Influence of Cyclic Cylinder Pressure Variability in the Numerical Transfer Path Analysis of Multi-Body Engine

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Abstract

Practical mechanical systems often operate with some degree of uncertainty. The uncertainties can result from poorly known or variable parameters, from uncertain inputs or from rapidly changing forcing that can be best described in a stochastic framework. In automotive applications, cylinder pressure variability is one of the uncertain parameters that engineers have to deal with when designing and analyzing internal combustion engines. Multi-body dynamics is a powerful numerical tool largely implemented during the design of new engines. In this paper the influence of cylinder pressure cyclic variability on the results obtained from the multi-body simulation of engine dynamics is investigated. Particular attention is paid to the influence of these uncertainties on the analysis and the assessment of the different engine vibration sources. A numerical transfer path analysis, based on system dynamic sub structuring is used to derive and assess the internal engine vibration sources. In order to investigate the cyclic variability of cylinder pressure, a Monte Carlo approach is adopted. Starting from measured cylinder pressure that exhibits cyclic variability, random Gaussian distribution of the equivalent force applied on the piston is generated. The aim of this paper is to outline a methodology which can be used to derive correlations between cyclic variability and statistical distribution of results. The statistical information derived can be used to advance the knowledge of the multi-body analysis and the assessment of system sources when uncertain inputs are considered.

Keywords: Cyclic variability, numerical transfer path analysis, multi-body dynamics.

Introduction

Transfer Path Analysis (TPA) is a test-based method, which allows to trace the flow of vibro-acoustic energy from a source, through a set of known structure- and air borne pathways, to a given receiver location. It is commonly implemented for the analysis of vibrating active systems and the transmission of these vibrations to connected passive structures [1,2]. In recent years TPA tends to be particularly associated with noise, vibration and harshness (NVH) engineering, with particular interest in the automotive industry driven by the increasing customer expectations on acoustic comfort [3-6]. The TPA technique is adopted in automotive industry since 1980's and so far it is still a useful tool for the NVH assessment of vehicles and mechanical structures. The first examples of techniques nowadays denoted as classical TPA are often attributed to the work of Verheij around 1980s who studied the transmission of ship machinery vibrations through resilient mounts [7,8]. In 1981 Magran studied the transmissibility between terminals in a network [9]. The so called Global Transfer Direct Transfer (GTDT) method, further investigated by Guasch [10-12], was put into practice as the advanced TPA [13]. Independently, Liu and Ewins [14] and Varoto and Mc Connel [15] explored properties of transmissibility matrices for structural vibration problems, followed by Ribeiro, Maia, Silva and Fontul [16-18]. The transmissibility based method known as Operational TPA, developed to overcome the disadvantage of having individual measures of each single path from a source to a receiver, was developed by Nomura and Yoshida [19]; further reviews and benchmark studies can be found in [20,21]. Recent applications combine the TPA methodology with the novelty of numerical simulations. The Hybrid TPA formulation (HTPA) uses simulated excitation forces as input for TPA analysis, where the vibro-acoustic transfer functions are measured [22], while the Numerical TPA (NTPA) proposed in [23,24] is based on the Dynamic Sub structuring (DS) of

flexible multi-body systems.

Flexible multi-body systems are non-linear systems that exhibit large rigid body motion with associated small flexible deformations of the bodies. In industrial automotive applications, it is widely implemented during the development process of internal combustion engines and power unit systems. Crankshaft and engine dynamics, radial slider bearings dynamics or noise, vibration and harshness are examples of problems that can be investigated with this methodology. Multi-body dynamic models of internal combustion engines may have inaccuracies, which are caused by parameters uncertainties: joint clearances, friction, lubrication, load estimation, material non-uniformities, manufacturing and assembly errors are examples of factors influencing the model uncertainties. Uncertainties can be divided into model parameter uncertainties and excitation uncertainties. In literature, a discussion on the investigation of model parameter uncertainties can be found in [25]. This paper focuses on uncertainties related to system excitations.

In literature, several methods can be found that can be used to formally assess the effects of uncertainties in mechanical systems. A very general approach is to solve the Fokker-Plank equation that governs the evolution of uncertainty distribution under system dynamics, but this method requires high-dimensional probability space and cannot be easily implemented in systems of practical interest [26,27]. The Monte Carlo (MC) approach is maybe the most extensively used method in dynamic models. It consists of repeated random sampling of a set of system parameters to obtain the uncertainty distribution *via* numerical simulation results [28-30]. Being computationally demanding, Latin Hypercube Sampling [31] and Bayesian methods [32,33] were introduced to overcome classical MC limitations. The perturbation approach, useful when the perturbations are small, uses first and higher

order sensitivity coefficients to derive low order moments of the simulation uncertainty [34,35]. Other methodologies such as the Neumann series expansion, statistical linearization, stochastic averaging and generalized polynomial chaos methods, together also with some applications on numerical modeling, are described in [36-46]. In this paper, a Monte Carlo methodology is implemented to determine the influence of the variability of statistically independent excitations on numerical results of multibody dynamic system. In particular it focuses on the influence of cylinder cyclic variability on the application of DS techniques for NTPA on internal combustion engines.

Cyclic variability has been observed since the earliest scientific studies of internal combustion engines [47,48] Although in the beginning the predominant understanding of cyclic variability was that cycle-to-cycle variations were of stochastic nature [49,50], there have been many attempts to analyze the cycle-to-cycle variations by applying deterministic methods from nonlinear dynamical systems and chaos theory [51-55]. In general, it can be said that the physiochemical processes in an internal combustion engines are influenced by many factors like composition of fuel air mixtures, amount of recycled gases in the combustion chamber, engine aerodynamics or engine operating conditions. In this paper the cylinder pressure is measured in experimental data and the cylinder variability is statistically treated to define the input for the multi-body dynamic simulations.

In this paper, a methodology to correlate excitation uncertainties and sources assessment in Multibody dynamic systems is described. The influence of cyclic variability in the numerical transfer path analysis of multi-body engine systems was investigated. The variability of cylinder pressure was measured from running operating conditions of a test engine and the measured data were used as input excitation for the analysis on a numerical multi-body systems of the engine

tested. The aim of this paper is to outline a methodology which can be used to derive correlations between cyclic variability and statistical distribution of results. The statistical information derived can be used to advance the knowledge of the multi-body analysis and the assessment of system sources when uncertain inputs are considered.

Investigation of vibration paths in internal combustion engine using numerical models

Flexible multi-body dynamics

A multi-body system consists of discrete bodies, representing the components of the investigated system, having known elastic and inertia properties and connected each other *via* joints [56-58]. In classical multi-body dynamics (MBD), these bodies are rigid and only their gross motion is considered. In order to enhance the modeling of the reality, deterministic models based on finite element method (FEM) are combined with equations of MBD. These modeling strategies are addressed as flexible multi-body dynamic (FMBD) methods [59-63]. The equations describing the motion of the flexible multi-body system are derived from linear elasticity theory using a floating frame of reference formulation. The mathematical modeling of each body is based on Newton's equation of momentum and Euler's equation of angular momentum. Transforming these fundamental equations into a body fixed coordinate system and introducing linear elastic structural properties, results in the classical equation of motion for linear systems, reading

$$M_b \ddot{q}_b(t) + D_b \dot{q}_b(t) + K_b q_b(t) = f_b(t) + g_b(t). \quad (1)$$

Where M_b , D_b and K_b denotes respectively mass, damping and stiffness of the body b , q_b is the vector of degrees of

freedom (DoFs) of the considered body and $\dot{q}_b$ and $\ddot{q}_b$ denotes respectively its first and second time derivative, f_b is the vector of external forces directly applied on the considered body and g_b is the vector of joint forces, resulting from contact with other bodies. In the equations that follow the time dependency (t) is omitted to improve readability.

From equation (1) both global motion quantities and local deformations are calculated. For the simulation of the structural dynamics of internal combustion engines, FMBD offers a wide variety of possible nonlinear joints, which can model different contact forces between bodies: piston-liner contacts, radial slider bearings, axial bearings, engine mounts, rotational coupling of shafts, etc. [64-68]. Since these joints introduce non-linear functional relations, it is not possible to solve the equation system in the frequency domain as well as it is also not possible to derive the analytical solution in the time domain. Hence, an efficient time integration method has to be used. The derivation of equations in this paragraph follows the notation used in [25].

Sub structuring of multibody system for numerical transfer path analysis

In powertrain analysis it can be useful to divide the powertrain multi-body model into several substructures. Each substructure can be a collection of bodies and linear joints or can be represented by a single body. For example, in acoustic analysis engine block, gearbox, oil pan and cover head are bodies that are in direct contact with the acoustic medium (surrounding air) and can represent a unique substructure, while all other bodies (i.e. crankshaft connected with conrods and pistons) can be seen as another substructure. In common FMBD systems for industrial applications, the contacts between different bodies are often nonlinear. In order to analyze the dynamics of a substructure considering nonlinear connections at interfaces, a numerical transfer path analysis,

based on the sub structuring of a multibody system, was proposed [23,24]. The aim of this methodology is to decompose the original system in order to identify vibro-acoustic sources and noise paths of a vibrating subsystem. The main difference between this method and classical TPA is that it is not required to evaluate the FRFs because the partial contributions are evaluated applying directly external forces and interface forces on the considered subsystem.

Consider a system with a single target substructure s (e.g. engine block connected with linear engine mounts to the chassis) on which are applied a total of n different excitations. The total number of excitations n can be divided into m different external forces directly applied on subsystem s and n-m different contact forces at interfaces with the other substructures of the global system. The motion contributions at the receiving locations of interest are obtained from the following equations system written in the physical domain (see [25])

$$\begin{cases} M_s \ddot{q}_{s,1} + D_s \dot{q}_{s,1} + K_s q_{s,1} = f_{s,1} \\ \vdots \\ M_s \ddot{q}_{s,m} + D_s \dot{q}_{s,m} + K_s q_{s,m} = f_{s,m} \\ M_s \ddot{q}_{s,m+1} + D_s \dot{q}_{s,m+1} + K_s q_{s,m+1} = g_{s,m+1} \\ \vdots \\ M_s \ddot{q}_{s,n} + D_s \dot{q}_{s,n} + K_s q_{s,n} = g_{s,n} \end{cases} \quad (2)$$

The components of the motion vectors, $q_{s,i}$, $\dot{q}_{s,i}$ and $\ddot{q}_{s,i}$ with $i = 1, \dots, n$, are the partial contributions obtained applying on the substructure s the n different external and interface sets of forces.

The model investigated

In this paper a FMBD system of an internal combustion engine is investigated. It is an in-line four-cylinder internal combustion engine running at a full load conditions with constant engine cycle averaged speed of 3000 rpm. It is a four-stroke engine, with a stroke of 92.8mm and cylinder bore of 82.5mm. The system consists of one engine block mounted on 3 different mount positions, a crankshaft, four con-rods,

two balancer shafts in correspondence of intake and exhaust systems and a dual-mass-flywheel (DMF). The joints connecting gearbox and engine block are linear, while all others are nonlinear joints with exponential increasing stiffness as function of the displacement. No gear transmissions are included into the model. The pistons are excited with moving forces representing the combustion pressure. More information about this excitation is provided in section 3.

Figure (1): Schematic 2D representation of the multi-body dynamic system of the engine investigated.

The torque applied on the crankshaft from the pistons is balanced with an external torque applied in correspondence of the DMF. In (Figure 1), a schematic 2D representation of the system investigated is shown. Pistons, cylinders, bearings and other joints are numbered from front to rear. The simulation is carried out in time domain using the software AVL EXCITE [69]. The simulation procedure consists of three main steps: pre-processing, simulation and post-processing. In the first step the geometry and the structural data of all components as well as all joint data are prepared and entered in the model. Since the total amount of time needed for the computation is significantly influenced by the number of DoFs of the system, a condensed model with a reduced number of DoFs, equivalent to the full model [63,70-71], is considered for the simulation. The condensed number of DoFs for the

investigated model is 4285. In the second step, the dynamic system of equations is solved in time domain. In the last step, result data can be extracted in time and can be transformed into frequency domain. The results are analyzed in frequency domain up to 2.5 kHz.

Once the full FMBD simulation is performed, the methodology described in section 2.2 can be applied. The substructure investigated is the engine block. All forces acting on the engine block are collected and grouped in sets. The forces collected are the ones transmitted from joints and the combustion pressure acting in the combustion chambers. Each set of forces contains the forces applied on the structure from the same joint, i.e. from the same excitation. 21 different load cases, i.e. 21 different paths, are analyzed. The following cases are identified:

- 4 combustion chambers (corresponding to the vibrations transmitted directly from the walls of the combustion chambers; from "Comb1" to 4 in the figures)
- 5 radial main bearings (corresponding to the main bearings of the crankshaft; from "Bearing1" to 5 in the figures)
- 4 guidance for connecting rod small ends (which represent the piston-liner interaction; from "Liner1" to 4 in the figures)
- 6 bearings for balancer shafts (respectively from "BalInt1" to 3 for the supports in correspondence of intake system and from "BalExh1" to 3 for supports in correspondence of exhaust system in the figures)
- Axial bearings
- DMF supports

Finally, when all cases are computed, the vibration contributions are evaluated in the nodes of interest. In order to investigate the cyclic variability of cylinder pressure, a MC approach is adopted. Starting from measured

cylinder pressure that exhibits cyclic variability, random Gaussian distribution of the equivalent force applied on the piston is generated. According with [72], 31 different multi-body simulations are investigated to get relevant statistical results. The aim of this paper is to describe a methodology to derive correlations between cyclic variability and statistical distribution of results; therefore the number of simulations here performed was chosen to optimize the statistical relevance of results and calculation time required. In real industrial applications it is strongly suggested to perform a convergence analysis related to the statistics of the quantities of interest.

Cyclic variability of cylinder pressure

In this paper, the influence of cyclic variability on the results obtained from a FMBD simulation is investigated. Particular attention is paid to the contributions of the different vibration sources of the engine. Therefore, the effects of cyclic variability are investigated using the NTPA methodology described in section (Substructuring of multibody system for numerical transfer path analysis) As described in section (The model investigated), the cylinder pressure is given as an external excitation to the FMBD system investigated. This pressure was measured on a GDI engine running under full load operating conditions with constant engine cycle averaged speed of 3000 rpm and firing crank angle of 0 deg. The measured data consist of 16 different periods (one cycle correspond to two complete crankshaft rotations being a 4-stroke engine). In (Figure 2), the blue line shows the mean value of the cylinder pressure measured, while the orange line corresponds to the standard deviation, σ , of the measured data. The maximum pressure measured exhibits a Gaussian distribution with a standard deviation of 8.6 bar. From these pressure data, an equivalent force is

derived, which is applied in vertical direction both on piston head and on the walls of the combustion chambers. Each time history is obtained by mixing the 16 measured cycles in random sequence. Each excitation in a cylinder has a time history which is independent from the excitations in the other cylinders and the statistical dispersion of the excitations correspond to the one shown in (figures 2-3). (Figure 3) shows the statistical distribution of the force applied in the combustion chamber in vertical direction. This distribution is obtained from a MC approach consisting of 31 simulations. The frequency spectra, together with the coefficient of variation, here indicated with σ_n , defined as the ratio of the standard deviation and the mean value of the measured data, are plotted in (Figure 3).

This frequency spectrum is obtained from the FMBD simulation considering a period of 3 engine periods, i.e. 0.12 s, resulting in a frequency step of 8.3 Hz. From (Figure 3), it can be seen that the frequency spectra of the applied force is characterized by a series of tones. These tones depend on the engine speeds. The first peak in the frequency spectra occur at the 0.5th engine order, 25 Hz, and the other harmonics at its multiples. Most of the energy is contained below 250 Hz. concerning the statistical distribution of the data, it can be said that the coefficient of variation σ_n is relatively low in correspondence of the cylinder orders, when the frequency spectra exhibits tonal peaks. Comparing narrowband data and third octave bands data, it can be noticed that the $[\sigma]_n$ is drastically reduced when the data are investigated in bands. In detail, σ_n is lower than 0.1 up to nearly 1 kHz third octave band. Conversely, in narrowband σ_n stays below 0.1 only up to 250-300 Hz. The influence of this cyclic variability on the results obtained from the multi-body dynamic simulation is discussed in the Appendix 1.

Figure (2): Cyclic variability of measured cylinder pressure. In blue, the mean value. In orange, the standard deviation of measured data.

Figure (3): Cyclic variability of piston force frequency spectra. Left, narrowband. Right, third octave band. In blue, the mean value. In orange, the coefficient of variation of measured data. In grey, all the different samples of the MC ap

The dispersion in the frequency spectra is higher compared with the one in time domain. The main reason is that higher frequency content is influenced by the shape and the smoothness of the excitation time history [73]. In fact, the variability of the

cylinder pressure spectrum is not only influenced by the maximum pressure but also (and especially) by the shape of the cylinder pressure time history, as confirmed in (Figure 4) where first and second time derivative of the pressure are plotted.

Figure (4): First and second time derivative of cylinder pressure.

Influence of cyclic variability on the assessment of vibration source contribution

Results of numerical transfer path analysis

In this section, the influence of cyclic variability on the ranking of the different source contributions is discussed. The assessment of the vibration contributions is done using the methodology described in section (Substructuring of multibody system for numerical transfer path analysis). In fig. 5, the mean values of the contributions to acceleration in vertical direction at nodal position of Mount1 are plotted. In (Figure 6), the acceleration spectrum of Mount1 in the vertical direction is shown. Both in (Figure 5-6), the quantities shown consider the mean values obtained from all the different samples of the Monte Carlo method. In order to account for their phase relationship, the sources are ranked in terms of projected contributions and their variability is studied. The projected contributions, discussed in [25], are defined as

$$a_p = a_n \cos(\varphi_{tot} - \varphi_n) \quad (3)$$

Where a_p is the projected contribution, i.e. the projection of the complex acceleration vector of the investigated contribution ($a_n e^{i\varphi_n}$) on the direction of the total resulting complex acceleration vector ($a_{tot} e^{i\varphi_{tot}}$). The projected contribution gives an idea of the investigated configuration: the higher its value, the higher is the component of the contribution on the direction of the overall vibration. (Figure 7) shows the variability of the projected contributions at the frequencies shown in (Figure 6). These frequencies were chosen to show how to use the results obtained from the NTPA method. (Figure 7) can help to understand how the different sources interact, e.g. at 100 Hz the vibrations derived from the excitations applied in combustion chambers are opposite in phase with the main bearings contributions. Moreover, it can be seen that the assessment of the sources is not straightforward. For instance, at 1140 and 1625 Hz, some relevant contributions may change direction. Unfortunately, with increasing frequency, the variability also increases and may happen that some projected contributions change sign.

Figure (5): Acceleration contributions in vertical direction at Mount1. Mean values.

Figure (6): Mean value of acceleration in vertical direction at Mount1. In red, the frequencies investigated considering phase information and plotted in Fig. 7.

This suggests that the variability of the phase is not negligible. To show how the variability of the phase affects the projected contributions, in (Figure 8) the projected contribution of Bearing3 is investigated in frequency domain. The coefficient of variation of the cosine of the phase of the complex vector of this contribution is also considered. For the sake of simplicity only Bearing3 is investigated. From (Figure 8), it is possible to conclude that if the coefficient of variation of the phase cosine is lower than 0.2, the

projected contribution do not change its sign (i.e. the complex vector of the contribution and the complex vector of the overall acceleration preserve their directions). This condition is fulfilled at low frequencies and in correspondence of the main engine orders. Between engine orders and with increasing frequency, the variability is high and it is difficult to properly rank the different contributions considering the phase information.

Figure (7): Variability of projected contributions at 100Hz, 200Hz, 333Hz, 767Hz, 1140 Hz and 1625 Hz. The bars around the mean value indicate to the 95% confidence interval of data.

Figure (8): Cyclic variability projected contributions of Bearing3 with 95% confidence interval of data. In orange, the coefficient of variation of the phase of the contribution of Bearing3.

Given the high variability of the results with increasing frequency, the cross correlation between the sources is investigated. (Figure 9) shows the Pearson correlation coefficients between the different projected contributions for a given frequency. The Pearson correlation coefficient is defined

as the ratio between the covariance of two random variables, A and B, and the product of their standard deviations, respectively σ_A and σ_B , [74]

$$cc = \frac{cov(A, B)}{\sigma_A \sigma_B}. \quad (4)$$

Figure (9): Pearson Correlation coefficient between projected contributions at 100 and 1625 Hz.

Analyzing (Figure 9), three things can be observed:

- Although the high variability of some sources, their projected contributions are correlated. E.g., at 1625 Hz, bearing contributions are strongly correlated. Therefore, it rarely happens that the contribution from bearing 3 is less relevant than the others.
- Despite of the high variability, comparing the correlation coefficients between the overall acceleration and all other projected contributions, it is still possible to rank the sources. For example, again at 1625 Hz, the contribution from Bearing3 is the most relevant. The conclusion is that at this frequency, the variability mainly influences the magnitude of all contributions and that Bearing 3 remains the most relevant source in most of the cases.
- Studying the correlation between contributions also when the variability is low, gives more insights on the source assessment. This is the case for the second order, 100 Hz, where bearings and balancers contributions are inversely correlated with combustion chambers

contributions and cancel each other. In fact, only liners contributions and combustion chamber 1 are correlated with the overall noise. This suggests that it is more convenient to act on piston-liner contacts to influence the acceleration measured in the investigated mount position.

Given two or more correlated sources, one may intuitively think that they can be ranked using only mean values despite of high variability. To quantify how much the ranking based on mean values of projected contributions is affected from system variability, the difference between projections is investigated. Considering two projected contributions A and B, their difference (A-B) has mean value μ_{AB} and standard deviation σ_{AB} . The probability that A is greater than B is proportional to the integral between zero and $+\infty$ of the Gaussian probability function generated using μ_{AB} and σ_{AB} . The elements in Fig. 10 indicate the probability that the projected contribution i on the row is greater or equal to the projected contribution j on the column. For the sake of simplicity, Fig. 10 refers to the same frequencies of Fig. 9.

Figure (10): Comparisons between projected contributions at 100 and 1625 Hz. The data indicate the probability that the contribution on the row is greater or equal compared with the contribution on the column.

If the variability is low, the comparison is quite obvious: (Figure 10) shows how projected contributions almost never change their ranking at 100 Hz. At 100 Hz, Comb1 is always the highest projected contribution, Comb2 the second, Comb3 the third and so on, in accordance with the results shown in (Figure 7). At 1625 Hz, the discussion arising are more interesting and less intuitive. As shown in (Figure 10), at 1625 Hz, Bearing3 is often bigger than all other projected contributions, but it is not always the highest. In the 86% of the cases, Bearing 3 is higher than Bearing 5 and only in the 76% of the cases it is higher than Bearing 4. In conclusion, the ranking based on mean values of projected contributions is valid, although correlations between sources and data dispersion has to be investigated if the variability increase.

An alternative to rank the system sources at high frequencies is to investigate only the acceleration magnitude, in particular, can be useful to study results in

broad bands. (Figure 11) shows the variability of the ranking of the contributions in third octave bands. For the sake of simplicity, results are shown only in bands from 800 to 1600 Hz, where the variability is higher. Lower frequencies bands are mainly driven from the tones appearing at engine orders (see Figure 6) and, therefore, it is more convenient to study directly the tones with the projected contributions. Studying Figure 11, it is clear that, despite of the variability that the model exhibits, it is possible to focus on the sources which have the highest absolute amplitude and neglect the ones which are orders of magnitude lower than the overall acceleration level. The results presented in this section refer only to the vertical acceleration at the nodal position of Mount1, because the vertical direction is the one that exhibits the highest magnitude acceleration and joint contact forces. The conclusions reached in this section can also be derived from the analysis of the other directions of the mount acceleration

Figure (11): Influence of cylinder pressure variability on sources ranking at mid-high frequency. The bars around the mean value indicate to the 95% confidence interval of data.

Comparison of vibration contribution with system forces

The variability of the acceleration contributions, evaluated in the investigated mount position, can be correlated to the variability of the different source excitations acting on the engine block. Comparing the results obtained from the multi-body dynamic simulations (and plotted in Appendix 1), it can be observed how the coefficient of variation of the forces and the coefficient of variation of the respective acceleration contributions has similar trend, especially in octave bands. For this reason, (Figure 12) shows a comparison of these coefficients of variation. The data are presented in octave bands. Although the system investigated consists of 21 different sources, i.e. contributions, for the sake of readability only 14 of them are presented in (Figure 12). Never the less, the conclusions made on these contributions have general validity and the discussion can be extended also to the others omitted. In (Figure 12), the acceleration refers to the vertical direction at nodal position of Mount1, while the forces are considered as sum of all forces applied in the same direction at the same joint. For each source, 3 directions are considered, though not always all directions are excited.

In (Figure 12), it can be observed how the coefficient of variation of the acceleration follows the one of the corresponding exciting force. In particular, if the excitation acts in more than one direction, the variation of the acceleration follows the variation of the predominant component of the force. For example, at main bearings and at balancers supports the force acting in vertical direction is higher in magnitude compared to the force acting in the thrust direction. Therefore, the coefficient of variation of the acceleration contribution most likely follows the trend of the coefficient of variation of the corresponding source forces acting in the vertical direction.

Conclusions

In this paper, a methodology to correlate excitation uncertainties and sources assessment in Multibody dynamic systems is described. The influence of cyclic variability in the numerical transfer path analysis of multi-body engine systems was investigated. The variability of cylinder pressure was measured from running operating conditions of a test engine and the measured data were used as input excitation for the analysis on a numerical multi-body systems of the engine tested. A Monte Carlo method was adopted to

perform a series of multi-body simulations which differ in terms of statistical distribution of the excitation, in accordance with the measured cyclic variability. The results obtained from this analysis are used to derive statistical information on the variability of the results of a numerical multi-body analysis and on the variability of the vibration contribution of the different vibration sources. The results obtained show how the variability of the results of the multi-body analysis is in accordance with the variability of the measured cylinder pressure. In general, the variability increases with frequency and is minimal at main engine orders. If the results are considered in terms of bands, e.g. third octave bands, the variability is reduced and the robustness of the results obtained is higher. As concern the investigation of the source contribution, this study shows how the variability of the vibration contributions strictly depend on the variability of the corresponding exciting sources. In particular, if the excitation acts in more than one direction, the variation of the acceleration follows the variation of the predominant component of the force.

The statistical information derived can be used to advance the knowledge of the multi-body analysis and the assessment of system sources when uncertain inputs are considered. From this study, the following considerations can be made:

- The variability of cylinder pressure can affect the results of multi-body analysis simulation at high frequency range;
- In the low frequency range and at main engine orders the variability is limited;
- The variability of the different sources contributions is connected to the variability of the corresponding exciting forces;
- In order to rank the different source contributions at low frequencies and at main engine orders it is convenient to use both magnitude and phase information. Conversely, at higher frequencies, where the variability of both magnitude and phase is higher, the assessment made using the phase is not straightforward. It is still possible to consider the phase at high frequency, although it can be required to also include information regarding the correlation terms between the different sources. As an alternative, it can be possible to rank the sources in terms of band results (octave or third octave bands).
- The investigation of the cross correlation between source terms can be used to analyze the correlation between sources. Also sources relationship with the overall vibrations can be investigated with correlation coefficients, giving more insight on the effective contributions to the global response.

Figure (12): Comparison of coefficient of variation of source excitations and source contributions to acceleration at Mount1, DoF 3.

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Appendix 1: Influence of cyclic variability on multi-body dynamic systems

In this appendix, the results of the multi-body dynamic system are discussed.

Figure (13): Cyclic variability of joint forces at Bearing1 and 3. Left, narrowband. Right, third octave band.

The results presented refer to the MC series of simulation performed. In order to show the variability of the joint forces, (Figure 13-14) illustrate the results obtained from the FMBD simulation in some of the joints; these joints are chosen as representative of all different phenomena occurring in the different bodies contacts.

The forces shown in (Figure 13-14) are the sum of all forces applied in the same direction in all nodes belonging to the same joint. For the sake of readability, results are presented only for few DoFs, although the similar conclusion can be made studying all directions. DoFs 1, 2 and 3 are associated with rotation axis, thrust and vertical directions respectively. Studying the cyclic variability of the joint forces, it is evident that the coefficient of variation is lower than 0.1 in correspondences of tonal excitations; in particular at low frequencies and at main engine orders. If the frequency differ from the ones at which the tones appear, the variability can increase and the coefficient of variation can also exceed 0.5.

Influence of cyclic cylinder pressure variability in the numerical transfer path analysis of multi-body engine.

Figure (14): Cyclic variability of joint forces from top: Axial and BallInt2. Left, narrowband. Right, third octave band.

E.g., in (Figure 13), where the vertical force at Bearing3 exhibits a tonal behavior and between the tones the coefficient of variation is high also below 200 Hz. In general, above 1 kHz the energy of the signal is orders of magnitude lower compared with the low frequencies and the variability is high. Comparing narrowband and third octave band results, it is evident that the variability decrease when the data are treated in bands. Similar considerations arise also analyzing the frequency spectra of the combustion excitation, (Figure 3).

(Figure 15) shows the acceleration in the node where the Mount1 (see Figure 1) is connected to the engine. All three directions are considered in frequency domain. The acceleration in vertical direction (DoF 3) is

the highest in terms of magnitude, while the acceleration in the direction of the axis of the crankshaft (DoF 1) is the lowest. The considerations, which can be done analyzing the acceleration on a response node (in this case an engine mount position), are similar to the ones made studying the forces. Indeed, the coefficient of variation increase with the frequency and it is relatively low in correspondence of the peaks occurring at main engine orders. The variability of the acceleration is also lower when considering frequency band results. Comparing narrowband and third octave band results, it is evident that the variability decrease when the data are treated in bands. These considerations suggest a direct correlation between the variability of joint forces and source contributions.

Figure (15): Cyclic variability of mount acceleration. Form top: DoF 1, 2 and 3. Left, narrowband. Right, third octave band

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