Applications of Homotopy Perturbation Transform Method for Solving Newell-Whitehead-Segel Equation

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Abstract. In the present work, we apply an analytical method known as Homotopy Perturbation Transform Method, is used for solving Newell-Whitehead-Segel Equation. Five tests are examined to validate the accuracy of this algorithm. Numerical results show that the proposed method is a more reliable, efficient and convenient one for solving different cases of Newell-Whitehead-Segel equation.

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1 Introduction

The Newell-Whitehead-Segel equations have wide applicability in mechanical and chemical engineering, ecology, biology and bio-engineering. Newell-Whitehead-Segel (NWS) equation is solved by using the Adomian decomposition and multi-quadric quasi-interpolation methods [2], an approximate solution to the Newell-Whitehead-Segel equation by the Adomian decomposition method [8], homotopy perturbation method [9], new iterative method [10], He's variational iteration method [11], Laplace Adomian Decomposition Method [12] and a comparison between the reduced differential transform method and the Adomian decomposition method for the Newell-Whitehead-Segel equation [13]. In this article, different types of nonlinear Newell-Whitehead-Segel equations are solved by the Homotopy Perturbation Transform Method (HPTM) which is introduced by Khan and Wu [7] by combining the Homotopy perturbation method [1]-[5]-[6] and Laplace transform method for solving various types of linear and nonlinear partial differential equations. The Newell-Whitehead-Segel equation models written as:

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}(x, t) + au(x, t) - bu^m(x, t), \quad (1)$$

where $u(x, t)$ is a function of the spatial variable x and the temporal variable t with $x \in \mathbb{R}$ and $t \geq 0$. The function $u(x, t)$ may be thought of as the nonlinear distribution of temperature in an infinitely thin and long rod or as the flow velocity of a fluid in an infinitely long pipe with small diameter a, b and k are real numbers with $k > 0$ and m is a positive integer.

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2 Analysis of Homotopy perturbation transform method (HPTM)

To illustrate the basic idea of (HPTM) [7], we consider a general nonlinear partial differential equation with the initial conditions of the form

$$\begin{aligned} Du(x, t) + Ru(x, t) + Nu(x, t) &= g(x, t), \\ u(x, 0) &= h(x), u_t(x, 0) = f(x), \end{aligned} \quad (2)$$

where D is the second order linear differential operator $D = \partial^2/\partial t^2$, R is the linear differential operator of less order than D , N represents the general non-linear differential operator and $g(x, t)$ is the source term. Taking the Laplace transform (denoted throughout this paper by L) on both sides of Eq. (2):

$$L[Du(x, t)] + L[Ru(x, t)] + L[Nu(x, t)] = L[g(x, t)]. \quad (3)$$

Using the differentiation property of the Laplace transform, we have

$$L[u(x, t)] = \frac{h(x)}{s} + \frac{f(x)}{s^2} - \frac{1}{s^2}L[Ru(x, t)] + \frac{1}{s^2}L[g(x, t)] - \frac{1}{s^2}L[Nu(x, t)]. \quad (4)$$

Operating with the Laplace inverse on both sides of Eq. (4) gives

$$u(x, t) = G(x, t) - L^{-1} \left[\frac{1}{s^2} L [Ru(x, t) + Nu(x, t)] \right], \quad (5)$$

where $G(x, t)$ represents the term arising from the source term and the prescribed initial conditions. Now, we apply the homotopy perturbation method

$$u(x, t) = \sum_{n=0}^{+\infty} p^n u_n(x, t), \quad (6)$$

and the nonlinear term can be decomposed as

$$Nu(x, t) = \sum_{n=0}^{+\infty} p^n H_n(u), \quad (7)$$

for some He's polynomials $H_n(u)$ see([3]-[4]) that are given by

$$H_n(u_0, \dots, u_n) = \frac{1}{n!} \frac{\partial^n}{\partial p^n} \left[N \left(\sum_{i=0}^{+\infty} p^i u_i \right) \right]_{p=0}, \quad n = 0, 1, 2, 3, \dots \quad (8)$$

Substituting Eqs.(6) and (7) in Eq. (5) we get

$$\sum_{n=0}^{+\infty} p^n u_n(x, t) = G(x, t) - p \left(L^{-1} \left[\frac{1}{s^2} L \left[R \left(\sum_{n=0}^{+\infty} p^n u_n(x, t) \right) + \sum_{n=0}^{+\infty} p^n H_n(u) \right] \right] \right), \quad (9)$$

which is the coupling of the Laplace transform and the homotopy perturbation method using He's polynomials. Comparing the coefficient of like powers of p , the following approximations are obtained

$$\begin{aligned} p^0 &: u_0(x, t) = G(x, t), \\ p^1 &: u_1(x, t) = -L^{-1} \left[\frac{1}{s^2} L [Ru_0(x, t) + H_0(u)] \right], \\ p^2 &: u_2(x, t) = -L^{-1} \left[\frac{1}{s^2} L [Ru_1(x, t) + H_1(u)] \right], \\ p^3 &: u_3(x, t) = -L^{-1} \left[\frac{1}{s^2} L [Ru_2(x, t) + H_2(u)] \right], \\ &\vdots \end{aligned} \quad (10)$$

Then the solution is

$$u(x, t) = \lim_{p \rightarrow 1} \sum_{i=0}^{+\infty} p^i u_i(x, t) = u_0(x, t) + u_1(x, t) + u_2(x, t) + \cdots \quad (11)$$

For later numerical computation, the expression of ψ_n for $n \geq 1$ below denotes the n - term approximation to $u(x, t)$.

$$\psi_n(x, t) = \sum_{i=0}^{n-1} u_i(x, t). \quad (12)$$

Absolute error = $|u(x, t) - \psi_n(x, t)|$.

Remark: Consider the following linear operators and their invers operators.

$$\begin{aligned} L_t &= \frac{\partial}{\partial t}, & L_{xx} &= \frac{\partial^2}{\partial x^2}, & \frac{\partial^2 u_n}{\partial x^2} &= (u_n)_{xx}, \\ L^{-1} \left[\frac{1}{s} L(t^n) \right] &= L^{-1} \left[\frac{n!}{s^{n+2}} \right] = \frac{t^{n+1}}{n+1}, & \text{for } n &= 0, 1, 2, 3, \dots \\ L^{-1} \left[\frac{1}{s} L(\cdot) \right] &= \int_0^t (\cdot) ds. \end{aligned}$$

3 Numerical results

To illustrate the accuracy and the effectiveness of the present method, several test examples are considered in this section.

Example 1: Consider (NWS) equation if $k = 5$, $a = 2$, $b = -1$ and $m = 2$:

$$\frac{\partial u(x, t)}{\partial t} = 5 \frac{\partial^2 u(x, t)}{\partial x^2} + 2u(x, t) + u^2(x, t), \quad (13)$$

subject to the initial condition

$$u(x, 0) = \alpha, \quad \alpha \in \mathbb{R}. \quad (14)$$

Taking Laplace transform both of sides, subject to the initial condition we have

$$L[u(x, t)] = \frac{\alpha}{s} + \frac{1}{s} L[5u_{xx} + 2u + u^2]. \quad (15)$$

Taking Inverse Laplace transform we get

$$u(x, t) = \alpha + p L^{-1} \left[\frac{1}{s} L[5u_{xx} + 2u + u^2] \right]. \quad (16)$$

By homotopy perturbation method, we get

$$u(x, t) = \sum_{n=0}^{+\infty} p^n u_n(x, t). \quad (17)$$

Substitute Eq. (17) in Eq. (16), we get

$$\sum_{n=0}^{+\infty} p^n u_n(x, t) = \alpha + p L^{-1} \left\{ \frac{1}{s} L \left[\begin{aligned} &5 \left(\sum_{n=0}^{+\infty} p^n u_n(x, t) \right)_{xx} \\ &+ 2 \left(\sum_{n=0}^{+\infty} p^n u_n(x, t) \right) \\ &+ \left(\sum_{n=0}^{+\infty} p^n H_n(u) \right) \end{aligned} \right] \right\}. \quad (18)$$

Where H_n are He's polynomials that represents the nonlinear terms. The first few components of He's polynomials are given by

$$\begin{aligned} H_0(u) &= u_0^2 \\ H_1(u) &= 2u_0u_1 \\ H_2(u) &= 2u_0u_2 + u_1^2 \\ H_3(u) &= 2u_0u_3 + 2u_1u_2 \\ &\vdots \end{aligned}$$

By comparing the coefficient of like powers of p , we have:

$$\begin{aligned} p^0 &: u_0(x, t) = \alpha, \\ p^1 &: u_1(x, t) = L^{-1} \left[\frac{1}{s} L [5(u_0)_{xx} + 2u_0 + H_0(u)] \right] \\ &= \alpha(\alpha + 2)t, \\ p^2 &: u_2(x, t) = L^{-1} \left[\frac{1}{s} L [5(u_1)_{xx} + 2u_1 + H_1(u)] \right] \\ &= \alpha(\alpha + 2)(\alpha + 1)t^2, \\ p^3 &: u_3(x, t) = L^{-1} \left[\frac{1}{s} L [5(u_2)_{xx} + 2u_2 + H_2(u)] \right] \\ &= 2\alpha(\alpha + 2)(\alpha + 1)^2 \frac{t^3}{3}, \\ p^4 &: u_4(x, t) = L^{-1} \left[\frac{1}{s} L [5(u_3)_{xx} + 2u_3 + H_3(u)] \right] \\ &= \alpha(\alpha + 2)(\alpha + 1)(5\alpha^2 + 10\alpha + 2) \frac{t^4}{6} \\ &\vdots \end{aligned}$$

the approximate solution of equation (13)-(14) is given as:

$$\begin{aligned} u(x, t) &= \alpha(\alpha + 2)t + \alpha(\alpha + 2)(\alpha + 1)t^2 + 2\alpha(\alpha + 2)(\alpha + 1)^2 \frac{t^3}{3} \\ &\quad + \alpha(\alpha + 2)(\alpha + 1)(5\alpha^2 + 10\alpha + 2) \frac{t^4}{6} + \dots \end{aligned}$$

the closed form solution will be as follows:

$$u(x, t) = \frac{2\alpha e^{2t}}{2 + \alpha(1 - e^{2t})}. \quad (19)$$

which is an exact solution and is same as obtained by [10].

Example 2: Consider (NWS) equation if $k = 1$, $a = 2$, $b = 3$ and $m = 2$:

$$\frac{\partial u(x, t)}{\partial t} = \frac{\partial^2 u(x, t)}{\partial x^2} + 2u(x, t) - 3u^2(x, t), \quad (20)$$

with the initial condition

$$u(x, 0) = \beta, \quad \beta \in \mathbb{R}. \quad (21)$$

As before, taking Inverse Laplace transform we find

$$u(x, t) = \beta + pL^{-1} \left[\frac{1}{s} L [u_{xx} + 2u - 3u^2] \right]. \quad (22)$$

By applying homotopy perturbation transform method, subject to initial condition we have

$$\sum_{n=0}^{+\infty} p^n u_n(x, t) = \beta + pL^{-1} \left\{ \frac{1}{s} L \left[\begin{array}{c} \left(\sum_{n=0}^{+\infty} p^n u_n(x, t) \right)_{xx} \\ + 2 \left(\sum_{n=0}^{+\infty} p^n u_n(x, t) \right) \\ - 3 \left(\sum_{n=0}^{+\infty} p^n u_n(x, t) \right)^2 \end{array} \right] \right\}.$$

By comparing the coefficient of like powers of p , we get:

$$\begin{aligned} p^0 & : u_0(x, t) = \beta, \\ p^1 & : u_1(x, t) = L^{-1} \left[\frac{1}{s} L [(u_0)_{xx} + 2u_0 - 3u_0^2] \right] \\ & = \beta(2 - 3\beta)t, \\ p^2 & : u_2(x, t) = L^{-1} \left[\frac{1}{s} L [(u_1)_{xx} + 2u_1 - 6u_0u_1] \right] \\ & = 2\beta(2 - 3\beta)(1 - 3\beta) \frac{t^2}{2}, \\ p^3 & : u_3(x, t) = L^{-1} \left[\frac{1}{s} L [(u_2)_{xx} + 2u_2 - 6u_0u_2 - 3u_1^2] \right] \\ & = 2\beta(2 - 3\beta)(27\beta^2 - 18\beta + 2) \frac{t^3}{3!}, \\ & \vdots \end{aligned}$$

Then the series solution is given by

$$\begin{aligned} u(x, t) &= \beta + \beta(2 - 3\beta)t + 2\beta(2 - 3\beta)(1 - 3\beta) \frac{t^2}{2} \\ &\quad + 2\beta(2 - 3\beta)(27\beta^2 - 18\beta + 2) \frac{t^3}{3!} + \cdots \\ &= \frac{-2\beta e^{2t}}{-2 + 3\beta(1 - e^{2t})}. \end{aligned}$$

which is an exact solution and is the same as obtained by [9] and [11]

Example 3: Consider (NWS) equation if $k = 1$, $a = 1$, $b = 1$ and $m = 2$:

$$\frac{\partial u(x, t)}{\partial t} = \frac{\partial^2 u(x, t)}{\partial x^2} + u(x, t) - u^2(x, t), \quad (23)$$

subject to the initial condition

$$u(x, 0) = \frac{1}{\left(1 + e^{\frac{x}{\sqrt{6}}}\right)^2}. \quad (24)$$

By applying aforesaid method subject to initial condition we have

$$\sum_{n=0}^{+\infty} p^n u_n(x, t) = \frac{1}{\left(1 + e^{\frac{x}{\sqrt{6}}}\right)^2} + pL^{-1} \left\{ \frac{1}{s} L \left[\begin{array}{c} \left(\sum_{n=0}^{+\infty} p^n u_n(x, t) \right)_{xx} \\ + \left(\sum_{n=0}^{+\infty} p^n u_n(x, t) \right) \\ - \left(\sum_{n=0}^{+\infty} p^n u_n(x, t) \right)^2 \end{array} \right] \right\}.$$

By comparing the coefficient of like powers of p , we have:

$$\begin{aligned}
 p^0 &: u_0(x, t) = \frac{1}{\left(1 + e^{\frac{x}{\sqrt{6}}}\right)^2}, \\
 p^1 &: u_1(x, t) = L^{-1} \left[\frac{1}{s} L \left[(u_0)_{xx} + u_0 - u_0^2 \right] \right] \\
 &= \frac{5}{3} \frac{e^{\frac{x}{\sqrt{6}}}}{\left(1 + e^{\frac{x}{\sqrt{6}}}\right)^3} t, \\
 p^2 &: u_2(x, t) = L^{-1} \left[\frac{1}{s} L \left[(u_1)_{xx} + u_1 - 2u_0u_1 \right] \right] \\
 &= \frac{25}{36} \frac{e^{\frac{x}{\sqrt{6}}} \left(2e^{\frac{x}{\sqrt{6}}} - 1 \right)}{\left(1 + e^{\frac{x}{\sqrt{6}}}\right)^4} t^2,
 \end{aligned}$$

Proceeding in a similar manner, we obtain

$$\begin{aligned}
 p^3 &: u_3(x, t) = L^{-1} \left[\frac{1}{s} L \left[(u_2)_{xx} + u_2 - 2u_0u_2 - u_1^2 \right] \right] \\
 &= \frac{125}{648} \frac{e^{\frac{x}{\sqrt{6}}} \left(4e^{\frac{2x}{\sqrt{6}}} - 7e^{\frac{x}{\sqrt{6}}} + 1 \right)}{\left(1 + e^{\frac{x}{\sqrt{6}}}\right)^5} t^3, \\
 &\vdots
 \end{aligned}$$

Then the series solution of equation (23)-(24) in closed form is given by

$$\begin{aligned}
 u(x, t) &= \frac{1}{\left(1 + e^{\frac{x}{\sqrt{6}}}\right)^2} + \frac{5}{3} \frac{e^{\frac{x}{\sqrt{6}}}}{\left(1 + e^{\frac{x}{\sqrt{6}}}\right)^3} t \\
 &+ \frac{25}{36} \frac{e^{\frac{x}{\sqrt{6}}} \left(2e^{\frac{x}{\sqrt{6}}} - 1 \right)}{\left(1 + e^{\frac{x}{\sqrt{6}}}\right)^4} t^2 \\
 &+ \frac{125}{648} \frac{e^{\frac{x}{\sqrt{6}}} \left(4e^{\frac{2x}{\sqrt{6}}} - 7e^{\frac{x}{\sqrt{6}}} + 1 \right)}{\left(1 + e^{\frac{x}{\sqrt{6}}}\right)^5} t^3 + \dots \\
 &= \frac{1}{\left(1 + e^{\frac{x}{\sqrt{6}} - \frac{5t}{6}}\right)^2}.
 \end{aligned}$$

As presented by [9] and [11].

(a)

The behavior of the exact solution (a) in Example 3

(b)

The behavior of the approximate solution (b) in Example 3

(c)

The behavior of the absolute Error (c) in Example 3

Table 1: Comparison of the exact solution and ψ_4 when $t = 0.01$ in Example 3.

x	Exact value	ψ_4	$ u - \psi_4 $
-20	9.99476799E-1	9.99476800E-1	1.11679902E-9
-10	9.69673588E-1	9.69673631E-1	4.29068732E-8
-5	7.97929279E-1	7.97929085E-1	1.93993830E-7
0	2.71254811E-1	2.71255305E-1	4.93486634E-7
5	1.53001105E-2	1.52999789E-2	1.31618083E-7
10	3.24044532E-4	3.24036591E-4	7.94194895E-9
20	9.55244894E-8	9.55218070E-8	2.68256912E-12

Example 4: Consider linear Newell-Whitehead-Segel equation if $k = 1$, $a = 1$, $b = 1$ and $m = 4$:

$$\frac{\partial u(x, t)}{\partial t} = \frac{\partial^2 u(x, t)}{\partial x^2} + u(x, t) - u^4(x, t), \quad (25)$$

subject to the initial condition

$$u(x, 0) = \frac{1}{\left(1 + e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{2}{3}}}. \quad (26)$$

By applying the present method (HPTM) subject to initial condition we have

$$\sum_{n=0}^{+\infty} p^n u_n(x, t) = \frac{1}{\left(1 + e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{2}{3}}} + pL^{-1} \left\{ \frac{1}{s} L \left[\begin{array}{l} \left(\sum_{n=0}^{+\infty} p^n u_n(x, t) \right)_{xx} \\ + \left(\sum_{n=0}^{+\infty} p^n u_n(x, t) \right) \\ - \left(\sum_{n=0}^{+\infty} p^n H_n(u) \right) \end{array} \right] \right\}.$$

Where

$$\sum_{n=0}^{+\infty} p^n H_n(u) = u^4,$$

the first few components of He's polynomials are given by

$$\begin{aligned} H_0(u) &= u_0^4 \\ H_1(u) &= 4u_0^3 u_1 \\ H_2(u) &= 2u_0^2(2u_0 u_2 + u_1^2) + 4u_0^2 u_1^2. \end{aligned}$$

By comparing the coefficient of like powers of p ,

$$\begin{aligned}
 p^0 & : u_0(x, t) = \frac{1}{\left(1 + e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{2}{3}}}, \\
 p^1 & : u_1(x, t) = L^{-1} \left[\frac{1}{s} L[(u_0)_{xx} + u_0 - H_0(u)] \right] \\
 & = \frac{7}{5} \frac{e^{\frac{3x}{\sqrt{10}}}}{\left(1 + e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{5}{3}}} t, \\
 p^2 & : u_2(x, t) = L^{-1} \left[\frac{1}{s} L[(u_1)_{xx} + u_1 - H_1(u)] \right] \\
 & = \frac{49}{100} \frac{e^{\frac{3x}{\sqrt{10}}} \left(2e^{\frac{3x}{\sqrt{10}}} - 3\right)}{\left(1 + e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{8}{3}}} t^2, \\
 p^3 & : u_3(x, t) = L^{-1} \left[\frac{1}{s} L[(u_2)_{xx} + u_2 - H_2(u)] \right] \\
 & = \frac{343}{3000} \frac{e^{\frac{3x}{\sqrt{10}}} \left(4e^{\frac{6x}{\sqrt{10}}} - 27e^{\frac{3x}{\sqrt{10}}} + 9\right)}{\left(1 + e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{11}{3}}} t^3, \\
 & \vdots
 \end{aligned}$$

The other components of the (HPTM) can be determined in a similar way. The 4-term approximate solution of equation (25)-(26) in is given as:

$$\begin{aligned}
 \psi_4(x, t) & = \frac{1}{\left(1 + e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{2}{3}}} + \frac{7}{5} \frac{e^{\frac{3x}{\sqrt{10}}}}{\left(1 + e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{5}{3}}} t \\
 & + \frac{49}{100} \frac{e^{\frac{3x}{\sqrt{10}}} \left(2e^{\frac{3x}{\sqrt{10}}} - 1\right)}{\left(1 + e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{8}{3}}} t^2 \\
 & + \frac{343}{3000} \frac{e^{\frac{3x}{\sqrt{10}}} \left(4e^{\frac{6x}{\sqrt{10}}} - 27e^{\frac{3x}{\sqrt{10}}} + 9\right)}{\left(1 + e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{11}{3}}} t^3.
 \end{aligned}$$

Hence the exact solution is given as;

$$u(x, t) = \left\{ \frac{1}{2} \tanh \left(\frac{-3}{2\sqrt{10}} \left(x - \frac{7t}{\sqrt{10}} \right) \right) + \frac{1}{2} \right\}^{\frac{2}{3}},$$

which is in full agreement with [9] and [11].

Table 2: Absolute Error $|u - \psi_4|$ in Example 4 .

x	$ u - \psi_4 $ $t = 0.1$	$ u - \psi_4 $ $t = 0.3$	$ u - \psi_4 $ $t = 0.5$
0	1.221229E-4	3.022038E-3	1.245533E-2
0.2	1.328287E-4	3.415236E-3	1.462723E-2
0.4	1.352957E-4	3.602846E-3	1.597269E-2
0.6	1.296021E-4	3.570756E-3	1.636664E-2
0.8	1.168616E-4	3.33532E-3	1.580592E-2
1	9.893015E-5	2.929772E-3	1.440339E-2

(e)

The behavior of the exact solution (e) in Example 4

(f)

The behavior of the approximate solution (f) in Example 4

(g)

The behavior of the absolute Error (g) in Example 4

Example 5: Consider (NWS) equation if $k = 1$, $a = 3$, $b = 4$ and $m = 1$:

$$\frac{\partial u(x, t)}{\partial t} = \frac{\partial^2 u(x, t)}{\partial x^2} + 3u(x, t) - 4u^3(x, t), \quad (27)$$

subject to the initial condition

$$u(x, 0) = \sqrt{\frac{3}{4}} \frac{e^{\sqrt{6}x}}{e^{\sqrt{6}x} + e^{\frac{\sqrt{6}}{2}x}}. \quad (28)$$

According to Homotopy Perturbation Transform Method procedures , we have

$$\sum_{n=0}^{+\infty} p^n u_n(x, t) = \sqrt{\frac{3}{4}} \frac{e^{\sqrt{6}x}}{e^{\sqrt{6}x} + e^{\frac{\sqrt{6}}{2}x}} + pL^{-1} \left\{ \frac{1}{s} L \left[\begin{aligned} & \left(\sum_{n=0}^{+\infty} p^n u_n(x, t) \right)_{xx} \\ & + 3 \left(\sum_{n=0}^{+\infty} p^n u_n(x, t) \right) \\ & - 4 \left(\sum_{n=0}^{+\infty} p^n H_n(u) \right) \end{aligned} \right] \right\}.$$

Where

$$\sum_{n=0}^{+\infty} p^n H_n(u) = u^3,$$

Table 3: Comparison of the exact solution and ψ_4 for different time when $x = 1$ in Example 5.

t	Exact value	ψ_4	$ u - \psi_4 $
0	6.693489E-1	6.693489E-1	0
0.1	7.293730E-1	7.292306E-1	1.424098E-4
0.2	7.736076E-1	7.715752E-1	2.032386E-3
0.3	8.047268E-1	7.956462E-1	9.080522E-3
0.4	8.259108E-1	8.007073E-1	2.520344E-2
0.5	8.400105E-1	7.860220E-1	5.398846E-2

By comparing the coefficient of like powers of p , we find

$$\begin{aligned}
p^0 : \quad u_0(x, t) &= \sqrt{\frac{3}{4}} \frac{e^{\sqrt{6}x}}{e^{\sqrt{6}x} + e^{\frac{\sqrt{6}}{2}x}}, \\
p^1 : \quad u_1(x, t) &= L^{-1} \left[\frac{1}{s} L \left[(u_0)_{xx} + 3u_0 - 4u_0^3 \right] \right] \\
&= \frac{9}{2} \sqrt{\frac{3}{4}} \frac{e^{\sqrt{6}x} e^{\frac{\sqrt{6}}{2}x}}{\left(e^{\sqrt{6}x} + e^{\frac{\sqrt{6}}{2}x} \right)^2} t, \\
p^2 : \quad u_2(x, t) &= L^{-1} \left[\frac{1}{s} L \left[(u_1)_{xx} + 3u_1 - 12u_0^2 u_1 \right] \right] \\
&= \frac{81}{8} \sqrt{\frac{3}{4}} \frac{e^{\sqrt{6}x} e^{\frac{\sqrt{6}}{2}x} \left(-e^{\sqrt{6}x} + e^{\frac{\sqrt{6}}{2}x} \right)}{\left(e^{\sqrt{6}x} + e^{\frac{\sqrt{6}}{2}x} \right)^3} t^2, \\
p^3 : \quad u_3(x, t) &= L^{-1} \left[\frac{1}{s} L \left[(u_2)_{xx} + 3u_2 - 4u_0(2u_0 u_2 + u_1^2) - 8u_0 u_1^2 - 4u_2 u_0^2 \right] \right] \\
&= \frac{243}{48} \sqrt{\frac{3}{4}} \frac{e^{\sqrt{6}x} e^{\frac{\sqrt{6}}{2}x} \left(-4e^{\sqrt{6}x} + e^{2\sqrt{6}x} + e^{\sqrt{6}x} \right)}{\left(e^{\sqrt{6}x} + e^{\frac{\sqrt{6}}{2}x} \right)^4} t^3, \\
&\vdots
\end{aligned}$$

Thus, the solution in series form is given by:

$$\begin{aligned}
u(x, t) &= \sqrt{\frac{3}{4}} \frac{e^{\sqrt{6}x}}{e^{\sqrt{6}x} + e^{\frac{\sqrt{6}}{2}x}} + \frac{9}{2} \sqrt{\frac{3}{4}} \frac{e^{\sqrt{6}x} e^{\frac{\sqrt{6}}{2}x}}{\left(e^{\sqrt{6}x} + e^{\frac{\sqrt{6}}{2}x} \right)^2} t \\
&\quad + \frac{81}{8} \sqrt{\frac{3}{4}} \frac{e^{\sqrt{6}x} e^{\frac{\sqrt{6}}{2}x} \left(-e^{\sqrt{6}x} + e^{\frac{\sqrt{6}}{2}x} \right)}{\left(e^{\sqrt{6}x} + e^{\frac{\sqrt{6}}{2}x} \right)^3} t^2 \\
&\quad + \frac{243}{48} \sqrt{\frac{3}{4}} \frac{e^{\sqrt{6}x} e^{\frac{\sqrt{6}}{2}x} \left(-4e^{\sqrt{6}x} + e^{2\sqrt{6}x} + e^{\sqrt{6}x} \right)}{\left(e^{\sqrt{6}x} + e^{\frac{\sqrt{6}}{2}x} \right)^4} t^3 + \dots \\
&= \sqrt{\frac{3}{4}} \frac{e^{\sqrt{6}x}}{e^{\sqrt{6}x} + e^{\left(\frac{\sqrt{6}}{2}x - \frac{9}{2}t \right)}}.
\end{aligned}$$

As presented by [9]

(h)

The behavior of the exact solution (h) in Example 5

(i)

The behavior of the approximate solution (i) in Example 5

(j)

The behavior of the absolute Error (j) in Example 5

It can be observed through tables (1-3) and figures (1-9) that this method is efficient and accurate for different values of time and place.

4 Conclusion

In this paper, the homotopy perturbation transforms method (HPTM) is successfully applied to solve Newell-Whitehead-Segel equations. It is worth mentioning that the nonlinear terms can be easily handled by the use of Hes Polynomials. which reduces the computational work as compared to other methods, without using discretization, linearization, or restrictive assumptions, while still maintaining the fast convergence and accuracy of this method. Finally, we conclude that the (HPTM) is considered as a best and accurate method to finding analytic as well as numerical solutions for wider classes of linear and nonlinear differential equations.

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