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IMPROVEMENT OF APPROACHES AND METHODS OF TURBULENT FLOW THEORY IN THE PIPES

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Abstract. The article consideres the analysis of the literature about the development of the water turbulent flow theory in the pipes. According to the results of analysis and theoretical studies, we obtained mathematical models. These models described the kinematic structure of the water turbulent flow in the pipes for different regions of turbulence.

For the first time, the hypothesis was accepted that the dependence obtained from the Navier-Stokes differential equation for constructing the velocity profile in the laminar regime is suitable for calculating the average velocities in the turbulent regime of flow, but for this, it is necessary to replace the molecular kinematic viscosity with the total turbulent kinematic viscosity, which includes kinematic viscosity on the inner surface of the pipe ν_s and turbulent kinematic viscosity ν_t, which occurs due to the movement of masses from one layer into another, as recommended in J.V. Boussinesq.

Based on experimental data I. Nikuradze and F.O. Shevelev, we obtained a distribution of the total kinematic viscosity in the pipes, including the kinematic viscosity on the pipe inner surface and the kinematic turbulent viscosity.

For the first time, we used the kinematic viscosity distribution equation in the pipes and obtained the averaged velocity profile equation. This equation corresponds to the boundary conditions on the pipe inner surface and on the axis of the pipe. The equation of maximum averaged velocity, the equation of distance from the axis of the pipe to the points having average velocity, the equation of the ratio of maximum velocity to average velocity was obtained.

For the first time, the equation of the tangent stresses components ($\vec{\tau}_{yx}$, $\vec{\tau}_{zx}$) and the tangent stresses equation in radial coordinates ($\vec{\tau}_{rx}$) were obtained. The equation of the maximum value of the tangent stresses located on the inner surface of the pipe was obtained. The tangent stresses assume a zero value on the pipe axis. The equation of the vortex components ($\vec{\omega}_y$, $\vec{\omega}_z$) was obtained. We have shown that vortex lines are concentric circles whose centers are located on the pipe axis. The equation of angular velocity of flow particles rotation relative to the vortex lines was obtained. The maximum value of the particle rotation angular velocity on the pipe inner surface is determined. It decreases monotonically to zero on the axis of the pipeline. It is zero on the pipe axis.

In this article, all equations reveal the kinematic structure of the water flow. We described these equations by the Reynolds number and the pipe friction factor. Such equations are adopted to show the dependencies between the regimes and the flow kinematic structure. These equations make it possible to calculate the distribution profile of the total kinematic viscosity, averaged velocity, tangential stresses and angular velocity of flow particle rotation.

Keywords: flow theory, laminar flow, laminar flow regime, flow kinematic structure, pipes.

Introduction. During the development of heat, hydropower and hydraulic engineering and water engineering, there is a growing need to increase the reliability and efficiency of structures, namely pipelines. Transportation of liquids in pipelines is widely used in engineering practice. The purpose of hydraulic calculations of pressure pipelines is the calculation of their hydraulic parameters. Accordingly, the hydraulic calculation of pipes mainly solves three problems: the pressure loss in the pipe, the flow of fluid during its transportation, and finding the diameter of the pipe.

Pipes are part of facilities with a high class of consequences (responsibility) and must provide the reliability of operation in different operating conditions. Therefore, there is a need to improve the approach to the hydraulic calculation of pressure pipes with different purposes, namely the development of the theory of the kinematic structure of the flow in them [1, 2].

Analysis of recent research. In the areas of hydraulic flow on the Nikuradze graph, the pipe friction number depends on the Reynolds number and the inner surface of the pipe. All equations are recommended only for certain conditions (the type of roughness of the inner surface, material, height of roughness, and the distance between them, their shape, and location on the surface). An adequate equation for determining the pipe friction number from all main factors can be established only experimentally [1, 3-6].

For example, the value of the pipe friction number for the smooth pipes of the turbulent flow can be calculated by the dependence of I. Nikuradze, which is obtained on the basis of experimental data [7-9]:

$$\frac{1}{\sqrt{\lambda}} = 2\,lg\left(Re\sqrt{\lambda}\right) - 0{,}8 . \tag{1}$$

For round pipes from the region of hydraulically smooth turbulence to quadratic resistance region inclusive I. Nikuradze recommends the following dependence on the basis of the experimental researches:

$$\frac{1}{\sqrt{\lambda}} = 2\,lg\,\frac{R}{K_s} + 1{,}74 , \tag{2}$$

where K_s – equivalent roughness height, m; R – hydraulic radius, m.

I.K. Nikitin recommended formula for turbulent flow from the region of hydraulically smooth turbulence to quadratic resistance region [10, 11]:

$$\frac{1}{\sqrt{\lambda}} = Re_{*\delta\infty}\left(0{,}813\,lg\,\frac{h}{\varDelta} + 0{,}706\right), \tag{3}$$

where $Re_{*\delta\infty}$ – the coefficient of proportionality of the thickness of the laminar layer; $\varDelta$ – absolute height of roughness, m; h – flow depth, m.

It should be noted that the graph of the dependence of the change of pipe friction number on the Reynolds number reveals only the regimes, but does not reveal the kinematic structure of the flow, which must be taken into account in hydraulic calculations of pipes [1].

One of the first scientists of the 19th century to develop the theoretical foundations of the flow structure was S. Navier, who introduced additional terms into L. Euler's differential equation to take into account tangential stresses that occur in the presence of a velocity gradient.

J. Stokes obtained the solution of the differential equation of S. Navier and his theoretical results completely coincided with the data of experiments conducted in pipes of small diameter at low speeds (laminar flow). After closing the Navier Stokes differential equation, J. Stokes took into account the continuity equation and accepted the boundary conditions. As a result, the equation of distribution of the averaged velocity of the fluid for the laminar flow was obtained [1, 2]:

$$u_{xl} = \lambda\,Re^2\,\frac{\nu}{8d^3}\left(r_0^2 - r^2\right). \tag{4}$$

It should be noted that for turbulent flow (large pipes diameters and large Reynolds numbers $Re > 2320$) a significant deviation of the theoretically determined initial parameters according to the above dependences on the results of experimental studies was revealed.

Many scientists have worked on the solution of this problem: M.A. Velikanov, J. Taylor, V.M. Makaveev, A.M. Kolmogorov, O.M. Obukhov, T. Karman, L.G. Лойцянський, O.O. Friedman, G. Reichardt, D. Rotta, R. Deisler, G.W. Zheleznyakov, S. Kullupailo, A.D. Altshul, W.W. Smyslov, P.G. Kiselyov, L. Prandtl, V. Tolmin, V.F. Durenda, H.L. Dryden, G. Schlichting, I.K. Nikitin, and others. [2-8]. They proposed semi-empirical theories of turbulent flows, in which the unknown connections between turbulent stresses and averaged strain velocities are specified on the basis of hypotheses, qualitative physical conjectures, and dimensional theories obtained by generalizing experimental materials.

Setting objectives. The obtained semi-empirical theories cannot be universal because they are limited by the range of conditions under which assumptions are accepted.

Models were proposed for power-law profiles and logarithmic profiles. The logarithmic profile has become widespread, despite the fact that this dependence does not correspond to the boundary conditions on the axis and on the inner surface of the pipe. In order to ensure the boundary conditions on the inner surface of the pipe, scientists have developed two-layer and three-layer models. But along the axis of the pipe, they do not meet the limit conditions.

Due to the fact that the equations of distribution of averaged velocities in the pipe are given do not correspond to the boundary conditions at the pipe wall (power equations), and the proposed logarithmic equations do not correspond to the limit conditions not only at the wall but also on the pipeline axis, the following solution to this problem is proposed.

Research methodology. The accepted hypothesis is that dependence (4) obtained from the Navier-Stokes differential equation is suitable for calculating averaged velocities also in the turbulent regime of flow, but for this, it is necessary to take into account the total turbulent kinematic viscosity, which includes kinematic, in the Navier-Stokes equation instead of molecular kinematic viscosity on the inner surface of the pipe and turbulent kinematic viscosity, which occurs due to the movement of masses from one layer to another, as recommended by J.V. Boussinesq. Then equation (4) takes the form:

$$u_{xt} = \lambda\, Re^2 \frac{\nu^2}{64 \nu_{tot}\, r_0^3}\left(r_0^2 - r^2\right). \tag{5}$$

where ν_{tot} – total turbulent kinematic flow viscosity, which takes into account the molecular kinematic viscosity on the inner surface of the pipe ν_s and the turbulent kinematic viscosity between the flow layers ν_t .

The movement of liquid molecules on the inner surface of the pipe is limited, so the kinematic viscosity on the inner surface of the pipe ν_s is less than the molecular viscosity of the liquid ν . As shown by statistical studies, we can assume that it depends on the coefficient of hydraulic resistance and is determined by:

$$\nu_s = \lambda \nu \,. \tag{6}$$

The turbulent kinematic viscosity between the flow layers, as shown by statistical studies, is also not a constant physical quantity, and requires the necessary studies.

To obtain a graph of the dependence of the relative total turbulent kinematic viscosity along the radius based on the experimental data, equations (5) are reduced to:

$$\frac{\nu_{tot}}{\nu} = \frac{\nu \lambda\, Re^2}{64 u_{xt} r_0^3}\left(r_0^2 - r^2\right). \tag{7}$$

Figure 1 shows a graph of the relative total turbulent kinematic viscosity along the radius.

The graph shows that the relative turbulent kinematic viscosity $\frac{\nu_t}{\nu}$ takes its maximum value on the axis of the pipeline, and the minimum zero value at $r = r_0$. At intermediate points, the graph has a shape similar to an ellipse.

Therefore, it is advisable, taking into account the canonical equation of the ellipse, to accept the hypothesis that the sum of the relative turbulent kinematic viscosity at the flow point and the relative distance to this point in the corresponding order is equal to unity:

$$\left(\frac{\nu_t}{\nu_{t\,max}}\right)^{1/m} + \left(\frac{r}{r_0}\right)^n = 1\,. \tag{8}$$

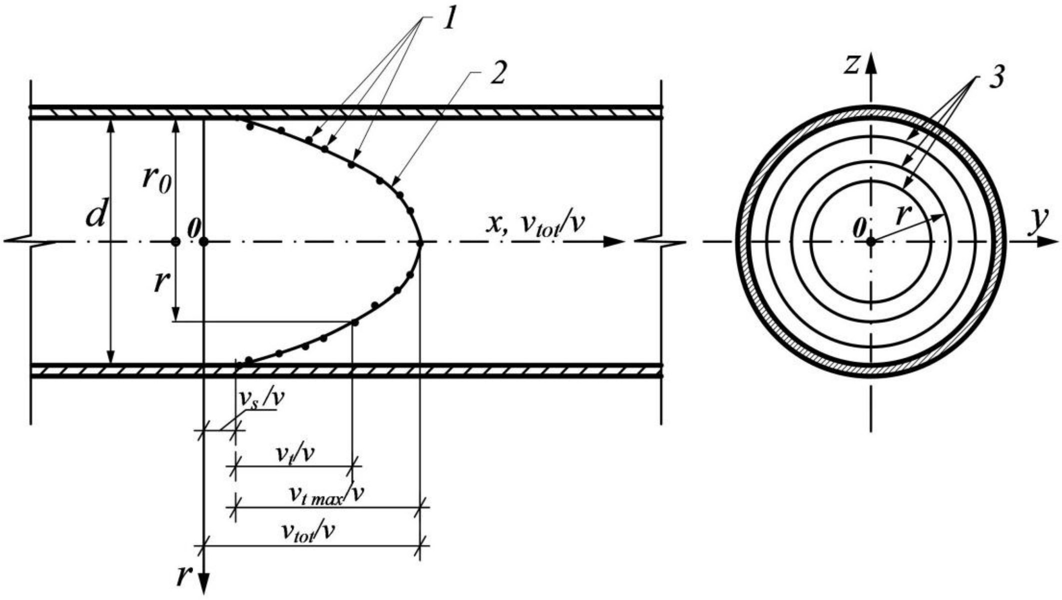

Fig. 1. Distribution of relative total turbulent kinematic viscosity in the pipe:
1 – experimental points of relative total turbulent kinematic viscosity obtained by the equation (7);
2 – the profile of the relative total turbulent kinematic viscosity is obtained by the equation (13);
3 – circular surfaces with the same total turbulent kinematic viscosity

Given that the flow interacts with the inner surface of the pipe, it is necessary to take into account the kinematic viscosity on the inner surface of the pipe $\nu_s = \lambda\nu$ in equation (8).

Then equation (8) is reduced to the form:

$$\left(\frac{\nu_{tot}}{\nu_{t\,max}}\right)^{1/m} + \left(\frac{r}{r_0}\right)^{n} = 1 + \left(\frac{\nu_s}{\nu_{t\,max}}\right)^{1/m}. \tag{9}$$

The equation of total turbulent kinematic viscosity will take the form:

$$\nu_{tot} = \frac{1}{r_0^{nm}}\left(\nu_{t\,max}^{1/m}\left(r_0^n - r^n\right) + \nu_s^{1/m} r_0^n\right)^m, \tag{10}$$

where unknown parameters are determined by a system of equations:

$$\left.\begin{array}{l} \nu_{t\,max} = k\nu\lambda\,Re \\ k = a\,lg\,Re\,lg(100\lambda) + b \\ m = c\,lg\,Re\,lg(100\lambda) + d \\ n = 2/m \\ \nu_s = \lambda\nu \end{array}\right\}, \tag{11}$$

where $\nu_{t\,max}$ – the maximum value of turbulent kinematic viscosity that occurs on the axis of the pipe; k, m i n – constant parameters for a certain flow regime; $(lg\,Re;\,lg(100\lambda))$ – coordinates on the Nikuradze graph, which take into account the areas of hydraulic resistance; a, b, c, d – constant coefficients, which are determined on the basis of experimental data.

If we take into account in equation (11) equation (10), then we obtain:

$$\nu_{tot} = \frac{\nu\lambda}{r_0^{nm}}\left((k\,Re)^{1/m}\left(r_0^n - r^n\right) + r_0^n\right)^m. \tag{12}$$

The dependence for determining the relative total turbulent kinematic viscosity will take the form:

$$\frac{\nu_{tot}}{\nu} = \frac{\lambda}{r_0^{nm}}\left((k\,Re)^{1/m}\left(r_0^n - r^n\right) + r_0^n\right)^m. \tag{13}$$

From equation (12) it follows that on the axis of the pipeline at $r = 0$ we have the maximum value of the total turbulent kinematic viscosity in the pipe:

$$\nu_{tot\ max} = \nu\lambda\left(\left(k\ Re\right)^{1/m} + 1\right)^m, \tag{14}$$

and at $r = r_0$ we have the minimum value of the total turbulent kinematic viscosity in the pipe, which arises due to the velocity gradient on the inner surface of the pipe:

$$\nu_{tot\,min} = \nu_s = \lambda\nu\ . \tag{15}$$

1. If we take into account the equation of the total turbulent kinematic viscosity (12), the dependence of the distribution of the averaged velocity of the turbulent flow (5) will be:

$$u_{xt} = \frac{\nu\, Re^2\left(r_0^2 - r^2\right)}{64\left(\left(k\ Re\right)^{1/m}\left(r_0^n - r^n\right) + r_0^n\right)^m r_0}\ . \tag{16}$$

Figure 2 shows the profiles of the average velocities of the fluid for laminar flow, for the region of hydraulically smooth turbulence, almost quadratic resistance region, and quadratic resistance region of turbulent flow.

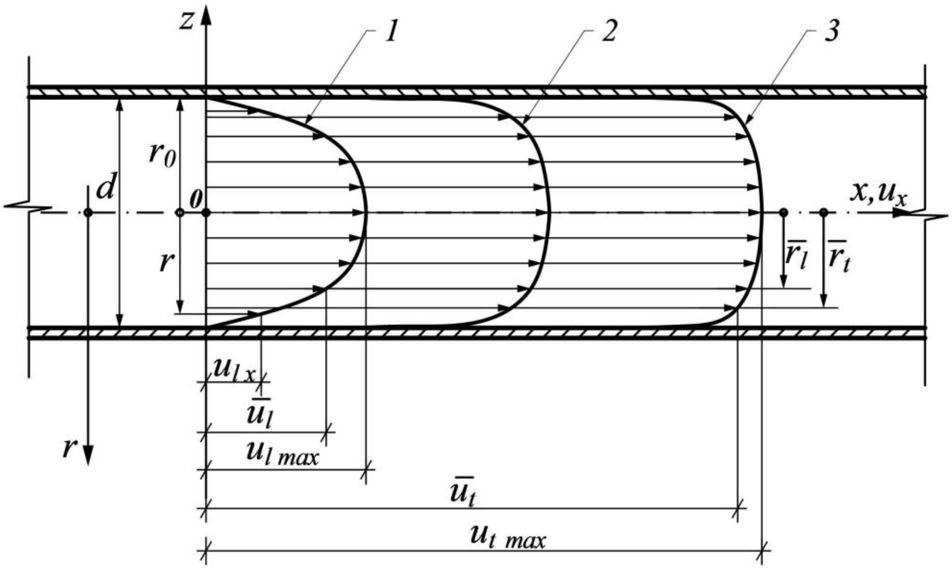

Fig. 2. Distribution of the averaged velocity of the flow:
1 – laminar flow; 2 – the region of hydraulically smooth turbulence;
3 – almost quadratic resistance region and quadratic resistance region of turbulent flow

2. The maximum averaged flow velocity is determined from equation (16) at $r = 0$.

$$u_{xt\,max} = \frac{\nu\, Re^2}{64\, r_0\left(\left(k\, Re\right)^{1/m} + 1\right)^m}, \tag{17}$$

and on the inner surface of the pipe - at $r = r_0$, then $u_{xt} = 0$.

The maximum averaged velocity of the laminar flow is:

$$u_{xl\ max} = \lambda\, Re^2\, \frac{\nu}{8d^3}\, r_0^2\ .$$

3. The relative distance from the axis of the pipe to the points having the averaged velocity $\bar{r}$ is expressed by the equation in implicit form:

$$\frac{Re\left(r_0^2 - \bar{r}^2\right)}{32\left(\left(k\ Re\right)^{1/m}\left(r_0^n - \bar{r}^n\right) + r_0^n\right)^m} = 1\ . \tag{18}$$

It is not constant and depends on the pipe friction factor and the Reynolds number. For laminar flow, the distance from the axis of the pipe to the points having an average velocity is:

$$\bar{r}_l = \frac{r_0}{\sqrt{2}}.$$

4. The ratio of the maximum flow velocity to the average velocity in the pipe is:

$$\frac{u_{xt\,max}}{\bar{u}_x} = \frac{\left((k\,Re)^{1/m}\left(r_0^n - \bar{r}^n\right) + r_0^n\right)^m}{\left((k\,Re)^{1/m}+1\right)^m \left(r_0^2 - \bar{r}^2\right)}. \quad (19)$$

For laminar flow, the ratio of the maximum flow rate to the average velocity in the pipe is:

$$\frac{u_{xl\,max}}{\bar{u}_{xl}} = 2.$$

5. An important parameter that characterizes the kinematic structure of the flow is the tangential stresses. Using the equation proposed by J.V. Bossinessq, without theoretical justification for the laminar regime by analogy with Newton's law, can be written:

$$\tau_{yx} = \rho \nu_{tot} \frac{du_x}{dy}, \quad (20)$$

$$\tau_{zx} = \rho \nu_{tot} \frac{du_x}{dz} \quad (21)$$

$$\tau_{rx} = \rho \nu_{tot} \frac{du_x}{dr}, \quad (22)$$

where $\vec{\tau}_{yx}$ – the tangential stresses on the line, which is normal to axis $0Y$ and parallel to axis $0X$; $\vec{\tau}_{zx}$ – the tangential stresses on the line, which is normal to axis $0Z$ and parallel to axis $0X$; $\vec{\tau}_r$ – the tangential stresses on a circular surface of radius r regardless of the azimuth component and parallel to the axis $0X$.

The stress on a circular surface of the radius r (Fig. 3) is independent of the azimuth component and the parallel to axis $0X$ has the form:

$$\tau_{rx} = \rho \frac{\lambda}{r_0^{nm}} \frac{\nu^2\,Re^2}{32\,r_0} \frac{(k\,Re)^{1/m}\left(\left(r_0^2 - r^2\right)\left(r^{n-1}\right) - r\left(r_0^n - r^n\right)\right) - r r_0^n}{\left((k\,Re)^{1/m}\left(r_0^n - r^n\right) + r_0^n\right)}, \quad (23)$$

The maximum value of tangential stresses in turbulent regime is obtained from equation (23) at $r = r_0$:

$$\tau_0 = \rho \,\mathrm{Re}^2 \frac{\lambda \nu^2}{32 r_0^2}, \quad (24)$$

and this equation has the same form as for the laminar flow:

$$\tau_0 = \rho \frac{2\,\mathrm{Re}\,\nu^2}{r_0^2} = \rho\,\mathrm{Re}^2 \frac{\lambda \nu^2}{32 r_0^2}.$$

The tangential stresses on the axis of the pipe, if $r = 0$ for turbulent and laminar regimes is zero.

The equation of a circular surface with tangential stress $\vec{\tau}_{rx}$ has the form:

$$y^2 + z^2 = r^2. \quad (25)$$

Thus, the surfaces with tangential stress $\vec{\tau}_{rx}$ are concentric surfaces with radius r, the centers of which are located on the axis of the pipe.

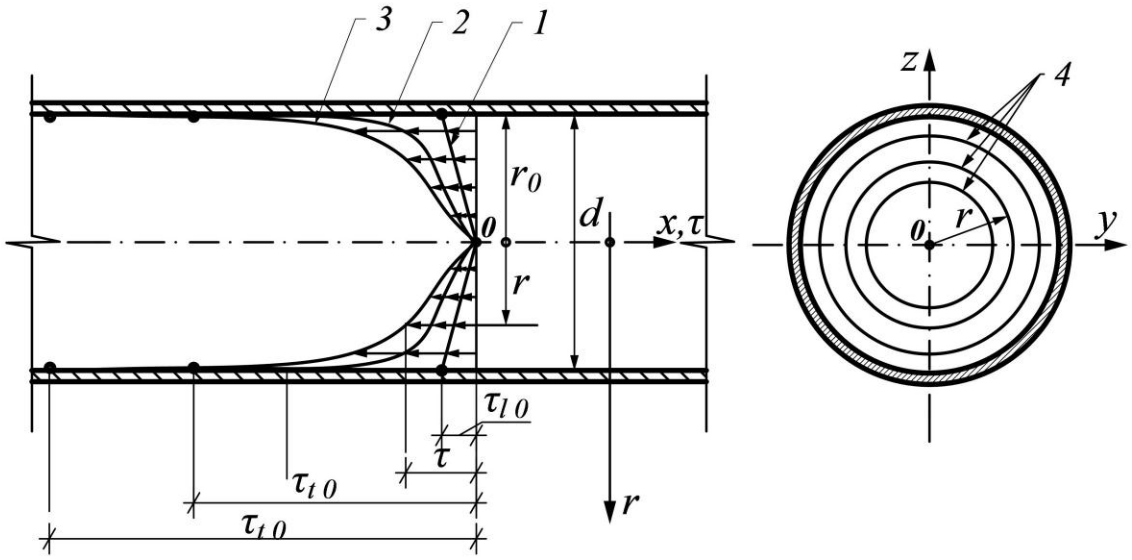

Fig. 3. Distribution of tangential stresses:
1 – laminar flow; 2 – the region of hydraulically smooth turbulence; 3 – almost quadratic resistance region and quadratic resistance region of turbulent flow; 4 – circular surfaces with the same tangential stresses

7. The value of the components of vortices (Fig. 4):

$$\left.\begin{aligned}
&\omega_x = \frac{1}{2}\left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right) = 0 \\
&\omega_y = \frac{1}{2}\left(\frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x}\right) = \frac{1}{2}\frac{\nu\,\mathrm{Re}^2}{32 r_0} z \frac{(k\,\mathrm{Re})^{1/m}\left(\left(r_0^2-\left(y^2+z^2\right)\right)\left(\left(y^2+z^2\right)^{n-2/2}\right)-\left(r_0^n-\left(y^2+z^2\right)^{n/2}\right)\right)-r_0^n}{\left((k\,\mathrm{Re})^{1/m}\left(r_0^n-\left(y^2+z^2\right)^{n/2}\right)+r_0^n\right)^{m+1}} \\
&\omega_z = \frac{1}{2}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right) = -\frac{1}{2}\frac{\nu\,\mathrm{Re}^2}{32 r_0} y \frac{(k\,\mathrm{Re})^{1/m}\left(\left(r_0^2-\left(y^2+z^2\right)\right)\left(\left(y^2+z^2\right)^{n-2/2}\right)-\left(r_0^n-\left(y^2+z^2\right)^{n/2}\right)\right)-r_0^n}{\left((k\,\mathrm{Re})^{1/m}\left(r_0^n-\left(y^2+z^2\right)^{n/2}\right)+r_0^n\right)^{m+1}}
\end{aligned}\right\} \quad (26)$$

Vortex lines are concentric circles with radius r, the centers of which are located on the axis of the pipe:

$$y^2 + z^2 = r^2. \quad (27)$$

The angular velocity of rotation of liquid particles of the flow relative to the vortex lines is determined by the equation:

$$\omega = \sqrt{\omega_x{}^2 + \omega_y{}^2 + \omega_z{}^2}, \quad (28)$$

$$\omega = \frac{\nu\,Re^2}{64 r_0} \frac{r r_0^n - (k\,Re)^{1/m}\left(\left(r_0^2-r^2\right)\left(r^{n-1}\right)-r\left(r_0^n-r^n\right)\right)}{\left((k\,Re)^{1/m}\left(r_0^n-r^n\right)+r_0^n\right)^{m+1}}. \quad (29)$$

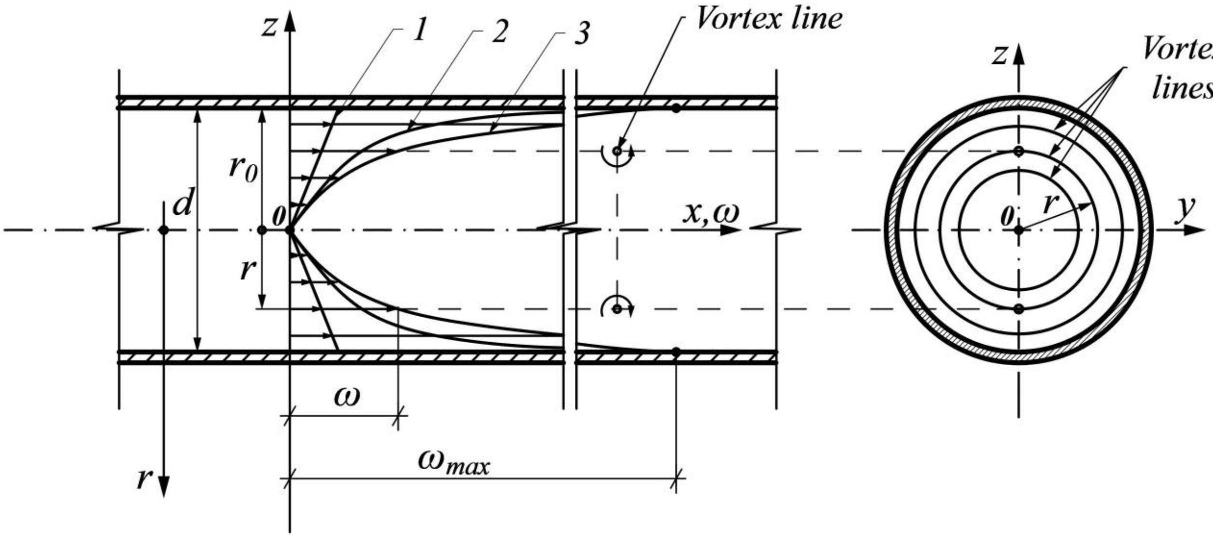

Fig. 4. Distribution of the angular velocity of rotation of liquid particles
in the cross section of the pipe:
1 – laminar flow; 2 – the region of hydraulically smooth turbulence; 3 – almost quadratic resistance region and quadratic resistance region of turbulent flow

The value of the angular velocity of rotation of the particles on the inner surface of the pipe will have the maximum value:

$$\omega_{max} = Re^2 \frac{\nu}{64 r_0^2}, \tag{30}$$

and the value of the angular velocity of rotation of the particles on the axis has zero value.

The angular velocity of rotation of the particles of the laminar flow on the surface of the pipe is:

$$\omega_{max} = \lambda\, Re^2 \frac{\nu}{64 r_0^2} = Re \frac{\nu}{r_0^2} .$$

Conclusions and prospects for further research. For the first time, the kinematic structure of the flow in a turbulent regime is revealed. The hypothesis was accepted that the dependence obtained from the Navier-Stokes differential equation is suitable for calculating the averaged velocities for a turbulent flow, but for this, it is necessary to take into account the total kinematic viscosity in Navier-Stokes equation, which includes kinematic viscosity on the inner surface, instead of molecular kinematic viscosity pipe ν_s and kinematic turbulent viscosity ν_t, arising due to the movement of masses from one layer to another, as recommended by J.V. Boussinesq.

For the first time on the basis of experimental data I. Nikuradze and F.O. Shevelyov obtained a distribution profile of the total kinematic viscosity in the pipe, which includes the kinematic viscosity on the inner surface of the pipe and the kinematic turbulent viscosity between the flow layers.

For the first time, taking into account the equation of distribution of the total kinematic viscosity in the pipe, the equation of the distribution profile of the averaged velocity was obtained, which corresponds to the boundary conditions on the inner surface and on the axis of the pipe. The equations of the maximum averaged flow velocity, the distance from the pipe axis to the points having the average velocity, and the equation of the ratio of the maximum velocity to the average are obtained.

For the first time, the components of tangential stresses $\vec{\tau}_{yx}$ and $\vec{\tau}_{zx}$, and tangential stresses in radial coordinates $\vec{\tau}_{rx}$ were obtained. An equation to determine the maximum value of tangential stresses located on the inner surface of the pipe was obtained. The tangential stresses monotonically decreasing take a zero value on the axis of the pipe. For the first time, the equations of the components of vortices $\vec{\omega}_y$ and $\vec{\omega}_z$ are obtained. It is proved that vortex lines are concentric circles, the centers of which are located on the axis of the pipe. The obtained equation of the angular velocity of rotation of liquid particles of the flow relative to the vortex lines. The maximum value of the angular velocity of the rotation of particles on the surface of the pipe was determined. The angular velocity of rotation of the particles decreases monotonically and takes zero value on the axis of the pipe.

In this paper, all the dependences that describe the kinematic structure are represented by the Reynolds number and the pipe friction number. This form of formula is adopted in order to show the relationship between the flow regime of movement and the kinematic structure of the flow. Thus, having a graph of the dependence of the pipe friction number on the Reynolds number (which can be obtained only experimentally), the proposed theory makes it possible for any point of the Nikuradze graph to reveal the kinematic structure of the flow in the pipe. Namely, to construct a distribution profile of total kinematic viscosity, averaged velocity, tangential stresses, and angular velocity of rotation of liquid particles.

Future papers will describe linear and angular deformations of liquid flow particles according to the Cauchy-Helmholtz theorem. The adequacy of the proposed equations, which express the kinematic structure of the flow depending on the turbulent flow regime, is proved on the basis of experimental studies by I. Nikuradze and F.O. Shevelyova and will be given in future papers.

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УДОСКОНАЛЕННЯ ПІДХОДІВ ТА МЕТОДІВ ТЕОРІЇ РУХУ ПОТОКУ В ТРУБОПРОВОДАХ ПРИ ТУРБУЛЕНТНОМУ РЕЖИМІ

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Анотація. У статті наведено аналіз літературних джерел щодо розвитку теорії руху потоку в трубопроводах при турбулентному режимі. За узагальненими результатами аналізу та за допомогою проведених теоретичних досліджень отримані математичні моделі, які розкривають структуру потоку в залежності від області гідравлічного опору при турбулентному режимі руху потоку в трубопроводах.

Вперше було прийнято гіпотезу, що отримана з диференціального рівняння Нав'є-Стокса, залежність придатна для розрахунку усереднених швидкостей і для турбулентного режиму руху рідини, але для цього необхідно враховувати в рівнянні Нав'є-Стокса замість молекулярної кінематичної в'язкості загальну турбулентну кінематичну в'язкість, яка включає кінематичну в'язкістю на внутрішній поверхні трубопроводу ν_s і кінематичну турбулентну в'язкість ν_t, яка виникає за рахунок переміщення мас з одного шару в інший, як рекомендував Ж.В. Буссінеска.

На основі експериментальних даних І. Нікурадзе і Ф.А. Шевельова було отримано профіль розподілу загальної турбулентної кінематичної в'язкості в трубопроводі, яка включає кінематичну в'язкістю на внутрішній поверхні трубопроводу і кінематичну турбулентну в'язкість між шарами рідини.

Вперше, з огляду на рівняння розподілу загальної кінематичної в'язкості в трубопроводі, було отримано рівняння профілю осередненої швидкості, яке відповідає граничним умовам на внутрішній поверхні і на осі трубопроводу. Отримано рівняння максимальної осередненої швидкості, відстані від осі трубопроводу до точок, що мають середню швидкість, відношення максимальної швидкості до середньої швидкості.

Вперше були отримані компоненти дотичних напружень $\vec{\tau}_{yx}$ і $\vec{\tau}_{zx}$, а також дотичні напруження в радіальних координатах $\vec{\tau}_{rx}$. Отримано рівняння для визначення максимального значення дотичних напружень, які розташовуються на внутрішній поверхні трубопроводу. Дотичні напруження, монотонно зменшуючись, приймають нульове значення на осі трубопроводу. Отримано рівняння компонентів вихорів $\vec{\omega}_y$ і $\vec{\omega}_z$. Доведено, що вихрові лінії є концентричними колами, центри яких знаходяться на осі труби. Отримане рівняння кутової швидкості обертання частинок рідини потоку щодо вихрових ліній. Визначено максимальну величину кутової швидкості обертання частинок на стінці трубопроводу, яка монотонно зменшуючись, приймає нульове значення на осі трубопроводу.

У даній роботі всі залежності, що розкривають структуру потоку, виражені через число Рейнольдса і коефіцієнт гідравлічного опору. Такий запис формул прийнято з метою, щоб показати зв'язок між режимом руху і структурою потоку.

Таким чином, маючи графік залежності зміни коефіцієнта гідравлічного опору від числа Рейнольдса (який можна отримати тільки досвідченим шляхом) запропонована теорія дає можливість для будь-якої точки графіка Нікурадзе розкрити кінематичну структуру потоку в трубопроводі. А саме: побудувати профіль розподілу повної турбулентної кінематичної в'язкості, осередненої швидкості, дотичних напружень і кутової швидкості обертання частинок рідини.

Ключові слова: теорія руху потоку, турбулентний режим, кінематична структура потоку, загальна турбулентна кінематична в'язкість, трубопроводи.

СОВЕРШЕНСТВОВАНИЕ ПОДХОДОВ И МЕТОДОВ ТЕОРИИ ДВИЖЕНИЯ ПОТОКА В ТРУБОПРОВОДАХ ПРИ ТУРБУЛЕНТНОМ РЕЖИМЕ

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Аннотация. В статье приведен анализ литературных источников по развитию теории движения потока в трубопроводах при турбулентном режиме. По обобщенным результатам анализа и с помощью проведенных теоретических исследований получены математические модели, которые раскрывают структуру потока в зависимости от области гидравлического сопротивления при турбулентном режиме движения потока в трубопроводах.

Впервые было принято гипотезу, что полученная из дифференциального уравнения Навье-Стокса, зависимость пригодна для расчета осредненных скоростей и для турбулентного режима движения жидкости, но для этого необходимо учитывать в уравнении Навье-Стокса вместо молекулярной кинематической вязкости общую турбулентную кинематическую вязкость, которая включает кинематическую вязкость на внутренней поверхности трубопровода ν_s и кинематическую турбулентную вязкость ν_t, которая возникает за счет перемещения масс из одного слоя в другой, как рекомендовал Ж.В. Буссинеск.

На основе экспериментальных данных И. Никурадзе и Ф.А. Шевелева было получено профиль распределения общей турбулентной кинематической вязкости в трубопроводе, которая включает кинематическую вязкостью на внутренней поверхности трубопровода и кинематическую турбулентную вязкость между слоями жидкости.

Впервые, учитывая уравнение распределения общей кинематической вязкости в трубопроводе, было получено уравнение профиля осредненной скорости, которое соответствует граничным условиям на внутренней поверхности и на оси трубопровода. Получено уравнение максимальной осредненной скорости, расстояния от оси трубопровода до точек, имеющих среднюю скорость, отношение максимальной скорости к средней скорости.

Впервые были получены компоненты касательных напряжений $\vec{\tau}_{yx}$ и $\vec{\tau}_{zx}$, а также касательные напряжения в радиальных координатах $\vec{\tau}_{rx}$. Получено уравнение для определения максимального значения касательных напряжений, которые располагаются на внутренней поверхности трубопровода. Касательные напряжения, монотонно уменьшаясь, принимают нулевое значение на оси трубопровода. Получены уравнения компонентов вихрей $\vec{\omega}_y$ и $\vec{\omega}_z$. Доказано, что вихревые линии являются концентрическими окружностями, центры которых находятся на оси трубы. Полученное уравнение угловой скорости вращения частиц жидкости потока относительно вихревых линий. Определена максимальная величина угловой скорости вращения частиц на стенке трубопровода, которая монотонно уменьшаясь, принимает нулевое значение на оси трубопровода.

В данной работе все зависимости, раскрывающие структуру потока, выражены через число Рейнольдса и коэффициент гидравлического сопротивления. Такая запись формул принята с целью, чтобы показать связь между режимом движения и структурой потока.

Таким образом, имея график зависимости изменения коэффициента гидравлического сопротивления от числа Рейнольдса (который можно получить только опытным путем) предложенная теория дает возможность для любой точки графика Никурадзе раскрыть кинематическую структуру потока в трубопроводе. А именно: построить профиль распределения полной турбулентной кинематической вязкости, осредненной скорости, касательных напряжений и угловой скорости вращения частиц жидкости.

Ключевые слова: теория движения потока, турбулентный режим, кинематическая структура потока, общая турбулентная кинематическая вязкость, трубопроводы.

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