An Extend Kalman Filter based Extremum Seeking Control (EKF based ESC) is proposed, which can make the amplitude of perturbation signal variable and ensure convergence to zero, i.e. without steady-state oscillation. The proposed ESC algorithm makes also possible to obtain more accurate gradient estimate and more rapid ESC convergence. The proposed EKF based ESC algorithm is applied to a fifth-order model of anaerobic digestion process and its performances are compared with the performances of the classical ESC algorithm.

Key Words: ESC, EKF, steady state oscillation, anaerobic digestion processes

I. INTRODUCTION

To ensure resource-saving and environment protection, more and more attention is paid nowadays to the anaerobic digestion (AD) processes [1]. In order to obtain maximum biogas and biofuels production, a range of control methods [2 - 4] are dedicated to AD process in which Extremum Seeking Control (ESC) does not require any knowledge of the model while most optimization controllers [5 - 7] need complete model information of the controlled system.

It is precisely because of the characteristics of non-model based that ESC has been developed and applied as early as a few decades ago. But it was until 2000, when Krstić and Wang [8, 9] proposed the first proof of stability theory for a series of classical ESCs (perturbation based ESC). These articles have inspired scholars' interests in ESC, and a large number of important developments and application results have begun to appear, such as multi-input ESC [10], sliding mode ESC [11], time-varying ESC [12], Newton based ESC [13], etc. More detailed summary of ESC can be found in the article [14].

Since the model and/or cost function of the optimized system are not available, ESC transforms the optimization problem into a control problem ensuring convergence to zero of the estimated gradient [15]. Therefore, obtaining gradient information for an unknown function is a key element for ESC. The classic ESC uses a combination of high- and low-pass filters or a low-pass filter alone to perform gradient estimation of unknown functions by modulation and demodulation. In order to eliminate the influence of the Hessian matrix contained in the gradient on the convergence speed of ESC, article [13] proposed a Newton-based ESC, which solves the problem by estimating the gradient of the cost function and the

inverse of the Hessian matrix, but it is essentially a combination of filters. A recursive least square with forgetting factor estimation algorithm is used to estimate the gradient of the cost function [16], which can eliminate a time-scale separation to speed up the convergence, but only when the optimized system accurately approximates the Wiener-Hammerstein model.

In this paper, the EKF based ESC is developed from perturbation based ESC to optimize for nonlinear dynamic systems with static maps and its performance by simulations taking into account an AD process is studied. The proposed control method only need the optimization target of the system, and contains an Extend Kalman Filter (EKF) and a variable amplitude perturbation signal. The EKF is used to obtain more accurate estimated gradient, instead of a series of high- and low-pass filters in perturbation based ESC. In addition, the proposed ESC can eliminate the shortcoming of steady-state oscillation in perturbation based ESC because the amplitude of the perturbation signal has converged to zero during steady state (this part is mainly referred to [17]). This construction method for ESC without steady-state oscillation is simpler and more effective than methods mentioned in papers [18, 19], which incorporate some complex mathematical modules to distinguish between the transient and steady state phases of the ESC optimization process or require some prior knowledge (such as convergence time).

The article is organized as follows. In Section 2 is the problem formulation and introduction of perturbation based ESC. Section 3 focuses on EKF based ESC. Models and descriptions of fifth-order anaerobic digestion processes, as well as related application simulations, are given in Section 4. Finally, Section 5 contains the summary.

II. PERTURBATION BASED EXTREMUM SEEKING CONTROL

Consider a general single-input single-output nonlinear system:

$$\begin{cases} \dot{x} = f(x, u) \\ y = h(x) \end{cases}, \quad u = \alpha(x, \theta) \quad (1)$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}$ is the input, $y \in \mathbb{R}$ is the output, $f: \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}^n$ and $h: \mathbb{R}^n \rightarrow \mathbb{R}$ is the smoothing function, $u = \alpha(u, \theta)$ is the assumption that there is a smoothing law.

Perturbation based Extremum Seeking Control (classic ESC) [8], [9] is shown in Fig. 1. It should be noted that the objective function should be

available or directly measured, $y = J(x, u)$. It can be seen from Fig. 1 that the input θ is obtained by superimposing the perturbation signal $aS(t)$ ($S(t)$ is usually a sinusoidal signal, $\sin wt$) on the estimated $\hat{\theta}$, $\theta = \hat{\theta} + aS(t)$.

The estimated $\hat{\theta}$ is given by:

$$\dot{\hat{\theta}} = k\zeta \quad (2)$$

where ζ is an estimate for the gradient of the function y in (1) evaluated at θ , i.e. $\zeta = f'(\theta)$. Since there is a static map between the output and the input of the system, the output of the system can

Fig. 1. Perturbation based ESC

be written as:

$$y \approx f(\theta^*) + \frac{f''(\theta^*)}{2}(\theta - \theta^*)^2 \quad (3)$$

Without loss of generality, the system has a maximum value, i.e. θ^* .

Assumption 1. The system admits at least one stable equilibrium point θ^* in which

$$\frac{\partial f}{\partial \theta}(\theta^*) = 0, \quad \frac{\partial^2 f}{\partial \theta^2}(\theta^*) < 0 \quad (4)$$

For more analysis, the error $\tilde{\theta} = \hat{\theta} - \theta^*$ is introduced. Bring it into (4) to get (For the convenience of analysis, let $S(t) = \sin wt$):

$$y = f(\theta^*) + \frac{f''(\theta^*)}{2}(\tilde{\theta} + a \sin wt)^2 \quad (5)$$

When the system output y passes through the

high-pass filter of the cutoff frequency w_h , isolates the change of the optimized variable from its average value:

$$y - \eta = \frac{f''(\theta^*)}{2}(\tilde{\theta} + a \sin wt)^2 \quad (6)$$

The signal is then demodulated, multiplying by $\sin wt$, resulting in

$$\begin{aligned} (y - \eta) \sin wt &= \frac{f''(\theta^*)}{2}(\tilde{\theta} + a \sin wt)^2 \\ &= \frac{f''(\theta^*)}{2}(a\tilde{\theta} + 2a \sin^2 wt \tilde{\theta}^2 \\ &\quad - \cos 2wt \tilde{\theta} + a^2 \sin^3 wt) \end{aligned} \quad (7)$$

Finally we can get

$$\dot{\hat{\theta}} = k\zeta = \frac{kaf''(\theta^*)}{2}\tilde{\theta} \quad (8)$$

Since $k > 0$, $a > 0$, according to equation (4), we know $\tilde{\theta} < 0$. So the controller is stable.

III. EKF BASED EXTREMUM SEEKING CONTROL

In order to obtain faster and more accurate gradient estimation, an Extended Kalman filter (EKF) [20] is used to replace the linear filters combination estimation method in the classical ESC scheme, which can speed up the convergence of the algorithm. There is a static map between the input and output of the controlled system, and the output can be approximated as

$$y \approx y_0 + f'(\theta)\theta(t) \quad (9)$$

where y_0 is the constant component of y , $f'(\theta)$ is the slope at $\theta(t)$, i.e. the gradient information. In order to use the EKF to estimate the unknown parameter, y_0 and $f'(\theta)$. Let $x_1 = f'(\theta)$ and $x_2 = y_0$, then the corresponding discrete state and measurement equations are as follows:

$$\begin{aligned} x(t_{k+1}) &= \begin{pmatrix} x_1(t_{k+1}) \\ x_2(t_{k+1}) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} x(t_k) + \omega_k \\ y(t_{k+1}) &= \begin{pmatrix} y_1(t_k) \\ y_2(t_{k-n}) \end{pmatrix} = \begin{pmatrix} \theta(t_k) & 1 \\ \theta(t_{k-n}) & 1 \end{pmatrix} x(t_k) + v_k \end{aligned} \quad (10)$$

where stochastic signals ω_k and v_k represent process excitation noise and observation noise, respectively. The EKF algorithm assumes that they are white noises that are independent of each other and normally distributed. More details on the EKF used to estimate the state in real time can be found in [21].

In addition, by adjusting the structure of the classical ESC, the amplitude a of the perturbation signal is no longer a constant value, but is proportional to high frequency component of the system output:

$$a = \frac{rw_l}{s + w_l}(y - \zeta) \quad (11)$$

We can see that a varies as the estimation error changes and approach zero eventually, which can eliminate the steady state oscillations of the system. The structure of EKF based ESC is shown in Fig. 2.

Fig. 2. The structure of EKF based ESC

From Fig. 2, we can get the system that

$$\dot{\hat{\theta}} = kF(\theta, y) \quad (12)$$

where $F(\theta, y)$ is the functions of EKF, which is used to estimate $f'(\theta)$, i.e. the gradient between system output and input.

Let $\tilde{\theta} = \hat{\theta} - \theta^*$. Because θ^* is constant, we can get

$$\dot{\tilde{\theta}} = k\tilde{F}(\theta, y) \quad (13)$$

Without loss of generality, the nonlinear system to be optimized is considered to have a maximum (ie, static maps is concave function). At this time, k is a positive constant. It can be known from the literature [20] that the Kalman filtering process must be convergent as long as the appropriate filter

parameters are selected, which means the estimated gradient gradually approaches the true value and the error of estimated gradient $\tilde{F}(\theta, y) < 0$. Under the action of EKF, the system of Fig. 2 is stable.

IV. APPLICATION TO AD PROCESS

4.1 Description of the anaerobic digestion process

The AD process is a very complex multi-stage biochemical process. The complex mathematical models, such as Anaerobic Digestion Model 1 (ADM1) [22] proposed by International Water Association (IWA), are not convenient for control applications. Thus in this paper, a fifth-order anaerobic digestion model [23] is adopted. The mathematical model consists of the following state equations:

$$\begin{aligned} \frac{dS_0}{dt} &= -DS_0 - \beta X_1 S_0 + DY_p S_{0in} \\ \frac{dX_1}{dt} &= (\mu_1 - D)X_1 \\ \frac{dS_1}{dt} &= -DS_1 + \beta X_1 S_0 - \frac{\mu_1 X_1}{Y_1} \\ \frac{dX_2}{dt} &= (\mu_2 - D)X_2 \\ \frac{dS_2}{dt} &= -DS_2 + Y_b \mu_1 X_1 - \frac{\mu_2 X_2}{Y_2} \\ Q &= Y_g \mu_2 X_2 \end{aligned} \quad (14)$$

where S_0 , S_1 and S_2 are the concentrations of macromolecular organics, soluble small molecule organics and acetate; X_1 and X_2 are respectively acidogenic bacteria and methanogenic bacteria concentration; Q represent the methane production rate; μ_1 and μ_2 are respectively the growth rate of acidogenic bacteria, methanogenic bacteria, characterized by Monod equation:

$$\mu_i = \frac{\mu_{i\max} S_i}{K_{S_i} + S_i} \quad i = 1, 2 \quad (15)$$

The objective of controlling the anaerobic digestion process is to obtain a maximum quantity of methane, which implies the maximization of the methane productivity. So, the optimization criterion is methane production rate Q .

The input-output static characteristic maps (dilution rate D and methane production rate Q) of the anaerobic digestion model for several values of the organics S_{0in} are shown in Fig. 3.

Fig. 3. Static characteristic D - Q under different S_{0in}

4.2 Simulation results

To demonstrate the effectiveness of EKF based ESC, two kinds of ESC schemes (classic ESC, and EKF based ESC) are applied to the above AD system, respectively. The comparison results (all the following simulation experiments are carried out under the condition of the inlet organics concentration of $S_{0in} = 25$ g/L) are shown in Fig. 4 and Fig. 5.

Fig. 4 is a comparison of the two ESCs with the initial dilution rate $D(0) = 0.01 \text{ h}^{-1}$. All two ESC schemes can control the output to converge to the maximum point of the methane outflow rate (about 0.29 L/h). But in terms of convergence time, EKF based ESC (about 130h) is faster than the classic ESC (about 500h).

(4.a) Input dilution rate D

(4.b) Methane production rate

Fig. 4. The behavior of AD process with Initial dilution rate $D(0) = 0.01 \text{ h}^{-1}$

The plot on Fig. 5 shows the inputs and outputs with initial dilution rate $D(0)=0.16\text{ h}^{-1}$, which is on the right side of the maximum methane yield Q^* corresponding to D^* . Similarly, the methane production rates can converge to the about 0.29 L/h. The relative speed of convergence time is the same as that of Fig. 4, which the classic ESC leads to a convergence time of about 650h, and EKF based ESC allows reducing the convergence time to approximatively 200h.

Summarizing these results, EKF based ESC proposed in this paper can converge faster than the classical ESC, especially when the initial input is on the right of the extreme point. Due to an improved estimation of the local gradients by using EKF, the EKF based ESC can get the fastest convergence speed. Moreover, It is easy to see that in all simulation experiments, the input and output of EKF based ESC have almost no steady-state oscillation, instead of oscillating at the extreme point like the classic ESC. This means the smoothness of the dilution rate and the gas production rate, which is advantageous for actuators that control the dilution rate and gas collectors.

(5.a) Input dilution rate D

(5.b) Methane production rate

Fig. 5. The behavior of AD process with Initial dilution rate $D(0)=0.16\text{ h}^{-1}$

V. CONCLUSION

This paper proposes EKF based ESC, which can eliminate the problem of steady-state oscillation of input and output in the classical ESC during real-time optimization and achieve faster convergence. The new ESC uses EKF instead of a combination of high- and low-pass filters to get a more accurate gradient estimate, which can speed up the convergence. The proposed ESC was applied to the fifth-order model of an anaerobic digestion process. The simulation studies show that the EKF based ESC algorithm converges without steady-state oscillation and faster than the classical ESC algorithm.

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