Neighborhoods of certain p -valent analytic functions defined by using generalized differential operators

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Abstract

In this paper, by use of the familiar concept of neighborhoods of p -valently analytic functions, the authors show several inclusion relations associated with the (n, τ) -neighborhood of certain subclasses of analytic functions.

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1. Introduction and preliminaries

Let $\mathcal{A}(p)$ denote the class of functions f of the form:

$$f(z) = z^p + \sum_{n=p+1}^{\infty} a_n z^n \quad (z \in \mathbb{U}, p, n \in \mathbb{N} = \{1, 2, 3, \dots\}) \quad (1.1)$$

which are analytic and univalent in the open unit disc $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$.

The function $f(z)$ in $\mathcal{A}(p)$ is said to be p -valent starlike functions of order α and univalent convex of order α in $\mathbb{U}$ if it satisfies:

$$\operatorname{Re} \left\{ \frac{zf'(z)}{f(z)} \right\} > \alpha \quad (p \in \mathbb{N}, z \in \mathbb{U}, 0 \leq \alpha < p) \quad (1.2)$$

and

$$\operatorname{Re} \left\{ 1 + \frac{zf''(z)}{f'(z)} \right\} > \alpha \quad (p \in \mathbb{N}, z \in \mathbb{U}, 0 \leq \alpha < p) \quad (1.3)$$

respectively.

For the function $f \in \mathcal{A}(p)$, we define the following new generalized differential operator as

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follows:

$$\begin{aligned} A_{\mu,\lambda,p}^0(\alpha, \beta, \delta)f(z) &= f(z), \\ A_{\mu,\lambda,p}^1(\alpha, \beta, \delta)f(z) &= \left(\frac{\mu + p(1 - \beta(\lambda - \alpha))}{\mu + \lambda} \right) f(z) + \left(\frac{\beta(\lambda - \alpha) + (1 - p)\delta}{\mu + \lambda} \right) zf'(z) \\ &\quad + \frac{\delta}{\mu + \lambda} zf''(z), \end{aligned}$$

and for $m = 1, 2, 3, \dots$

generalises various operators as follows.

$$A_{\mu,\lambda,p}^m(\alpha, \beta, \delta)f(z) = A_{\mu,\lambda,p}^m(\alpha, \beta, \delta)(A_{\mu,\lambda,p}^{m-1}(\alpha, \beta, \delta)f(z)). \quad (1.4)$$

$$= z^p + \sum_{n=p+1}^{\infty} \left[1 + \frac{(n-p)[(\lambda - \alpha)\beta + n\delta]}{\mu + \lambda} \right]^m a_n z^n \quad (1.5)$$

for $f \in \mathcal{A}(p)$, $\alpha, \delta, \mu \geq 0$, $\beta, \lambda > 0$, $\lambda \neq \alpha$, $p \in \mathbb{N}$, $p \neq n$ and $m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$. This generalises various operators as follows.

(i) $A_{\mu,\lambda,1}^m(\alpha, \beta, \delta)f(z) = z + \sum_{n=2}^{\infty} \left[1 + \frac{(n-1)[(\lambda - \alpha)\beta + n\delta]}{\mu + \lambda} \right]^m a_n z^n$, the operator introduced and studied by Amourah and Darus (2016) [4].

(ii) $A_{0,\lambda,1}^m(\alpha, \beta, 0)f(z) = z + \sum_{n=2}^{\infty} [1 + (n-1)(\lambda - \alpha)\beta]^m a_n z^n$, the operator introduced and studied by Darus and Ibrahim (2009) [12].

(iii) $A_{0,\lambda,1}^m(0, 1, 0)f(z) = z + \sum_{n=2}^{\infty} [1 + (n-1)\lambda]^m a_n z^n$, the operator introduced and studied by Al-Oboudi (2004) [1].

(iv) $A_{0,1,1}^m(0, 1, 0)f(z) = z + \sum_{n=2}^{\infty} [n]^m a_n z^n$, the operator introduced and studied by Sălăgean (1983) [13].

Further, let $T^*(p)$ denoted the subclass of $\mathcal{A}(p)$ consisting functions of the form

$$f(z) = z^p - \sum_{n=p+1}^{\infty} a_n z^n \quad (a_n \geq 0, n, p \in \mathbb{N}, z \in \mathbb{U}). \quad (1.6)$$

Now, we define the (n, τ) -neighborhood of the function $f \in T^*(p)$ by

$$N_{n,\delta}(f) = \left\{ g \in T^*(p) : g(z) = z^p - \sum_{n=p+1}^{\infty} |b_n| z^n \text{ and } \sum_{n=p+1}^{\infty} n |a_n - b_n| \leq \tau \right\}. \quad (1.7)$$

In particular, for the identity function

$$h(z) = z^p \quad (1.8)$$

we immediately have

$$N_{n,\delta}(h) = \left\{ g \in T^*(p) : g(z) = z^p - \sum_{n=p+1}^{\infty} b_n z^n \text{ and } \sum_{n=p+1}^{\infty} n |b_n| \leq \tau \right\}. \quad (1.9)$$

The concept of the neighborhood was first presented by Goodman [14], and then generalized by Ruscheweyh [15], and applied recently to Neighborhoods of certain p -valent analytic functions by Amourah

and Darus [[3], [9], [2]], and others including Al-Hawary et al. [[10], [4], [6]] and Anakira et al. [[7], [11], [8]] .

We define and study some (n, τ) - neighborhood properties. The classes $G^*(b, p)$ and $G^*(b, p, \chi)$ are defined as follows:

Definition 1.1. A function $f \in T^*$ is said to be in the class $G^*(b, p)$ if and only if

$$\left| \frac{1}{b} \left[\frac{z(A_{\mu, \lambda, p}^m(\alpha, \beta, \delta)f(z))'}{A_{\mu, \lambda, p}^m(\alpha, \beta, \delta)f(z)} - p \right] \right| < \zeta, \quad (1.10)$$

for some $\alpha \geq 0, 0 < \zeta \leq 1, \beta, \lambda, \mu > 0, \lambda \neq \alpha, p \in \mathbb{N}, p \neq n, b \in \mathbb{C} - \{0\}$ and $m \in \mathbb{N}_0$ and for all $z \in \mathbb{U}$.

Definition 1.2. A function $f \in T^*(p)$ is said to be in the class $G^*(b, p, \chi)$ if and only if

$$\left| \frac{1}{b} \left[(1 - \chi) \frac{A_{\mu, \lambda, p}^m(\alpha, \beta, \delta)f(z)}{z^p} + \chi \frac{(A_{\mu, \lambda, p}^m(\alpha, \beta, \delta)f(z))'}{pz^{p-1}} - 1 \right] \right| < \zeta, \quad (1.11)$$

for some $\alpha, \chi \geq 0, 0 < \zeta \leq 1, \beta, \lambda, \mu > 0, \lambda \neq \alpha, p \in \mathbb{N}, p \neq n, b \in \mathbb{C} - \{0\}$ and $m \in \mathbb{N}_0$ and for all $z \in \mathbb{U}$.

2. Neighborhoods for the classes $G^*(b, p)$ and $G^*(b, p, \chi)$

In our investigation of the inclusion relations involving $N_{n, \delta}(h)$, we shall require theorem 2.1 and theorem 2.2 below:

Theorem 2.1. Let the function $f(z) \in T^*(p)$ be defined by (1.6). Then f is in the class $G^*(b, p)$ if and only if

$$\sum_{n=p+1}^{\infty} (n - p + \zeta|b|) \left[1 + \frac{(n - p)[(\lambda - \alpha)\beta + n\delta]}{\mu + \lambda} \right]^m a_n \leq \zeta|b|, \quad (2.1)$$

where $\alpha \geq 0, 0 < \zeta \leq 1, \beta, \lambda, \mu > 0, \lambda \neq \alpha, p \in \mathbb{N}, p \neq n, b \in \mathbb{C} - \{0\}$ and $m \in \mathbb{N}_0$.

Proof. We first suppose that $f \in G^*(b, p)$. Then by appealing to the condition (1.10), we readily obtain

$$\operatorname{Re} \left\{ \frac{z(A_{\mu, \lambda, p}^m(\alpha, \beta, \delta)f(z))'}{A_{\mu, \lambda, p}^m(\alpha, \beta, \delta)f(z)} - p \right\} > -\zeta|b|.$$

or, equivalently,

$$\operatorname{Re} \left\{ \frac{- \sum_{n=p+1}^{\infty} (n - p) \left[1 + \frac{(n - p)[(\lambda - \alpha)\beta + n\delta]}{\mu + \lambda} \right]^m a_n z^{n-p}}{1 - \sum_{n=p+1}^{\infty} \left[1 + \frac{(n - p)[(\lambda - \alpha)\beta + n\delta]}{\mu + \lambda} \right]^m a_n z^{n-p}} \right\} > -\zeta|b|. \quad (2.2)$$

We now choose values of z on the real axis and let $z \rightarrow 1^-$ through real values. Then the inequality (2.2) immediately yields the desired condition (2.1).

Conversely, by applying the hypothesis (2.1) and letting $|z| = 1$, we find from (1.10) that

$$\left| \frac{z(A_{\mu,\lambda,p}^m(\alpha, \beta, \delta)f(z))'}{A_{\mu,\lambda,p}^m(\alpha, \beta, \delta)f(z)} - p \right| = \left| \frac{\sum_{n=p+1}^{\infty} (n-p) \left[1 + \frac{(n-p)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda} \right]^m a_n z^{n-p}}{1 - \sum_{n=p+1}^{\infty} \left[1 + \frac{(n-p)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda} \right]^m a_n z^{n-p}} \right| \quad (2.3)$$

$$\leq \frac{\sum_{n=p+1}^{\infty} (n-p) \left[1 + \frac{(n-p)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda} \right]^m a_n}{1 - \sum_{n=p+1}^{\infty} \left[1 + \frac{(n-p)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda} \right]^m a_n} \quad (2.4)$$

$$\leq \frac{\zeta|b| \left(1 - \sum_{n=p+1}^{\infty} \left[1 + \frac{(n-p)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda} \right]^m a_n \right)}{1 - \sum_{n=p+1}^{\infty} \left[1 + \frac{(n-p)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda} \right]^m a_n} = \zeta|b|. \quad (2.5)$$

This implies

$$\sum_{n=p+1}^{\infty} (n-p + \zeta|b|) \left[1 + \frac{(n-p)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda} \right]^m a_n \leq \zeta|b|. \quad (2.6)$$

Then, we have $f \in G^*(b, p)$.

Similarly, we can prove the following result: □

Theorem 2.2. Let the function $f(z) \in T^*(p)$ be defined by (1.6). Then f is in the class $G^*(b, p, \chi)$ if and only if

$$\sum_{n=p+1}^{\infty} (p + \chi(n-p)) \left[1 + \frac{(n-p)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda} \right]^m a_n \leq p\zeta|b|. \quad (2.7)$$

Proof. We first suppose that $f \in G^*(b, p, \chi)$. Then by appealing to the condition (1.11), we readily obtain

$$\operatorname{Re} \left\{ (1-\chi) \frac{A_{\mu,\lambda,p}^m(\alpha, \beta, \delta)f(z)}{z^p} + \chi \frac{(A_{\mu,\lambda,p}^m(\alpha, \beta, \delta)f(z))'}{pz^{p-1}} - 1 \right\} > -\zeta|b|$$

or, equivalently

$$\operatorname{Re} \left\{ - \sum_{n=p+1}^{\infty} \left(1 - \chi + \frac{n}{p} \chi \right) \left[1 + \frac{(n-p)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda} \right]^m a_n z^{n-p} \right\} > -\zeta|b|, \quad (2.8)$$

We now choose values of z on the real axis and let $z \rightarrow 1$ through real values. Then the inequality (2.8) immediately yields the desired condition (2.7).

Conversely, by applying the hypothesis (2.7) and letting $z = 1$, we find from (1.11) that

$$\begin{aligned} & \left| (1-\chi) \frac{A_{\mu,\lambda,p}^m(\alpha, \beta, \delta)f(z)}{z^p} + \chi \frac{(A_{\mu,\lambda,p}^m(\alpha, \beta, \delta)f(z))'}{pz^{p-1}} - 1 \right| \\ &= \left| \sum_{n=p+1}^{\infty} (1-\chi + \frac{n}{p}\chi) \left[1 + \frac{(n-p)[(\lambda-\alpha)\beta + n\delta]}{\mu+\lambda} \right]^m a_n z^{n-p} \right| \\ &\leq \sum_{n=p+1}^{\infty} (1-\chi + \frac{n}{p}\chi) \left[1 + \frac{(n-p)[(\lambda-\alpha)\beta + n\delta]}{\mu+\lambda} \right]^m a_n |z^{n-p}| \\ &\leq \sum_{n=p+1}^{\infty} (1-\chi + \frac{n}{p}\chi) \left[1 + \frac{(n-p)[(\lambda-\alpha)\beta + n\delta]}{\mu+\lambda} \right]^m a_n < \zeta|b|. \end{aligned}$$

Thus, we have

$$\sum_{n=2}^{\infty} (p + \chi(n-p)) \left[1 + \frac{(n-p)[(\lambda-\alpha)\beta + n\delta]}{\mu+\lambda} \right]^m a_n \leq p\zeta|b|.$$

Hence, we have $f \in G^*(b, p, \chi)$.

Our first inclusion relation involving $N_{k,\delta}(h)$ is given by theorem 2.3 below.

□

Theorem 2.3. *If*

$$\tau = \frac{\zeta|b|(1+p)}{(1+\zeta|b|) \left[1 + \frac{(n-p)[(\lambda-\alpha)\beta + n\delta]}{\mu+\lambda} \right]^m} \quad (|b| < 1), \quad (2.9)$$

then

$$G^*(b, p) \subset N_{k,\delta}(h). \quad (2.10)$$

Proof. Let $f(z) \in G^*(b, p)$. Then, in view of the assertion (2.1) of theorem 2.1, we have

$$\begin{aligned} & (1+\zeta|b|) \left[1 + \frac{[(\lambda-\alpha)\beta + n\delta]}{\mu+\lambda} \right]^m \sum_{n=p+1}^{\infty} a_n \leq \\ & \sum_{n=p+1}^{\infty} (n-p + \zeta|b|) \left[1 + \frac{(n-p)[(\lambda-\alpha)\beta + n\delta]}{\mu+\lambda} \right]^m a_n \leq \zeta|b|, \end{aligned}$$

which readily yields

$$\sum_{n=p+1}^{\infty} a_n \leq \frac{\zeta|b|}{(1+\zeta|b|) \left[1 + \frac{[(\lambda-\alpha)\beta + n\delta]}{\mu+\lambda} \right]^m}. \quad (2.11)$$

Making use of (2.1) again, in conjunction with (2.11), we get

$$\begin{aligned} & \left[1 + \frac{[(\lambda-\alpha)\beta + n\delta]}{\mu+\lambda} \right]^m \sum_{n=p+1}^{\infty} na_n \leq \zeta|b| + \left[1 + \frac{[(\lambda-\alpha)\beta + n\delta]}{\mu+\lambda} \right]^m (1-\zeta|b|) \sum_{n=p+1}^{\infty} a_n \\ & \leq \frac{\zeta|b|(1+p)}{(1+\zeta|b|)}. \end{aligned}$$

Hence

$$\sum_{n=p+1}^{\infty} na_n \leq \frac{\zeta|b|(1+p)}{(1+\zeta|b|) \left[1 + \frac{[(\lambda-\alpha)\beta + n\delta]}{\mu+\lambda} \right]^m} = \tau, \quad (|b| < 1)$$

which, by means of the definition (1.9), establishes the inclusion (2.10) asserted by theorem 2.3.

Similarly, by applying theorem 2.2 instead of theorem 2.1, we now prove theorem 2.4 below.

□

Theorem 2.4. *If*

$$\tau = \frac{p\zeta|b|(1+p)}{(p+\chi) \left[1 + \frac{[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda}\right]^m} \quad (|b| < p),$$

then

$$G^*(b, p, \chi) \subset N_{n,\delta}(h). \quad (2.12)$$

Proof. Let $f(z) \in G^*(b, p, \chi)$. Then, in view of the assertion (2.7) of theorem 2.2, we have

$$\begin{aligned} (p+\chi) \left[1 + \frac{[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda}\right]^m \sum_{n=p+1}^{\infty} a_n &\leq \\ \sum_{n=p+1}^{\infty} (p+\chi(n-p)) \left[1 + \frac{(n-p)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda}\right]^m a_n &\leq p\zeta|b|, \end{aligned}$$

which readily yields

$$\sum_{n=p+1}^{\infty} a_n \leq \frac{p\zeta|b|}{(p+\chi) \left[1 + \frac{[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda}\right]^m}. \quad (2.13)$$

Making use of (2.7) again, in conjunction with (2.13), we get

$$\begin{aligned} \chi \left[1 + \frac{[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda}\right]^m \sum_{n=p+1}^{\infty} na_n &\leq p\zeta|b| + p(\chi-1) \left[1 + \frac{[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda}\right]^m \sum_{n=p+1}^{\infty} a_n \\ &\leq \frac{p\zeta|b|(p+1)\chi}{(p+\chi)}. \end{aligned}$$

Hence

$$\sum_{n=p+1}^{\infty} na_n \leq \frac{p\zeta|b|(p+1)}{(p+\chi) \left[1 + \frac{[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda}\right]^m} = \tau, \quad (|b| < p)$$

which, by means of the definition (1.9), establishes the inclusion (2.12) asserted by theorem 2.4. $\square$

3. Neighborhoods for the classes $G_\varepsilon^*(b, p)$ and $G_\varepsilon^*(b, p, \chi)$

In this section, we determine the neighborhood for the each classes $G_\varepsilon^*(b, p)$ and $G_\varepsilon^*(b, p, \chi)$, which we define as follows. A function $f(z) \in \mathcal{A}(p)$ is said to be in the class $G_\varepsilon^*(b, p)$ if there exists a function $g(z) \in G_\varepsilon^*(b, p)$ such that

$$\left| \frac{f(z)}{g(z)} - 1 \right| < p - \varepsilon, \quad (0 \leq \varepsilon < p). \quad (3.1)$$

Analogously, a function $f(z) \in \mathcal{A}(p)$ is said to be in the class $G_\varepsilon^*(b, p, \chi)$ if there exists a function $g(z) \in G_\varepsilon^*(b, p, \chi)$ such that the inequality (3.1) holds true.

Theorem 3.1. *If $g(z) \in G_\varepsilon^*(b, p)$ and*

$$\varepsilon = 1 - \frac{\tau(1+\zeta|b|) \left[1 + \frac{[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda}\right]^m}{(p+1)(1+\zeta|b|) \left[1 + \frac{[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda}\right]^m - \zeta|b|}, \quad (3.2)$$

then

$$N_{n,\delta}(g) \subset G_\varepsilon^*(b, p).$$

Proof. Suppose that $f(z) \in N_{n,\delta}(g)$. We find from (1.7) that

$$\sum_{n=p+1}^{\infty} n |a_n - b_n| \leq \tau, \quad (3.3)$$

which readily implies that

$$\sum_{n=p+1}^{\infty} |a_n - b_n| \leq \frac{\tau}{p+1}.$$

Next, since $g(z) \in G^*(b, p)$, we have [cf. equation 2.11]

$$\sum_{n=p+1}^{\infty} b_n \leq \frac{\zeta |b|}{(1 + \zeta |b|) \left[1 + \frac{[(\lambda - \alpha)\beta + n\delta]}{\mu + \lambda} \right]^m},$$

letting $|z| \rightarrow 1$, so we have

$$\begin{aligned} \left| \frac{f(z)}{g(z)} - 1 \right| &\leq \frac{\sum_{n=p+1}^{\infty} |a_n - b_n|}{1 - \sum_{n=p+1}^{\infty} |b_n|} \leq \frac{\tau}{p+1} \cdot \frac{(1 + \zeta |b|) \left[1 + \frac{[(\lambda - \alpha)\beta + n\delta]}{\mu + \lambda} \right]^m}{(1 + \zeta |b|) \left[1 + \frac{[(\lambda - \alpha)\beta + n\delta]}{\mu + \lambda} \right]^m - \zeta |b|} \\ &= p - \varepsilon \end{aligned} \quad (3.4)$$

provided that α is given by (3.2). Thus, by the above definition, $f(z) \in G_{\varepsilon}^*(b, p)$ for ε given by (3.2).

This evidently proves theorem 3.1. $\square$

Our proof of theorem 3.2 below is much akin to that of theorem 3.1.

Theorem 3.2. If $g(z) \in G_{\varepsilon}^*(b, p, \chi)$ and

$$\varepsilon = p - \frac{\tau(p + \chi) \left[1 + \frac{[(\lambda - \alpha)\beta + n\delta]}{\mu + \lambda} \right]^m}{(p + 1)(p + \chi) \left[1 + \frac{[(\lambda - \alpha)\beta + n\delta]}{\mu + \lambda} \right]^m - p\zeta |b|}, \quad (3.5)$$

then

$$N_{n,\delta}(g) \subset G_{\varepsilon}^*(b, p, \chi).$$

Proof. Suppose that $f(z) \in N_{n,\delta}(g)$. We find from (1.7) that

$$\sum_{n=p+1}^{\infty} n |a_n - b_n| \leq \tau, \quad (3.6)$$

which readily implies that

$$\sum_{n=p+1}^{\infty} |a_k - b_k| \leq \frac{\tau}{p+1}.$$

Next, since $g(z) \in G_{\varepsilon}^*(b, p, \chi)$, we have [cf. equation 2.13]

$$\sum_{n=p+1}^{\infty} b_n \leq \frac{p\zeta |b|}{(p + \chi) \left[1 + \frac{[(\lambda - \alpha)\beta + n\delta]}{\mu + \lambda} \right]^m},$$

letting $|z| \rightarrow 1$, so we have

$$\left| \frac{f(z)}{g(z)} - 1 \right| \leq \frac{\sum_{n=p+1}^{\infty} |a_n - b_n|}{1 - \sum_{n=p+1}^{\infty} |b_n|} \leq \frac{\delta}{p+1} \cdot \frac{(p+\chi) \left[1 + \frac{[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda} \right]^m}{(p+\chi) \left[1 + \frac{[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda} \right]^m - p\zeta|b|}$$

$$= p - \varepsilon \quad (3.7)$$

provided that ε is given by (3.5). Thus, by the above definition, $f(z) \in G_{\varepsilon}^*(b, p, \chi)$ for ε given by (3.5).

This evidently completes our proof of theorem 3.2. $\square$

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