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THE USE OF MECHANICAL FILTER MODELS IN THE ANALYSIS OF FORMING AND COMPACTION PROCESSES OF FORMATION AND COMPACTION OF BUILDING/CONCRETE MIXTURES BY VIBRATING FIELD

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Abstract. The paper describes the use of various types of mechanical filter models, which are used for the analysis of the processes of formation and compaction of the construction/concrete mixtures of building/concrete mixtures by means of vibrating fields. The values of resonant frequencies and equivalent masses for different resonators modeling the propagation in the latter of vibrating-wave formations have been established. The analysis of the influence of a vibrating field on the processes of formation and compaction of concrete/concrete mixtures in this study is based on the methods of mathematical physics, classical variation calculus, physics of oscillations and waves and methodology of solution of ordinary differential equations and partial differential equations. The conditions and main integral characteristics of resonance phenomena, the possibility of occurrence of which is conditioned by: 1) the geometry of the initial boundary-edge problem (it is The so-called "geometric resonances" of the considered system with distributed parameters simulating the mixture to be processed); 2) the working rheological model of the mixture involved in the study (these are the so-called "rheological resonances").

The approach developed and scientifically substantiated in this work allows us to establish the main parameters and opportunities for the use of energy-saving modes of operation of vibration systems intended for the formation and vibration compaction of the above mixtures. The results obtained in the work The results obtained can be further used to clarify and Improvement of existing engineering methods of calculation of vibration systems for the formation and compaction of concrete/concrete mixtures in order to optimize the operating modes of their functioning both at the design stage and in the modes of real operation.

Keywords: modeling, mechanical filters, vibration resonators, analysis, formation processes, compaction, construction and concrete mixtures, vibration field, resonances, equivalent masses.

Problem formulation. Mechanical circuit diagrams are quite often used in modeling the compaction and forming processes of concrete/construction mixtures. In particular, equivalent circuits simplify the calculation process or make the operation of mechanical filters more understandable [1]. Say, the distributed mechanical circuit of a piezoceramic transducer operating on flexural vibrations is often transformed in scientific research into a mechanical circuit with concentrated elements formed by springs and masses. The mechanical circuit is then converted into its electrical counterpart. The conversion from a plate resonator to a system of masses and springs simplifies the circuit, but this conversion is not exact. A plate resonator has an infinite number of natural frequencies, while the number of resonances of an equivalent circuit with concentrated parameters is determined by the number of springs, masses, and the way they are connected. Since mechanical filters are devices with a narrow frequency bandwidth, this fact does not cause large errors, and equivalent circuits with concentrated elements are very useful.

The second type of equivalent circuits are electrical analogs of mechanical circuits. This equivalence promotes understanding and facilitates the analysis of not only the converter but also the entire filter. In the initial stage of system design, the filter user may deal with the mechanical filter as if it were a LC – filter, or more generally, a resonator stage circuit. Let us first consider mechanical schematic diagrams and the very subject of analogies and equivalent circuits.

Both mechanical circuit diagrams and mechanical analogies may be used to describe physical systems. Mechanical circuit diagrams may have concentrated elements, such as masses and springs, or capacitances and inductances, or transmission lines (elements with common parameters), which are characterized by wave impedance and propagation constant (wave propagation).

Mathematical modeling of such systems can be expressed through differential equations or equations of change of state. In all cases, the starting point is a physical system, which in turn is expressed by a kinematic scheme [1].

A kinematic diagram is simply a representation in the form of a drawing of the main features of a real device. It is most often used to describe a mechanical device and acts as a bridge between the real device and its mechanical circuit diagram.

For example, the simplest kinematic diagram of a mechanical single resonance filter has the form of a mass M, which is connected to a stiffness spring K, and to this mass is connected in parallel damping element D.

The resonant frequency of such a system is determined from the relation:

$$\Omega = 2\pi f = \sqrt{K/M}\,, \tag{1}$$

where: Ω is cyclical, f – linear frequency.

The equivalent mass of such a system is the apparent mass of the resonator measured at a point on the resonator and in a particular direction. In other words, we have replaced the resonator with a spring-mass combination that has the same resistance near the resonant frequency. The equation of state for the resonator through the force is F and velocity $\dot{x} = \dfrac{dx}{dt}$, where x – displacement of mass M in space, t – is time, written as:

$$F = j\omega M\dot{x} + \left(\frac{K}{j\omega}\right)\cdot \dot{x} + D\dot{x},\ \ j^2 = -1. \tag{2}$$

The differential equation (essentially a Proportional-Integral-Differential (PID) controller equation for force F at its harmonic variation in time) will have the form:

$$F\sin\omega t = M\frac{d\dot{x}}{dt} + K\int \dot{x}dt + D\dot{x}\,. \tag{3}$$

Proceeding from equation (2), after passing to complex variables (find $\dot{x}(t) = x_0\exp(j\omega t)$), one has:

$$\dot{x}(t) = \frac{F(t)}{\left(j\omega M + \dfrac{K}{j\omega} + D\right)}. \tag{4}$$

Resonance value $\dot{x}_{res}$ will be under the condition of the minimum value of the denominator of formula (4), namely, when:

$$j\omega M + \frac{K}{j\omega} = 0 \Leftrightarrow \omega_{res} = \Omega_{res} = \sqrt{\frac{K}{M}};\ \dot{x}_{res}(t) = F(t)/D\,. \tag{5}$$

Consequently, such a simple model of the mechanical system exhibits resonant properties and has a resonant frequency $\omega_{res} = \Omega_{res}$. Note that the circuit diagram (for such a kinematic scheme) turns into three in parallel: mass M, stiffness K, and damping element D. In the electromechanical analogy, the equivalent of velocity becomes voltage V and force becomes current I. Therefore, writing (2) and (3) in electrical quantities, we obtain:

$$I = j\omega CV + \left(1/ j\omega L\right)V + GV\,; \tag{6}$$

$$I\sin\omega t = C\frac{dV}{dt} + \frac{1}{L}\int Vdt + GV\,, \tag{7}$$

where: G is permeability, L – inductiveness, C – capacitance. Therefore, the following equivalence of mechanical and electrical parameters arises: $M \leftrightarrow C;\ D \leftrightarrow G;\ K \leftrightarrow L^{-1}$.

Quite often in the literature, researchers also use longitudinal oscillation resonator models.

In the case when one of the resonator dimensions becomes much larger than the other two, the vibration types are simplified and become ideal types of longitudinal compression-extension vibrations. The wave equation for such vibrations of rods and strips have second order with respect to the length coordinate, i.e.:

$$\left(\frac{E}{\rho}\right)\cdot\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \tag{8}$$

where: $u\left(x,t\right)$ – moving the section point with the coordinate x, at a point in time t, ρ – material density, E – is the elasticity modulus of this material. After introducing complex variables, we consider that $u = A\cdot\exp(i\omega t)$, where $i^2 = -1$, ω – is the cyclic frequency of oscillations. Then from (8) we have:

$$\left(\frac{E}{\rho}\right)\cdot\frac{\partial^2 u}{\partial x^2} = -\omega^2 u\,. \tag{9}$$

We find the solution of (9) in the form:

$$u\left(x\right) = A\sin kx + B\cos kx\,, \tag{10}$$

where: x – distance from the rod end, k – is the wave propagation constant in the rod. We consider that the ends of the rod oscillate freely, then we have for the constants undefined in (10) A and B two boundary conditions if the rod has finite length l:

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = 0;\ \left.\frac{\partial u}{\partial x}\right|_{x=l} = 0\,. \tag{11}$$

Substituting (10) into the first limiting condition, we obtain:

$$A = 0\,. \tag{12}$$

Therefore,

$$u\left(x\right) = B\cos kx\,. \tag{13}$$

From the second boundary condition (11) we have:

$$\sin\left(kl\right) = 0\,. \tag{14}$$

Equation (14) is the resulting frequency equation and has the following roots:

$$k_n l = n\pi,\ n = 1,2,3,\ ...\left(n \in N\right). \tag{15}$$

Next, we establish the relationship between and frequency by substituting the solution (13) into the wave equation (9) and differentiating the left part by x. After contracting the value $\cos\left(kx\right)$ one obtains:

$$k = \omega\cdot\sqrt{\rho/E}\,, \tag{16}$$

that is, we obtain the so-called dispersion equation of the resonator. Note that the phase velocity of the waveforms:

$$v_{phase} = v_p = \frac{\omega}{k} = \left(\frac{E}{\rho}\right)^{1/2}, \tag{17}$$

and the group velocity of wave formation:

$$v_{group} = v_g = \frac{d\omega}{dk} = \left(\frac{E}{\rho}\right)^{1/2}, \tag{18}$$

independent of frequency ω, and therefore the longitudinal type of oscillations is nondispersive. Then we substitute the values k_n from (15) into the dispersion equation (16); this gives us a relation for determining the resonant frequencies of the rod:

$$f_n = \frac{\omega_n}{2\pi} = \left(\frac{n}{2l}\right) \cdot \sqrt{E/\rho},\ n \in N. \tag{19}$$

For each mode of wave formations in the rod by substituting the value of k_n from (15) into equation (13) the distribution of displacements can be found $u(x)$. The result is:

$$u(x) = B\cos\left(n\pi x/l\right). \tag{20}$$

The above approaches known in the scientific literature are used in this study.

Analyzing the study's publications. Rheological models of media to which concrete/construction mixtures belong are considered in [2-13], and the idea of using models of mechanical filters to analyze wave processes and determine the integral characteristics of mixtures (concrete or construction) is presented in [1]. However, the authors of these works practically do not investigate exactly the resonance properties of their proposed rheological models of media that support wave formation of various types (in particular, longitudinal waves). Therefore, this particular study is devoted to the problem of determining possible resonance phenomena (geometric and rheological) and the conditions for their occurrence in media that are treated by vibratory fields. Concrete/construction mixtures belong to such media.

The aim of the work is to establish the conditions and characteristics of resonances that are possible in concrete/construction mixtures when studying the processes of their vibration compaction and formation within the framework of classical rheological models of visco-elastic-plastic media in the presence of longitudinal wave formation in the latter. To achieve the goal of this work, the model of a rod supporting longitudinal wave formation is used.

Research Methodology. The following methods are chosen as research techniques: 1) models and mathematical apparatus of solid deformed body mechanics; 2) methods of solving differential equations describing linear wave processes in visco-elastic deformed bodies (in the presence of various dissipation mechanisms); 3) methods of mathematical physics; 4) methods and models of linear acoustics used in the analysis of impedance (including resonance) properties of media supporting waves of different physical nature; 5) discrete-continuum models and methods of analyzing linearly deformed media (including resonance) properties of media supporting waves of different physical nature; 6) discrete-continuum models and methods for analyzing linearly deformed media.

Outlining the main content of the study. Let us consider wave formation in a rod of finite length, when it is necessary to take into account dissipative processes that occur in this rod. It should be noted that the rod model is popular in studies of vibration compaction of concrete mixtures.

1. Investigation of absolute (in time) stability of wave formation in the rod.

Proceeding from (16), we feed the dispersion relation as follows:

$$\frac{\omega}{k} = \left(\frac{\tilde{E}}{\rho}\right)^{1/2},\ \omega = \omega' + i\omega'',\ i^2 = -1,\ \tilde{E} = E' + iE'', \tag{21}$$

where: ω' – characterizes the frequency of wave propagation in the rod material, ω'' – characterizes the attenuation of this wave formation in time t (specifically, $(\omega'')^{-1} = \tau_{att}$, where τ_{att}

– time period for which the amplitude of wave formation decays in the e-times), E' – characterizes the elastic properties of the rod (its material), E'' – its dissipative properties, with k we shall hereinafter understand the expression k_n, $n \in N$ (15). For the wave formation to be absolutely stable, i.e., not increasing with time, it is necessary that the $\omega'' > 0$. (It should be noted that, ω' describes the so-called frequency of (cyclic) filling of wave formation, and the true frequency of wave formation Ω is expressed as follows:

$$\Omega = \left\{ \left(\omega'\right)^2 - \left(\omega''\right)^2 \right\}^{1/2}, \tag{22}$$

that is, in fact, ω' characterizes the frequency (cyclic) of wave propagation in the rod (in its material) in the absence of dissipative processes – that is, we mean a rod of ideal material (in which there is no dissipation).

Relationship (21) can be represented as follows:

$$\frac{\left(\omega' + i\omega''\right)}{k_n} = \left(\frac{E' + iE''}{\rho}\right)^{1/2}. \tag{23}$$

The solution of this equation in the complex plane allows us to write the following (for each n-mode of wave formation):

$$\omega_n' = \left\{ \left[\left(\frac{E'}{\rho} \cdot \frac{k_n^{\ 2}}{2} \right)^2 + \left(\frac{E''}{\rho} \right)^2 \cdot \frac{k_n^{\ 4}}{4} \right]^{1/2} + \frac{E'}{\rho} \cdot \frac{k_n^{\ 2}}{2} \right\}^{1/2}, \quad n \in N. \tag{24}$$

$$\omega_n'' = \left\{ \left[\left(\frac{E'}{\rho} \cdot \frac{k_n^2}{2} \right)^2 + \left(\frac{E''}{\rho} \right)^2 \cdot \frac{k_n^4}{4} \right]^{1/2} - \frac{E'}{\rho} \cdot \frac{k_n^2}{2} \right\}^{1/2}, \ n \in N. \tag{25}$$

Expressions (24), (25) taking into account (15) can be presented as follows:

$$\omega_n' = \frac{n\pi}{\sqrt{2}l} \cdot \left\{ \frac{E'}{\rho} + \frac{\left[\left(E'\right)^2 + \left(E''\right)^2 \right]^{1/2}}{\rho} \right\}^{1/2}, \ n \in N, \tag{26}$$

$$\omega_n'' = \frac{n\pi}{\sqrt{2}l} \cdot \left\{ \frac{\left[\left(E'\right)^2 + \left(E''\right)^2 \right]^{1/2}}{\rho} - \frac{E'}{\rho} \right\}^{1/2}, \ n \in N. \tag{27}$$

For the value Ω_n we have the following expression (exact formula):

$$\Omega_n = \frac{n\pi}{l} \cdot \left\{ \frac{E'}{\rho} \right\}^{1/2}, \ n \in N. \tag{28}$$

We further consider that the deformation $\varepsilon(t)$ and $\sigma(t)$ – stresses related to each other by the linear Hooke's law, which takes into account dissipative processes in the rod material:

$$\sigma = \left(E' + iE''\right) \cdot \varepsilon; \ \sigma_0 = \varepsilon_0 \cdot \sqrt{\left(E'\right)^2 + \left(E''\right)^2}, \tag{29}$$

where: $\left(\sigma_0, \varepsilon_0\right)$ – stress and strain amplitudes, provided that $\sigma(t)$ and change over time as follows: $\varepsilon(t) = \varepsilon_0 \sin \omega t; \ \sigma(t) = \sigma_0 \sin\left(\omega t + \varphi\right); \ tg\varphi = \left(\frac{E''}{E'}\right).$ (30)

For the time variations given by (30) ε and σ let us calculate the work done by the wave formation/vibrations in the rod material during the time t (per unit area of the cross section of the rod S):

$$A=\int_0^t \sigma d\varepsilon=\omega\sigma_0\varepsilon_0\cdot\left[\cos\varphi\cdot\int_0^t\left(\sin\omega t\cdot\cos\omega t\right)dt+\sin\varphi\cdot\int_0^t\sin^2\omega t dt\right]= \qquad (31)$$
$$=U+Dt,$$

where: U – fully recoverable work of elastic forces, which can be written as follows:

$$U=\sigma_0\varepsilon_0\cdot\left[0,5\cos\varphi\cdot\sin^2\psi+0,25\sin\varphi\cdot\sin 2\psi\right],\ \varphi=\omega t; \qquad (32)$$

and D the following relationship can be found:

$$D=0,5\omega\sigma_0\varepsilon_0\cdot\sin\varphi. \qquad (33)$$

In essence Dt is irreversible/unrecoverable work per unit time.

The power of elastic forces can be found as follows:

$$\frac{dU}{dt}=0,5\omega\sigma_0\varepsilon_0\cdot\sin\left(2\omega t+\varphi\right). \qquad (34)$$

From (34) it can be seen that the maximum values of the elastic force power reach at the moments of time t^*, which can be found from the ratio:

$$2\omega t^*+\varphi=\frac{\pi}{2}+2\tilde{n}\pi,\ \tilde{n}=0,1,2,\ ...\ ; \qquad (35)$$

For t^* one has:

$$\left(\frac{dU}{dt}\right)_{\max}=0,5\omega\sigma_0\varepsilon_0. \qquad (36)$$

Maximum value U is achieved under the condition:

$$2\psi=\pi-\varphi. \qquad (37)$$

In this case we have:

$$U_{\max}=\frac{1}{4}\sigma_0\varepsilon_0\cdot\left(1+\cos\varphi\right). \qquad (38)$$

If we introduce the concept of inverse goodness of fit (Q^{-1}) as the ratio of the energy that is dissipated during the time when the phase of wave formation/vibration changes by 1 radian to the maximum value of free energy in the oscillation cycle (period of this cycle $T=2\pi/\omega$), and take into account that the change ψ by one radian occurs in the time ($1/\omega$), then you can have it:

$$Q^{-1}=\frac{D}{\omega U_{\max}}=\frac{0,5\omega\sigma_0\varepsilon_0\sin\varphi}{\omega\cdot\frac{1}{4}\sigma_0\varepsilon_0\cdot\left(1+\cos\varphi\right)}=\frac{2\sin\varphi}{\left(1+\cos\varphi\right)}=2\cdot tg\frac{\varphi}{2}. \qquad (39)$$

Since $tg\varphi=E''/E'$, with (39) one has:

$$Q^{-1}=-2\cdot\left(\frac{E'}{E''}\right)+2\cdot\sqrt{\left(\frac{E'}{E''}\right)^2+1}. \qquad (40)$$

Expression, for Q^{-1} (40) is exact, but $\varphi<<1$ one can write:

$$Q^{-1}=\frac{2\sin\varphi}{1+\cos\varphi}\approx\frac{2\sin\varphi}{2}=\sin\varphi\approx tg\varphi=\frac{E''}{E'}. \qquad (41)$$

D gains maximum value when:

$$\sin\varphi = 1 \Rightarrow \varphi = \frac{\pi}{2}. \tag{42}$$

Then:

$$Q^{-1} = 2, \tag{43}$$

and: $E'' >> E'$.

Also, it follows from relation (42) that: $E'' >> E'$. (44)

Then for Ω_n one gets the equality (28), which will be accurate for the resonant frequency of the waveformation/vibration propagating in the rod material. In this sense, such a frequency of wave formation can be called the resonant frequency, since it is at this frequency that the maximum energy absorption by the rod material from the existing wave formations in it occurs.For ω'_n and ω''_n one can obtain:

$$\omega'_n \approx \omega''_n \approx \frac{n\pi}{\sqrt{2}\cdot l}\cdot\left\{E''\middle/\rho\right\}^{1/2}. \tag{45}$$

So, the wave formation in the rod, which leads to the maximum absorption by its material of the energy of this wave formation, has all the features of a dissipative wave, with a "resonant" (in this sense of maximum absorption) frequency (28), the filling frequency of the envelope of this wave ω'_n (45) and amplitude, which decreases in time (envelope wave) according to the law proportional to $\exp(-\omega''_n \cdot t)$. Moreover, as the mode number increases, this damping of the envelope wave increases. Practically, only the first and second modes are excited, since all higher modes (in terms of the mode number n) will quickly decay. Therefore, the conclusion is as follows: in a rod of finite length l only a few dissipative waveforms are excited, which have the following parameters:

$$n = 1, 2, 3;\ \tilde{\Omega}_n = \frac{n\pi}{l}\cdot\left\{\frac{E'}{\rho}\right\}^{1/2};\ \tilde{\omega}'_n \approx \tilde{\omega}''_n \approx \frac{n\pi}{\sqrt{2}\cdot l}\cdot\left\{E''\middle/\rho\right\}^{1/2}. \tag{46}$$

Models of elastic-plastic material (rod) in the theory of internal friction are considered in detail by the author [2], where the values of the E' and E'' for many such models. The vast majority of works on amplitude-dependent friction, particularly in metals, are devoted to the study of harmonic or near-harmonic deformation laws (such as the one discussed above). This applies equally to both experimental and theoretical works. The main reason preventing the theoretical study of nonharmonic motions relevant for applied problems is the lack of an analytical expression for the external (forced) force at an arbitrary deformation law in time. Note that the available expressions of this force [2, 3] refer exclusively to the harmonic law of change of deformation, and it is not clear how to define or remake these dependencies so that they appear to be valid under an arbitrary law. At the same time, the ideas necessary for this purpose have been expressed in the literature.

In 1938, N.N. Davydenkov on the basis of the experimental works carried out at that time and at that level put forward a hypothesis according to which internal friction at significant material stresses is an effect of microplastic deformations. Even a direct indication that internal friction should be studied using the equations of Mises-Henky plasticity theory is known. However, this rational idea was realized by N.N. Davydenkov only for the case of cyclic deformation under uniaxial stress state and with a partial view of the material loading curve. As a result, the well-known formula of the hysteresis loop was proposed, according to which the energy losses in the material per oscillation cycle depend according to the step law on the amplitude of deformation or stress.

This view of the main role of the theory of plasticity for the applied theory of energy dissipation is explicitly or implicitly shared by many modern authors. Thus, the formula of N.N. Davydenkov and its generalization are intensively used in the studies of G.S. Pisarenko and other members of the Kiev school. The same formula was the basis for the creation of simpler applied theories of internal friction, of which the theory of J.G. Panovka is the most widespread.

The direct use of the equations of plasticity theory to analyze internal friction under uniaxial stress state and again for harmonic deformation was carried out by E.S. Sorokin in 1960 [2].

Thus, the most popular formulas of the theory of energy dissipation under intense stresses are based on the ideas of the theory of plasticity [2].

The fruitfulness of this approach also manifests itself in the fact that it is possible to unambiguously solve one more important problem for the applied theory of energy dissipation – generalization to the case of a complex stress state. The fact that this problem is relevant for the theory of energy dissipation is evidenced by direct indications in the book [2] and numerous ways of its solution exactly for the case of harmonic oscillations. In the literature it was suggested to carry out this generalization by methods of the theory of linear viscoelasticity [3], with the help of the superposition principle, using the hypothesis that the energy dissipation in a unit volume per oscillation cycle depends on the amplitude value of the potential energy density and, finally, by methods of the theory of plasticity . Only the last of these methods can be correctly generalized to nonharmonic motions [2, 3].

Let us dwell on the question of choosing the variant of plasticity theory that is suitable for the description of internal friction. The point of view of N.N. Davydenkov was mentioned above, according to which the amplitude-dependent internal friction represents the effect of microplastic deformations. Microplastic deformations are understood as such plastic deformations that occur at any stress level, including stresses less than the macroscopic yield strength of the material. From this point of view, the application of the theory of microplastic deformations by V.V. Novozhilov and Yu. Novozhilov and Y.I. Kadashevich, where, for example, the simplest of the microplasticity theories – the theory of elastic-plastic bodies – is systematically used, and the general theory of plasticity with linear hardening (which can be used to describe the cyclic deformation of concrete/construction mixtures) is considered in the works of this author.

In the following we will use expressions for E' and E'', which follow, in particular, from the consideration of the uniaxial stressed state of the material in the framework of the models of A.Y. Ishlinsky, E.S. Sorokin [2], G.S. Pisarenko, and Y.G. Panovka.

For the main parameters of wave formations arising in the rod material when dissipation processes are taken into account ($\tilde{\Omega}_n$, $\tilde{\omega}'_n$, $\tilde{\omega}''_n$ (46)) and (Ω_n (28)), (ω'_n (26), ω''_n (27)), one should determine E' and E'' – parameters of the complex modulus of elasticity (at known E – Young's modulus of the material, its density ρ geometric length of the rod l) for different models of visco-elastic-plastic materials. We have the following results:

a) A.Y. Ishlinsky's model:

$$E' = E\left(1 - ra^{\alpha}\right);\ E'' = Ega^{\alpha}, \tag{47}$$

where notations are introduced:

$$r = \frac{2H}{\left(2+\alpha\right)} \cdot n^{-\alpha};\ g = \frac{\alpha H}{2} \cdot n^{-\alpha} \cdot B\left(\frac{\alpha+1}{2}, \frac{3}{2},\right), n = \frac{4}{\pi}, \tag{48}$$

a – amplitude of cyclic (harmonic) deformation of material, B – Euler's beta function, $\left(H, \alpha\right) > 0$ – some positive constants describing the distribution function of defects in the region of their small values:

$$F\left(h\right) = Hh^{\alpha}, \tag{49}$$

where: h – dimensionless yield strength of the rod material;

b) E.S. Sorokin's model [2]:

$$E' = E \cdot \frac{(1-\gamma^2/4)}{(1+\gamma^2/4)},\ \ E'' = E \cdot \frac{\gamma}{(1+\gamma^2/4)}, \tag{50}$$

where: $\gamma = \gamma\left(a\right)$ – dissipation parameter (dissipation coefficient, characterizing the ratio of energy quantity ΔW, which is dissipated in the rod material during its oscillation cycle, to W – of all elastic energy stored in the rod material during the oscillation cycle, i.e. $\gamma = \Delta W / W$). As a general rule $0 < \gamma << 1$, therefore we can write:

$$E' \approx E\left(1 - \gamma^2/2\right),\ E'' \approx E\gamma . \tag{51}$$

c) G.S. Pisarenko's model:

$$\left\{E' = E \cdot (1 - r \cdot a^{\alpha}),\ \ E'' = E \cdot g \cdot a^{\alpha}\right\} \Leftrightarrow E' = E - \frac{r}{g} \cdot E'';$$
$$\frac{r}{g} = \frac{n \cdot 2^{2n}}{(n-1)} \cdot \frac{[G(n+1/2)]^2}{G(2n+1)}. \tag{52}$$

Note that in contrast to the model of A.Y. Ishlinsky, in the model of G.S. Pisarenko another value of $\frac{r}{g}$. In (52) G (z) – gamma function of the argument z.

In A.Y. Ishlinsky's model we have:

$$\frac{r}{g} = 4 \cdot \left[\alpha(2+\alpha) \cdot B\left(\frac{\alpha+1}{2}, \frac{3}{2}\right)\right]^{-1}. \tag{53}$$

d) Y.G. Panovka's model:

$$E' = E,\ E'' = Ega^{\alpha}. \tag{54}$$

In [3] it is shown that the theories of G.S. Pisarenko, Y.G. Panovk and A.Yu. Ishlinsky lead to practically the same results, and the existing differences in the formulas for the E' and E'' caused by different methods of averaging and linearization.

2. Investigation of convective (in space) stability of wave formations in a rod. We consider (in the field of complex quantities) the deformation, which propagates in the rod, as a quantity proportional to $\exp\left\{i\left(\omega t - \bar{k}_n x\right)\right\}$, where $\bar{k}_n = k'_n + ik''_n, 0x$ – axis along which the deformation wave propagates in the rod (thus ω – purely real value). For this deformation wave to be convectively stable, i.e., not growing in space along the propagation axis, the condition must be fulfilled:

$$k''_n < 0. \tag{55}$$

To determine k'_n and k''_n it is necessary to solve the dispersion equation in the complex plane:

$$\frac{\omega_n}{k'_n + ik''_n} = \left(\frac{E' + iE''}{\rho}\right)^{1/2}. \tag{56}$$

Physical meaning k'_n is that this quantity characterizes the wave vector of wave formation propagating in the rod material, and $\left(k''_n\right)^{-1}$ – at what length along the axis of the rod the amplitude 0x of wave formation in space decreases to $e-$ times.

For convectively stable wave formation we have:

$$k'_n = \left\{\frac{\omega_n^2 \rho}{2\left[\left(E'\right)^2 + \left(E''\right)^2\right]} \cdot \left[E' + \sqrt{\left(E'\right)^2 + \left(E''\right)^2}\right]\right\}^{1/2}; \tag{57}$$

$$k''_n = -\left\{\frac{\omega_n^2 \rho}{2\left[\left(E'\right)^2 + \left(E''\right)^2\right]} \cdot \left[\sqrt{\left(E'\right)^2 + \left(E''\right)^2} - E'\right]\right\}^{1/2}. \tag{58}$$

3. Equivalent mass and the method of its calculation.

The equivalent mass (of a resonator) is defined as the equivalent distributed parameter of a rod or plate, concentrated at a given point and determined for a particular natural frequency. In other words, we replace the equivalent transmission line circuit described above with a mass and spring that are resonant at frequency.

One method of finding the equivalent mass of a resonator that oscillates in the longitudinal direction is to calculate the kinetic energy, which is the product of the mass by half the square of the velocity in a given direction and at a given point [1]. This is a consequence of the fact that the kinetic energy of the rod is invariant, that is, the kinetic energy is independent of the point or direction of reference chosen to calculate the equivalent mass. Thus, it is possible to equate the kinetic energy in a spring-mass system to that of the rod resonator. From equation (13) and equation $V_x = du(x)/dt$ gives:

$$\frac{1}{2} \cdot M_{eqvx} \cdot V_x^2 = \frac{1}{2} \cdot \int_0^l (V_0 \cdot \cos k_n x)^2 \cdot \rho \cdot S dx, \qquad (59)$$

where: M_{eqvx} – equivalent mass in point x, linked to velocity V_x in the direction of axis x. V_0 is the velocity at the point $x = 0$, ρ – core material density, S – is the cross-sectional area. Therefore, the equivalent mass reduced to the ends of the rod is equal to:

$$M_{eqv0,l} = \frac{\frac{1}{2} \cdot \rho \cdot S \cdot V_0^2 \cdot \int_0^l \left(\cos^2 k_n x\right) dx}{\frac{1}{2} \cdot V_0^2}, \qquad (60)$$

and after substitution $k_n = n\pi/l$ in (60) and integration:

$$M_{eqv0,l} = \rho \cdot l \cdot S / 2 = M_{st} / 2. \qquad (61)$$

Consequently, the equivalent mass given to the end of a longitudinally vibrating resonator in the direction of the plate or rod axis is simply half its static mass ($M_{st} = \rho \cdot S \cdot l$).

Example. We solve the problem of finding the fundamental resonance frequency and equivalent mass of a longitudinal oscillating resonator with length, $l = (1...3)m,\ S = 0{,}4m^2,\ \rho = 2 \cdot 10^3 kg/m^3$.

Table 1 shows the values of the first resonant frequency, which is expressed by relation (19), for different grades of concrete (M200...M800), as well as the value of the $M_{eqv0,l}$ for different l.

Table 1 – Value of the first resonant frequency f_1, Hz and $M_{eqv0,l}$, kg, for different grades of concrete and different resonator lengths l, м

Type of concrete	$E,\ N/m^2 \equiv Pa$	l,m;/$M_{eqv0,l}$, kg		
		l =1m; 400 kg	l =2m; 800 kg	l =3m; 1200 kg
M200	$2 \cdot 10^7$	50 Hz	25 Hz	17 Hz
M300	$3 \cdot 10^7$	61 Hz	30.5 Hz	20.3 Hz
M400	$4 \cdot 10^7$	70.7 Hz	35.4 Hz	23.6 Hz
M500	$5 \cdot 10^7$	79.1 Hz	39.6 Hz	26.4 Hz
M600	$6 \cdot 10^7$	86.6 Hz	43.3 Hz	28.9 Hz
M700	$7 \cdot 10^7$	93.5 Hz	46.8 Hz	31.2 Hz
M800	$8 \cdot 10^7$	100 Hz	50 Hz	33 Hz

Formula (19) allows us to state that for the longitudinal wave resonators we have resonances at frequencies corresponding to the half-wave resonator, since wavelength λ_w and the length of the resonator l are related by the relations:

$$\lambda_w^{(n)} = \frac{2\pi}{k_n} = \frac{2\pi}{(n \cdot \pi / l)} = \frac{2l}{n}, \quad f_n = \frac{n}{2l} \cdot v_w, \quad v_w = \sqrt{\frac{E}{\rho}}, \qquad (62)$$

where: v_w – propagation velocity of the longitudinal wave in the resonator. Knowing f_n with (62) (they correspond to the conditions of excitation of standing waves and "resonances" of the half-wave resonator), it is possible to create conditions under which effectively, at these frequencies ($n = 1, 2, 3, \ldots$), where an integer number of half-waves is inserted, will be absorbed by the resonator material the energy from wave formation, which propagates in it. Let us finally consider the solution for the case of a non-thin rod. First we define the effective radius r_{eff} of a rod resonator whose cross section is rectangular with sides: b – width, h – height:

$$\pi r_{eff}^2 = bh \Leftrightarrow r_{eff} = \sqrt{\frac{bh}{\pi}}. \tag{63}$$

Consider the case when the radius $r_{eff} > 0{,}1 \cdot \lambda_w^{(n)}$, that is:

$$r_{eff} > 0{,}1 \cdot \frac{2l}{n} = \frac{0{,}2l}{n}, \; n = 1, 2, 3, \ldots. \tag{64}$$

In this case, the rod cannot be considered thin and a correction factor must be introduced into the wave equation. Relay [2] and Maison [3] showed that the displacement along the axis of the rod is expressed in the form:

$$u_x = z(r) \cdot \sin(k_n x), \tag{65}$$

where $z(r)$ – distance function r from the axis of the rod. At resonant frequencies f_n constant dissemination $k_n = n\pi/l$, $n \in N$, and $\lambda_w^{(n)}$ is found from (62). Taking into account only the main correction, which leaves only the compression-expansion inertia in the direction perpendicular to the rod axis, the frequency equation takes the following form:

$$f_n = \frac{n}{2l} \cdot \sqrt{\frac{E}{\rho}} \cdot \left[1 - \left(\frac{n\mu\pi r_{eff}}{2l}\right)^2\right], \tag{66}$$

where: μ – Poisson's ratio. The frequency equation transforms into the thin rod equation (equation (19)) when $r_{eff} \to 0$. Meson showed that when the rod diameter is very large. d_{eff} ($d_{eff} = 2r_{eff} >> 0{,}1 \cdot \lambda_w^{(n)}$) the higher order terms should be included in equation (66). From the point of view of calculating the equivalent mass, the effect of transverse inertia (i.e., the effect of taking into account the kinetic energy associated with motion in the direction perpendicular to the axis of the rod) is to increase this mass with respect to the value determined by the relation (61). The reason for the increase is as follows: the kinetic energy associated with oscillations along the axis is slightly less than the total kinetic energy of the rod (the numerator of relation (60), which determines the equivalent mass). Accordingly, the ratio of the total energy to the term in the denominator, which consists of the square of the velocity in the direction of the axis, increases. On the contrary, the equivalent mass in the direction perpendicular to the axis decreases when the ratio of the diameter to the term in the denominator is (d_{eff}) to the length increases (assuming that the static mass remains constant).

Let us consider further the influence of boundary conditions on the formation of the spectrum of natural frequencies of the resonator.

A. Both ends of the rod ($x = 0;\ x = l$) free of stress:

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = \left.\frac{\partial u}{\partial x}\right|_{x=l} = 0 \Leftrightarrow f_n = \frac{n}{2l} \cdot v_w = \frac{n}{2l} \cdot \sqrt{\frac{E}{\rho}}, \quad n \in N. \tag{67}$$

B. Both ends of the rod ($x = 0;\ x = l$) are fixed:

$$\left.u\right|_{x=0} = \left.u\right|_{x=l} = 0 \Leftrightarrow f_n = \frac{n}{2l} \cdot v_w = \frac{n}{2l} \cdot \sqrt{\frac{E}{\rho}}, \quad n \in N. \tag{68}$$

C. One end of the rod is fixed and the other end is stress free:

$$u\big|_{x=0} = \frac{\partial u}{\partial x}\bigg|_{x=l} = 0 \Leftrightarrow f_n = \frac{(2n-1)}{4l}\cdot v_w = \frac{(2n-1)}{4l}\cdot\sqrt{\frac{E}{\rho}},\ \ n \in N. \tag{69}$$

Consequently, the first two types of boundary conditions (67), (68) lead to the frequency (eigenfrequency) spectrum of the half-wave resonator, and conditions (69) lead to the frequency (eigenfrequency) spectrum of the quarter-wave resonator. f_n, except for the geometric dimensions of the resonator (l, r_{eff}) is also determined by the density of the rod material (ρ) and its elastic properties (Young's modulus – E). Note that $M_{eqv0,l}$ remains the same regardless of the boundary conditions and is described by (61).

4. Resonance properties of linear viscoelastic bodies.

4.1. Maxwell's viscoelastic fluid.

Defining dependence $\sigma_x(\varepsilon_x)$ gives:

$$\dot{\varepsilon}_x = \frac{1}{E_G}\cdot\dot{\sigma}_x + \frac{1}{\eta_v}\cdot\sigma_x,\ \ \dot{\varepsilon}_x = d\varepsilon/dt,\ \ \dot{\sigma}_x = d\sigma_x/dt, \tag{70}$$

where: E_G – Young's modulus of elasticity, η_v – coefficient of (bulk) viscosity.

With $(\sigma_x, \varepsilon_x) \sim \exp(i\omega t)$, $i^2 = -1$, from (70) one obtains:

$$i\omega\varepsilon_x = \frac{1}{E_G}\cdot i\omega\sigma_x + \frac{1}{\eta_v}\cdot\sigma_x \Leftrightarrow \sigma_x = \frac{\left(\omega^2/E_G\right)\cdot\varepsilon_x}{(1/\eta_v)^2 + (\omega/E_G)^2} + \frac{i\cdot(\omega/\eta_v)\cdot\varepsilon_x}{(1/\eta_v)^2 + (\omega/E_G)^2}. \tag{71}$$

Let us introduce the complex modulus of elasticity $\tilde{E}$ by ratios:

$$\tilde{E} = E' + iE'';\ \ E' = \frac{\left(\omega^2/E_G\right)}{(1/\eta_v)^2 + (\omega/E_G)^2};\ \ E'' = \frac{(\omega\cdot\eta_v)}{(1/\eta_v)^2 + (\omega/E_G)^2}. \tag{72}$$

Physical meaning of the dynamic modulus of elasticity E' (which coincides in phase with the applied strain ε_x) is that it characterizes the elastic dynamic properties of a material that is in a harmonic deformation field.

Physical meaning of the dynamic loss modulus E'' (which lags in phase from the applied strain ε_x at an angle φ) is that the tangent of the angle of loss φ or dissipative factor (ratio of mechanical energy dissipated during one cycle to the stored energy) is determined by the alternation between E'' and E':

$$tg\varphi = E''/E' \Leftrightarrow \varphi = arctg\left(E''/E'\right). \tag{73}$$

Since $E'' = E''(\omega)$, then its maximum value determines the "resonance frequency" at which the maximum amount of mechanical energy is absorbed during 1 cycle of oscillations. We call this "resonance frequency" based on the rheological model adopted for the study. It may differ from the "resonance frequency", which is caused by boundary conditions and dimensions, density and the given material as a half-wave or quarter-wave resonator specified above (formulas (67)-(69)). Studies show that such "rheological resonance frequency" within the framework of Maxwell's model for the medium has the form:

$$\omega_{res}{*} = \sqrt{E_G/\eta_v}. \tag{74}$$

4.2. The Kelvin-Feugt elastically coherent body.

$$\sigma_x = E_G\cdot\varepsilon_x + \eta_v\cdot\dot{\varepsilon}_x;\ \ E' = E_G;\ \ E'' = \omega\cdot\eta_v. \tag{75}$$

From the last expression in (75) it can be seen that the "rheological resonance frequency" for this model lies at infinity, i.e:

$$\omega_{res}^{*} \to \infty. \tag{76}$$

4.3. Poynting-Thomson elastically coherent body (A.Yu. Ishlinsky).

$$\sigma_x + \dot{\sigma}_x \cdot \tau_{re} = E_G \cdot (\varepsilon_x + \dot{\varepsilon}_x \cdot \tau_{sl}), \tag{77}$$

where: $\tau_{re} = (\eta_v / E_M)$– time of relaxation, E_M – modulus of elasticity in the viscous piston branch; $\tau_{sl} = \frac{(E_M + E_G)}{E_M \cdot E_G} \cdot \eta_v$ – during sliding.

For this model we have:

$$E' = \frac{E_G \cdot (1 + \omega^2 \cdot \tau_{re} \cdot \tau_{sl})}{(1 + \omega^2 \cdot \tau_{re}^2)}; \quad E'' = \frac{E_G \cdot \omega \cdot (\tau_{sl} - \tau_{re})}{(1 + \omega^2 \cdot \tau_{re}^2)}. \tag{78}$$

$$\omega_{res}^* = 1/\tau_{re} = E_M / \eta_v. \tag{79}$$

4.4. Ziner's elastically coherent body [2].

$$\sigma_x + \dot{\sigma}_x \cdot \tilde{\tau}_{re} = \tilde{E}_G \cdot (\varepsilon_x + \dot{\varepsilon}_x \cdot \tilde{\tau}_{sl}); \quad \tilde{\tau}_{re} = \frac{\eta_v}{E_f}; \quad \tilde{\tau}_{sl} = \frac{(\tilde{E}_G + E_f)}{\tilde{E}_G \cdot E_f} \cdot \eta_v, \tag{80}$$

where: E_f – elastic modulus of the branch parallel to the viscous piston branch; $\tilde{E}_G$ – elastic modulus of the branch, which is connected in series with two parallel branches (viscous piston and elastic branch with E_f).

For this model we have:

$$E' = \frac{\tilde{E}_G \cdot (1 + \omega^2 \cdot \tilde{\tau}_{sl} \cdot \tilde{\tau}_{re})}{(1 + \omega^2 \cdot \tilde{\tau}_{re}^2)}; \quad E'' = \frac{\tilde{E}_G \cdot \omega \cdot (\tilde{\tau}_{sl} - \tilde{\tau}_{re})}{(1 + \omega^2 \cdot \tilde{\tau}_{re}^2)}. \tag{81}$$

$$\omega_{res}^* = 1/\tilde{\tau}_{re} = E_f / \eta_v. \tag{82}$$

4.5 Brankov elastic-viscous body [4].

This rheological model consists of three parallel branches (connected in parallel), to which a fourth branch with a purely elastic element having an elastic modulus is connected in series E_G. In the first of the parallel branches there are elasticities with modulus of elasticity E_M, which is connected in series with a viscous piston η'_v. The second of the parallel branches contains only a viscous piston η''_v. In the third of the parallel branches there is a purely elastic element with modulus of elasticity E_f .

The constitutive equation for the (rheological) Brankov model has the form [4]:

$$a_1\ddot{\sigma}_x + a_2\dot{\sigma}_x + a_3\sigma_x = b_1\ddot{\varepsilon}_x + b_2\dot{\varepsilon}_x + b_3\varepsilon_x, \tag{83}$$

where:

$$\ddot{\sigma}_x = d^2\sigma_x / dt^2; \quad \ddot{\varepsilon}_x = d^2\varepsilon_x / dt^2; \quad a_1 = \frac{\eta''_v}{E_G \cdot E_M}; \quad a_2 = \left(\frac{1}{E_G} + \frac{1}{E_M} + \frac{E_f}{E_G \cdot E_M} + \frac{\eta''_v}{E_G \cdot \eta'_v}\right);$$

$$a_3 = \left(\frac{1}{\eta'_v} + \frac{E_f}{E_G \cdot \eta'_v}\right); \quad b_1 = \frac{\eta''_v}{E_M}; \quad b_2 = \left(1 + \frac{E_f}{E_M} + \frac{\eta''_v}{\eta'_v}\right); \quad b_3 = \frac{E_f}{\eta'_v}.$$

The values of the real and imaginary parts of the elastic modulus in the Brankov model are follows:

$$E'_{(\omega)} = \frac{\left[\left(-\omega^2 b_1 + b_3\right) \cdot \left(-\omega^2 a_1 + a_3\right) + \omega^2 a_2 b_2\right]}{\left[\left(-a_1\omega^2 + a_3\right)^2 + a_2^2\omega^2\right]}; \tag{84}$$

$$E''_{(\omega)} = \frac{\left[\left(-a_1 b_2 + a_2 b_1\right)\omega^3 + \left(a_3 b_2 - a_2 b_3\right)\omega\right]}{\left[\left(-a_1\omega^2 + a_3\right)^2 + a_2^2\omega^2\right]}. \tag{85}$$

Since $E''(\omega) \to 0$ with $\omega \to 0$ and $\omega \to \infty$, then $E''(\omega)$ there exists an extremum (like a maximum), which determines the ω^*_{res} for this model. Finding (ω^*_{res}) we need to solve the equation:

$$d\{E''(\omega)\}/d\omega = 0. \tag{86}$$

It can be shown that equation (86) with respect to ω^2 of the third degree:

$$\tilde{A}z^3 + \tilde{B}z^2 + \tilde{C}z^1 + \tilde{D} = 0,\ z = \omega^2, \tag{87}$$

where: $\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}$ – coefficients, which are expressed in terms of $a_j,\ j = \overline{(1,3)},\ b_k,\ k = \overline{(1,3)}$. Equation (87) can be solved using Cardano's formula. In this case, one of the possible valid (positive) solutions of this equation (87) will be the one that corresponds to the case: the (cubic) equation has three roots, of which one is valid and two are complex-conjugate. To establish and determine this real root it is necessary to investigate this cubic equation according to standard methods. (This solution is not given here because it is rather cumbersome).

5. Resonance properties of linear viscoelastic bodies, which are described by defining dependencies $\sigma_x(\varepsilon_x)$ integrally.

5.1. Buergers model. As per [5-8] the rheological properties of asphalt concrete are well described by the so-called Bürgers model, which consists of two successively connected links: the Maxwell model and the Kelvin-Feugt model [2]. The differential equations relating stress and strain for the Bürgers model are of the form [8]:

$$\frac{\eta_2}{E_1} \cdot \dot{\sigma} + \left(1 + \frac{E_2}{E_1} + \frac{\eta_2}{\eta_1}\right) \cdot \sigma + \frac{E_2}{\eta_1} \cdot \int_t \sigma dt = \eta_2 \dot{\varepsilon} + E_2 \varepsilon. \tag{88}$$

If we differentiate by t (88), then we obtain:

$$\frac{\eta_2}{E_1} \cdot \ddot{\sigma} + \left(1 + \frac{E_2}{E_1} + \frac{\eta_2}{\eta_1}\right) \cdot \dot{\sigma} + \frac{E_2}{\eta_1} \cdot \sigma = \eta_2 \ddot{\varepsilon} + E_2 \dot{\varepsilon}, \tag{89}$$

where: (η_1, E_1) – coefficient of dynamic viscosity and modulus of elasticity of the link, which corresponds to the Maxwell model; (η_2, E_2) – dynamic viscosity coefficient and elastic modulus of the link, which corresponds to the Kelvin-Foigt model. This Bürgers model fits into the reasoning of Sect. 4.5 related to the elastic-viscous body of Brankov [4], and therefore it also has an extremum of the maximum type for the $E''(\omega)$ and sets ω^*_{res}.

5.2. Brankow's model of $\tau_{re} << \tau_{sl}$ $(\ddot{\sigma}_x << E_G \cdot \ddot{\varepsilon}_x)$.

Single integration over t of the rheological law describing the deformation processes of the Brankov body [4], provided that $\tau_{re} << \tau_{sl}$ (relaxation time/duration is much smaller than the time/duration of the creep process), allows us to present this model of body deformation as an integral one, and in differential form it will already look as follows:

$$a_2\dot{\sigma}_x + a_3\sigma_x = b_1\ddot{\varepsilon}_x + b_2\dot{\varepsilon}_x + b_3\varepsilon_x, \tag{90}$$

where all coefficients (90) a_2, a_3, b_1, b_2, b_3 introduced in 4.5 above.

Let's define for this model ω^*_{res}. According to [4], in order for the rheological model to make physical sense of the degree polynomials of the differential operators σ_x and ε_x must either coincide, or for ε_x be 1 more than for σ_x.

The analysis shows that for this case. $\omega^*_{res} \to \infty$.

Findings:

1. In this study, the conditions of occurrence and the main integral characteristics of resonance phenomena (of geometric and rheological types) that are possible in the processes of vibro-compaction and formation of concrete/construction mixtures, which are modeled by visco-elastic-plastic rods of finite length, are determined. The (classical) rheological models known

in the scientific literature, which are scientifically valid and widely used, are used.

2. The results obtained in the paper can be further used to refine and improve the existing engineering methods of calculation of vibration systems for compaction of concrete/construction mixtures both at the design stage and during their actual operation. In addition, such an approach in the technologies of formation and vibration compaction of concrete/construction mixtures will be useful in establishing the conditions of energy-saving functioning and modes of operation of such systems.

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ВИКОРИСТАННЯ МОДЕЛЕЙ МЕХАНІЧНИХ ФІЛЬТРІВ У АНАЛІЗІ ПРОЦЕСІВ ФОРМУВАННЯ ТА УЩІЛЬНЕННЯ БУДІВЕЛЬНИХ/БЕТОННИХ СУМІШЕЙ ВІБРАЦІЙНИМ ПОЛЕМ

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Анотація. Використані моделі механічних фільтрів різних типів, які застосовуються для аналізу процесів формування та ущільнення будівельних/бетонних сумішей за допомогою вібраційних полів. Встановлені значення резонансних частот та еквівалентних мас для різноманітних резонаторів, що моделюють розповсюдження у останніх вібраційно-хвильових утворень. У основу аналізу впливу вібраційного поля на процеси формування та ущільнення бетонних/будівельних сумішей у даному дослідженні покладені методи математичної фізики, класичного варіаційного числення, фізики коливань і хвиль та методологія розв’язку звичайних диференціальних рівнянь й диференціальних рівнянь у частинних похідних. Встановлені умови та основні інтегральні характеристики резонансних явищ, можливість виникнення котрих обумовлена: 1) геометрією поставленої початково-крайової задачі (це так звані «геометричні резонанси» розглядуваної системи з розподіленими параметрами, що моделює оброблювану суміш); 2) задіяною у дослідженні робочою реологічною моделлю суміші (це так звані «реологічні резонанси»). Розвинутий і науково обґрунтований у роботі підхід дозволяє встановити основні параметри і можливості використання енергоощадних режимів функціонування вібраційних систем, призначених для формування й вібраційного ущільнення вказаних вище сумішей. Отримані у роботі результати можуть бути у подальшому використані для уточнення й вдосконалення існуючих інженерних методів розрахунку вібраційних систем для формування й ущільнення бетонних/будівельних сумішей з метою оптимізації робочих режимів їх функціонування як на стадії проектування, так і у режимах реальної експлуатації.

Ключові слова: моделювання, механічні фільтри, резонатори коливань, аналіз, процеси формування, ущільнення, будівельні та бетонні суміші, вібраційне поле, резонанси, еквівалентні маси.

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