On Binary Generalized Topological Spaces

Jamal M. Mustafa

Department of Mathematics, Al al-Bayt University, Mafrq, Jordan

jjmmrr971@yahoo.com

Abstract. In this paper, we introduce and study the new concept of binary generalized topological spaces. Also we examine some binary generalized topological properties. Furthermore, we define and study some forms of binary generalized continuous functions and we investigate the relationships between these functions and their relationships with some other functions.

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1 Introduction and preliminaries

In [1] - [12], Á. Császár founded the theory of generalized topological spaces, and studied the elementary character of these classes. Especially he introduced the notions of continuous functions on generalized topological spaces. We recall some notions defined in [3]. Let X be a non-empty set and $P(X)$ the power set of X . We call a class $\eta \subseteq P(X)$ a generalized topology [3] if $\phi \in \eta$ and the arbitrary union of elements of η belongs to η . A set X with a generalized topology η on it is called a generalized topological space and is denoted by (X, η) .

For a generalized topological space (X, η) , the elements of η are called generalized open sets and the complements of them are called generalized closed sets.

If A is a subset of X and B is a subset of Y , then the topological structures on X and Y provide a little information about the ordered pair (A, B) . In 2011, S. Jothi et al [13] introduced a single structure which carries the subsets of X as well as the subsets of Y for studying the information about the ordered pair (A, B) of subsets of X and Y . Such a structure is called a binary structure from X to Y . Mathematically a binary structure from X to Y is defined as a set of ordered pairs (A, B) where $A \subseteq X$ and $B \subseteq Y$. The concept of binary topology from X to Y was introduced by S. Jothi et al [13].

Definition 1.1. [13] If $f : Z \longrightarrow X \times Y$ is a function and $A \subseteq X$, $B \subseteq Y$. We define $f^{-1}(A, B) = \{z \in Z : f(z) = (x, y) \in (A, B)\}$.

2 Binary Generalized Topology

In this section, the concept of a binary generalized topology between two non-empty sets is introduced and its properties are studied. Also, in this section, the concepts of binary generalized closure and binary generalized interior are introduced and their properties are discussed.

Definition 2.1. Let X and Y be any two non empty sets. A binary generalized topology from X to Y is a binary structure $\mu_b \subseteq P(X) \times P(Y)$ that satisfies the following axioms:

- (i) $(\phi, \phi) \in \mu_b$.
- (ii) If $\{(A_\alpha, B_\alpha) : \alpha \in \Delta\}$ is a family of members of μ_b , then $(\cup A_\alpha, \cup B_\alpha) \in \mu_b$.

If μ_b is a binary generalized topology from X to Y then the triplet (X, Y, μ_b) is called a binary generalized topological space and the members of μ_b are called binary generalized open sets.

The complement of an element of $P(X) \times P(Y)$ is defined component wise. That is the binary complement of (A, B) is $(X - A, Y - B)$.

Definition 2.2. Let (X, Y, μ_b) be a binary generalized topological space and $A \subseteq X, B \subseteq Y$. Then (A, B) is called binary generalized closed if $(X - A, Y - B)$ is binary generalized open.

Definition 2.3. [13] Let $(A, B), (C, D) \in P(X) \times P(Y)$. Then

- (i) $(A, B) \subseteq (C, D)$ if $A \subseteq C$ and $B \subseteq D$.
- (ii) $(A, B) \cup (C, D) = (A \cup C, B \cup D)$.
- (iii) $(A, B) \cap (C, D) = (A \cap C, B \cap D)$.

Definition 2.4. Let (X, Y, μ_b) be a binary generalized topological space and $(x, y) \in X \times Y$, then a subset (A, B) of (X, Y) is called a binary generalized neighborhood of (x, y) if there exists a binary generalized open set (U, V) such that $(x, y) \in (U, V) \subseteq (A, B)$.

The proof of the next theorem is straight forward.

Theorem 2.5. Let (X, Y, μ_b) be a binary generalized topological space. Then

- (i) (X, Y) is a binary generalized closed set.
- (ii) If $\{(A_\alpha, B_\alpha) : \alpha \in \Delta\}$ is a family of binary generalized closed sets, then $(\cap A_\alpha, \cap B_\alpha)$ is binary generalized closed.

Definition 2.6. Let (X, Y, μ_b) be a binary generalized topological space and $A \subseteq X, B \subseteq Y$. Let $(A, B)^{1*} = \cap\{A_\alpha : (A_\alpha, B_\alpha) \text{ is binary generalized closed and } (A, B) \subseteq (A_\alpha, B_\alpha)\}$ and $(A, B)^{2*} = \cap\{B_\alpha : (A_\alpha, B_\alpha) \text{ is binary generalized closed and } (A, B) \subseteq (A_\alpha, B_\alpha)\}$. Then the pair $((A, B)^{1*}, (A, B)^{2*})$ is called the binary generalized closure of (A, B) and denoted by $\mu_b Cl(A, B)$.

Remark 2.7. The binary generalized closure $\mu_b Cl(A, B)$ is binary generalized closed and $(A, B) \subseteq \mu_b Cl(A, B)$.

Theorem 2.8. A set (A, B) is binary generalized closed in (X, Y, μ_b) if and only if $\mu_b Cl(A, B) = (A, B)$.

Theorem 2.9. In a binary generalized topological space (X, Y, μ_b) if $(A, B) \subseteq (X, Y)$ then $\mu_b Cl(A, B)$ is the smallest binary generalized closed set containing (A, B) .

Proof. Let $\{(A_\alpha, B_\alpha) : \alpha \in \Delta\}$ be the family of all binary generalized closed sets containing (A, B) . Let $(C, D) = \cap\{(A_\alpha, B_\alpha) : \alpha \in \Delta\}$. Then (C, D) is a binary generalized closed set. Now each (A_α, B_α) is a superset of (A, B) . Therefore, $(A, B) \subseteq (C, D)$. Now $(C, D) \subseteq (A_\alpha, B_\alpha)$ for each $\alpha \in \Delta$ and hence (C, D) is the smallest binary generalized closed set containing (A, B) . Therefore, $\mu_b Cl(A, B)$ is the smallest binary generalized closed set containing (A, B) . $\square$

Theorem 2.10. Let (X, Y, μ_b) be a binary generalized topological space and $(A, B), (C, D) \in P(X) \times P(Y)$ with $(A, B) \subseteq (C, D)$. Then $\mu_b Cl(A, B) \subseteq \mu_b Cl(C, D)$.

Proof. Follows from Remark 2.7 and Theorem 2.9. $\square$

Theorem 2.11. Let (X, Y, μ_b) be a binary generalized topological space and $(A, B), (C, D) \in P(X) \times P(Y)$. Then

- (i) $\mu_b Cl(X, Y) = (X, Y)$.
- (ii) $\mu_b Cl(\mu_b Cl(A, B)) = \mu_b Cl(A, B)$.
- (iii) $\mu_b Cl(A, B) \cup \mu_b Cl(C, D) \subseteq \mu_b Cl((A, B) \cup (C, D))$.
- (iv) $\mu_b Cl((A, B) \cap (C, D)) \subseteq \mu_b Cl(A, B) \cap \mu_b Cl(C, D)$.

Proof. (i) Since (X, Y) is a binary generalized closed set, then by Theorem 2.8, we have $\mu_b Cl(X, Y) = (X, Y)$.

(ii) Since $\mu_b Cl(A, B)$ is a binary generalized closed set, then by Theorem 2.8, we have $\mu_b Cl(\mu_b Cl(A, B)) = \mu_b Cl(A, B)$.

(iii) $(A, B) \subseteq (A, B) \cup (C, D)$ and $(C, D) \subseteq (A, B) \cup (C, D)$. Therefore, $\mu_b Cl(A, B) \subseteq \mu_b Cl((A, B) \cup (C, D))$ and $\mu_b Cl(C, D) \subseteq \mu_b Cl((A, B) \cup (C, D))$. Hence, $\mu_b Cl(A, B) \cup \mu_b Cl(C, D) \subseteq \mu_b Cl((A, B) \cup (C, D))$.

(iv) $(A, B) \cap (C, D) \subseteq (A, B)$ and $(A, B) \cap (C, D) \subseteq (C, D)$. Therefore, $\mu_b Cl((A, B) \cap (C, D)) \subseteq \mu_b Cl(A, B)$ and $\mu_b Cl((A, B) \cap (C, D)) \subseteq \mu_b Cl(C, D)$. Hence, $\mu_b Cl((A, B) \cap (C, D)) \subseteq \mu_b Cl(A, B) \cap \mu_b Cl(C, D)$. $\square$

Remark 2.12. The equality in (iii) and (iv) in the above theorem need not be true as shown in the following example.

Example 2.13. Let $X = \{a, b, c\}$ and $Y = \{1, 2\}$ with the binary generalized topology $\mu_b = \{(\phi, \phi), (\{a\}, \{2\}), (\{b\}, Y), (\{a, b\}, Y)\}$. Then $\mu_b Cl(\{a\}, \phi) \cup \mu_b Cl(\{b\}, \phi) = (\{a, c\}, \phi) \cup (\{b, c\}, \{1\}) = (X, \{1\})$, but $\mu_b Cl((\{a\}, \phi) \cup (\{b\}, \phi)) = (X, Y)$. Also, $\mu_b Cl((\{a, b\}, \{1\}) \cap (\{c\}, \{2\})) = (\{c\}, \phi)$, but $\mu_b Cl(\{a, b\}, \{1\}) \cap \mu_b Cl(\{c\}, \{2\}) = (X, Y)$.

Definition 2.14. Let (X, Y, μ_b) be a binary generalized topological space and $A \subseteq X$, $B \subseteq Y$. Let $(A, B)^{1^\circ} = \cup\{A_\alpha : (A_\alpha, B_\alpha) \text{ is binary generalized open and } (A_\alpha, B_\alpha) \subseteq (A, B)\}$ and $(A, B)^{2^\circ} = \cup\{B_\alpha : (A_\alpha, B_\alpha) \text{ is binary generalized open and } (A_\alpha, B_\alpha) \subseteq (A, B)\}$. Then the pair $((A, B)^{1^\circ}, (A, B)^{2^\circ})$ is called the binary generalized interior of (A, B) and denoted by $\mu_b Int(A, B)$.

Remark 2.15. The binary generalized interior $\mu_b Int(A, B)$ is binary generalized open and $\mu_b Int(A, B) \subseteq (A, B)$.

Theorem 2.16. A set (A, B) is binary generalized open in (X, Y, μ_b) if and only if $\mu_b Int(A, B) = (A, B)$.

Theorem 2.17. In a binary generalized topological space (X, Y, μ_b) if $(A, B) \subseteq (X, Y)$ then $\mu_b Int(A, B)$ is the largest binary generalized open set contained in (A, B) .

Proof. By Remark 2.15, $\mu_b Int(A, B)$ is a binary generalized open set contained in (A, B) . Now, let (C, D) be any binary generalized open set contained in (A, B) . Then, $C \subseteq \cup\{A_\alpha : (A_\alpha, B_\alpha) \text{ is binary generalized open and } (A_\alpha, B_\alpha) \subseteq (A, B)\}$ and $D \subseteq \cup\{B_\alpha : (A_\alpha, B_\alpha) \text{ is binary generalized open and } (A_\alpha, B_\alpha) \subseteq (A, B)\}$. Hence, $(C, D) \subseteq ((A, B)^{1^\circ}, (A, B)^{2^\circ})$. Then, $(C, D) \subseteq \mu_b Int(A, B)$. This means that $\mu_b Int(A, B)$ is the largest binary generalized open set contained in (A, B) . $\square$

Theorem 2.18. Let (X, Y, μ_b) be a binary generalized topological space and $(A, B), (C, D) \in P(X) \times P(Y)$ with $(A, B) \subseteq (C, D)$. Then $\mu_b Int(A, B) \subseteq \mu_b Int(C, D)$.

Proof. Follows from Remark 2.15 and Theorem 2.17. $\square$

Theorem 2.19. Let (X, Y, μ_b) be a binary generalized topological space and $(A, B), (C, D) \in P(X) \times P(Y)$. Then

- (i) $\mu_b Int(\phi, \phi) = (\phi, \phi)$.
- (ii) $\mu_b Int(\mu_b Int(A, B)) = \mu_b Int(A, B)$.
- (iii) $\mu_b Int(A, B) \cup \mu_b Int(C, D) \subseteq \mu_b Int((A, B) \cup (C, D))$.
- (iv) $\mu_b Int((A, B) \cap (C, D)) \subseteq \mu_b Int(A, B) \cap \mu_b Int(C, D)$.

Proof. (i) Since (ϕ, ϕ) is a binary generalized open set, then by Theorem 2.16, we have $\mu_b Int(\phi, \phi) = (\phi, \phi)$.

(ii) Since $\mu_b Int(A, B)$ is a binary generalized open set, then by Theorem 2.16, we have $\mu_b Int(\mu_b Int(A, B)) = \mu_b Int(A, B)$.

(iii) $(A, B) \subseteq (A, B) \cup (C, D)$ and $(C, D) \subseteq (A, B) \cup (C, D)$. Therefore, $\mu_b Int(A, B) \subseteq \mu_b Int((A, B) \cup (C, D))$ and $\mu_b Int(C, D) \subseteq \mu_b Int((A, B) \cup (C, D))$. Hence, $\mu_b Int(A, B) \cup \mu_b Int(C, D) \subseteq \mu_b Int((A, B) \cup (C, D))$.

(iv) $(A, B) \cap (C, D) \subseteq (A, B)$ and $(A, B) \cap (C, D) \subseteq (C, D)$. Therefore, $\mu_b Int((A, B) \cap (C, D)) \subseteq \mu_b Int(A, B)$ and $\mu_b Int((A, B) \cap (C, D)) \subseteq \mu_b Int(C, D)$. Hence, $\mu_b Int((A, B) \cap (C, D)) \subseteq \mu_b Int(A, B) \cap \mu_b Int(C, D)$. $\square$

Remark 2.20. The equality in (iii) and (iv) in the above theorem need not be true as shown in the following example.

Example 2.21. In Example 2.13, we have $\mu_b Int(\{a\}, \{1\}) \cup \mu_b Int(\{b\}, \{2\}) = (\phi, \phi)$, but $\mu_b Int((\{a\}, \{1\}) \cup (\{b\}, \{2\})) = (\{a, b\}, Y)$. Also, $\mu_b Int((\{a\}, \{2\}) \cap (\{b\}, Y)) = (\phi, \phi)$, but $\mu_b Int(\{a\}, \{2\}) \cap \mu_b Int(\{b\}, Y) = (\phi, \{2\})$.

3 Binary Generalized Continuity

In this section we define a new form of continuity called binary generalized continuity which is a map from a generalized topological space to a binary generalized topological space.

Definition 3.1. Let (X, Y, μ_b) be a binary generalized topological space and (Z, η) be a generalized topological space. A function $f : Z \rightarrow X \times Y$ is called binary generalized continuous at a point $z \in Z$ if for any binary generalized open set (U, V) in (X, Y, μ_b) with $f(z) \in (U, V)$ there exists a generalized open set G in (Z, η) such that $z \in G$ and $f(G) \subseteq (U, V)$. f is called binary generalized continuous if it is binary generalized continuous at each $z \in Z$.

Theorem 3.2. Let (X, Y, μ_b) be a binary generalized topological space and (Z, η) be a generalized topological space. A function $f : Z \rightarrow X \times Y$ is binary generalized continuous if and only if $f^{-1}(U, V)$ is generalized open in (Z, η) for every binary generalized open set (U, V) in (X, Y, μ_b) .

Proof. Let f be binary generalized continuous and (U, V) be a binary generalized open set in (X, Y, μ_b) . If $f^{-1}(U, V) = \phi$ then ϕ is generalized open. But if $f^{-1}(U, V) \neq \phi$ then let $z \in f^{-1}(U, V)$. Then $f(z) \in (U, V)$. Since f is binary generalized continuous at z , there exists a generalized open set G in (Z, η) such that $z \in G$ and $f(G) \subseteq (U, V)$. Hence, $z \in G \subseteq f^{-1}(U, V)$. Therefore, $f^{-1}(U, V)$ is generalized open in (Z, η) .

Conversely, to show that $f : Z \rightarrow X \times Y$ is binary generalized continuous, let $z \in Z$ and (U, V) be a binary generalized open set in (X, Y, μ_b) with $f(z) \in (U, V)$. Then $z \in f^{-1}(U, V)$ where $f^{-1}(U, V)$ is generalized open. Also, $f(f^{-1}(U, V)) \subseteq (U, V)$. Hence, f is binary generalized continuous at z . Therefore, f is binary generalized continuous. $\square$

Theorem 3.3. Let (X, Y, μ_b) be a binary generalized topological space and (Z, η) be a generalized topological space. Let $f : Z \rightarrow X \times Y$ be a function such that $Z - f^{-1}(A, B) = f^{-1}(X - A, Y - B)$ for all $A \subseteq X$ and $B \subseteq Y$. Then f is binary generalized continuous if and only if $f^{-1}(A, B)$ is generalized closed in (Z, η) for every binary generalized closed set (A, B) in (X, Y, μ_b) .

Proof. Let f be binary generalized continuous and (A, B) be binary generalized closed in (X, Y, μ_b) . Then, $(X - A, Y - B)$ is binary generalized open. Since f is binary generalized continuous, we have $f^{-1}(X - A, Y - B)$ is generalized open in (Z, η) . Therefore, $Z - f^{-1}(A, B)$ is generalized open. Hence, $f^{-1}(A, B)$ is generalized closed.

Conversely, let (U, V) be a binary generalized open set in (X, Y, μ_b) . Then, $(X - U, Y - V)$ is binary generalized closed. By assumption we have $f^{-1}(X - U, Y - V)$ is generalized closed in (Z, η) . Thus, $Z - f^{-1}(U, V)$ is generalized closed. Hence, $f^{-1}(U, V)$ is generalized open. This means that f is binary generalized continuous. $\square$

Lemma 3.4. [13] Let $f : Z \rightarrow X \times Y$ be a function. For $A \subseteq X$ and $B \subseteq Y$, we have $Z - f^{-1}(A, B) = f^{-1}(A, Y - B) \cup f^{-1}(X - A, B) \cup f^{-1}(X - A, Y - B)$.

Theorem 3.5. Let (X, Y, μ_b) be a binary generalized topological space and (Z, η) be a generalized topological space such that $(A, Y - B)$ and $(X - A, B)$ are binary generalized open sets in (X, Y, μ_b) whenever (A, B) is a binary generalized closed set in (X, Y, μ_b) . If $f : Z \rightarrow X \times Y$ is binary generalized continuous then $f^{-1}(A, B)$ is generalized closed in (Z, η) for every binary generalized closed set (A, B) in (X, Y, μ_b) .

Proof. Let f be binary generalized continuous and (A, B) be binary generalized closed in (X, Y, μ_b) . Then, $(X - A, Y - B)$ is binary generalized open. Since f is binary generalized continuous, we have $f^{-1}(X - A, Y - B)$ is generalized open in (Z, η) . Since $(A, Y - B)$ and $(X - A, B)$ are binary generalized open sets in (X, Y, μ_b) we have $f^{-1}(A, Y - B)$ and $f^{-1}(X - A, B)$ are generalized open in (Z, η) . Then, by Lemma 3.4, we have $Z - f^{-1}(A, B)$ is generalized open in (Z, η) . Hence, $f^{-1}(A, B)$ is generalized closed. $\square$

4 Some Other Types of Continuity in Binary Generalized Spaces

In this section we define and study some forms of binary generalized continuous functions and we investigate the relationships between these functions and their relationships with some other functions.

Definition 4.1. A subset (A, B) of a binary generalized topological space (X, Y, μ_b) is called

- (i) a binary generalized semiopen set if $(A, B) \subseteq \mu_b Cl(\mu_b Int(A, B))$.
- (ii) a binary generalized preopen set if $(A, B) \subseteq \mu_b Int(\mu_b Cl(A, B))$.
- (iii) a binary generalized α -open set if $(A, B) \subseteq \mu_b Int(\mu_b Cl(\mu_b Int(A, B)))$.
- (iv) a binary generalized regular open set if $(A, B) = \mu_b Int(\mu_b Cl(A, B))$.

The complement of a binary generalized semiopen (resp. binary generalized preopen, binary generalized α -open, binary generalized regular open) set is called binary generalized semiclosed (resp. binary generalized preclosed, binary generalized α -closed, binary generalized regular closed).

Remark 4.2. (1) Every binary generalized open set is binary generalized α -open.

- (2) Every binary generalized α -open set is binary generalized semiopen.
- (3) Every binary generalized α -open set is binary generalized preopen.
- (4) Every binary generalized regular open set is binary generalized open.

The converses of the statements in the above remark need not be true as shown in the following examples

Example 4.3. Let $X = \{a, b, c\}$ and $Y = \{1, 2\}$ with the binary generalized topology $\mu_b = \{(\phi, \phi), (\{a\}, \{2\}), (\{b\}, Y), (\{a, b\}, Y)\}$. Then, $(\{a\}, Y)$ is a binary generalized α -open set but not binary generalized open. Also the set $(\{a, c\}, \{2\})$ is a binary generalized semiopen set but not binary generalized α -open and the set $(\{a\}, \{1\})$ is a binary generalized preopen set but not binary generalized α -open.

Example 4.4. Let $X = \{a, b\}$ and $Y = \{1, 2, 3\}$ with the binary generalized topology $\mu_b = \{(\phi, \phi), (X, \phi), (X, \{1\}), (X, \{2\}), (X, \{1, 2\}), (\{a\}, \phi), (\{b\}, \phi)\}$. Then, $(X, \{1\})$ is a binary generalized open set but not binary generalized regular open.

Definition 4.5. Let (X, Y, μ_b) be a binary generalized topological space and (Z, η) be a generalized topological space. A function $f : Z \rightarrow X \times Y$ is called:

- (i) binary generalized semicontinuous if $f^{-1}(U, V)$ is generalized open in (Z, η) for every binary generalized semiopen set (U, V) in (X, Y, μ_b) .
- (ii) binary generalized precontinuous if $f^{-1}(U, V)$ is generalized open in (Z, η) for every binary generalized preopen set (U, V) in (X, Y, μ_b) .
- (iii) binary generalized α -continuous if $f^{-1}(U, V)$ is generalized open in (Z, η) for every binary generalized α -open set (U, V) in (X, Y, μ_b) .
- (iv) binary generalized regular continuous if $f^{-1}(U, V)$ is generalized open in (Z, η) for every binary generalized regular open set (U, V) in (X, Y, μ_b) .

Theorem 4.6. Every binary generalized α -continuous function is binary generalized continuous.

Proof. Let $f : Z \rightarrow X \times Y$ be a binary generalized α -continuous function and (U, V) be a binary generalized open set in (X, Y, μ_b) . Then (U, V) be binary generalized α -open. Since f is binary generalized α -continuous we have $f^{-1}(U, V)$ is generalized open in Z . Therefore, $f : Z \rightarrow X \times Y$ be binary generalized continuous. $\square$

Remark 4.7. The converse of the above theorem need not be true as shown in the following example.

Example 4.8. Let $X = \{a, b, c\}$ and $Y = \{1, 2\}$ with the binary generalized topology $\mu_b = \{(\phi, \phi), (\{a\}, \{2\}), (\{b\}, Y), (\{a, b\}, Y)\}$. Let $Z = \{p, q\}$ with the generalized topology $\eta = \{\phi, \{q\}, \{p, q\}\}$. Define a function $f : Z \rightarrow X \times Y$ by $f(p) = (a, 1)$ and $f(q) = (b, 1)$. Then, f is a binary generalized continuous function which is not binary generalized α -continuous since $(\{a\}, Y)$ is a binary generalized α -open set, but $f^{-1}(\{a\}, Y) = \{p\}$ is not generalized open.

The proofs of the following theorems are similar to the proof of Theorem 4.6.

Theorem 4.9. Every binary generalized semicontinuous function is binary generalized α -continuous.

Theorem 4.10. Every binary generalized precontinuous function is binary generalized α -continuous.

Theorem 4.11. Every binary generalized continuous function is binary generalized regular continuous.

Remark 4.12. The converses of the above theorems need not be true as shown in the following examples.

Example 4.13. Let $X = \{a, b, c\}$ and $Y = \{1, 2\}$ with the binary generalized topology $\mu_b = \{(\phi, \phi), (\{a\}, \{2\}), (\{b\}, Y), (\{a, b\}, Y)\}$. Let $Z = \{p, q, r\}$ with the generalized topology $\eta = \{\phi, \{p\}, \{q\}, \{p, q\}\}$. Define a function $f : Z \rightarrow X \times Y$ by $f(p) = (a, 2)$, $f(q) = (b, 1)$ and $f(r) = (c, 2)$. Then, f is a binary generalized α -continuous function which is not binary generalized semicontinuous since $(\{a, c\}, \{2\})$ is a binary generalized semiopen set, but $f^{-1}(\{a, c\}, \{2\}) = \{p, r\}$ is not generalized open.

Example 4.14. Let $X = \{a, b, c\}$ and $Y = \{1, 2\}$ with the binary generalized topology $\mu_b = \{(\phi, \phi), (\{a\}, \{2\}), (\{b\}, Y), (\{a, b\}, Y)\}$. Let $Z = \{p, q, r, s\}$ with the generalized topology $\eta = \{\phi, \{p\}, \{q\}, \{p, q\}, \{p, r\}, \{p, q, r\}\}$. Define a function $f : Z \rightarrow X \times Y$ by $f(p) = (a, 2)$, $f(q) = (b, 1)$, $f(r) = (a, 1)$ and $f(s) = (c, 2)$. Then, f is a binary generalized α -continuous function which is not binary generalized precontinuous since $(\{a\}, \{1\})$ is a binary generalized preopen set, but $f^{-1}(\{a\}, \{1\}) = \{r\}$ is not generalized open.

Example 4.15. Let $X = \{a, b\}$ and $Y = \{1, 2, 3\}$ with the binary generalized topology $\mu_b = \{(\phi, \phi), (X, \phi), (X, \{1\}), (X, \{2\}), (X, \{1, 2\}), (\{a\}, \phi), (\{b\}, \phi)\}$. Let $Z = \{p, q\}$ with the generalized topology $\eta = \{\phi, \{p, q\}\}$. Define a function $f : Z \rightarrow X \times Y$ by $f(p) = (a, 1)$ and $f(q) = (a, 2)$. Then, f is a binary generalized regular continuous function which is not binary generalized continuous since $(X, \{1\})$ is a binary generalized open set, but $f^{-1}((X, \{1\})) = \{p\}$ is not generalized open.

Theorem 4.16. Let (X, Y, μ_b) be a binary generalized topological space and (Z, η) be a generalized topological space. Let $f : Z \rightarrow X \times Y$ be a function such that $Z - f^{-1}(A, B) = f^{-1}(X - A, Y - B)$ for all $A \subseteq X$ and $B \subseteq Y$. Then f is binary generalized α -continuous if and only if $f^{-1}(A, B)$ is generalized closed in (Z, η) for every binary generalized α -closed set (A, B) in (X, Y, μ_b) .

Proof. The proof is obvious. □

Remark 4.17. The concepts of binary generalized semicontinuous functions and binary generalized precontinuous functions are independent of each others as shown in the following examples.

Example 4.18. Let $X = \{a, b\}$ and $Y = \{1, 2\}$ with the binary generalized topology $\mu_b = \{(\phi, \phi), (\{a\}, \{2\})\}$. Let $Z = \{p, q\}$ with the generalized topology $\eta = \{\phi, \{p\}\}$. Define a function $f : Z \rightarrow X \times Y$ by $f(p) = (a, 2)$ and $f(q) = (b, 1)$. Then, f is a binary generalized precontinuous function which is not binary generalized semicontinuous since $(\{a, b\}, \{1, 2\})$ is a binary generalized semiopen set, but $f^{-1}(\{a, b\}, \{1, 2\}) = \{p, q\}$ is not generalized open.

Example 4.19. Let $X = \{a, b, c\}$ and $Y = \{1, 2\}$ with the binary generalized topology $\mu_b = \{(\phi, \phi), (\{a\}, \{2\}), (\{b\}, Y), (\{a, b\}, Y)\}$. Let $Z = \{p, q, r\}$ with the generalized topology $\eta = \{\phi, \{p, q\}, \{p, q, r\}\}$. Define a function $f : Z \rightarrow X \times Y$ by $f(p) = (a, 2)$, $f(q) = (a, 2)$ and $f(r) = (a, 1)$. Then, f is a binary generalized semicontinuous function which is not binary generalized precontinuous since $(\{a\}, \{1\})$ is a binary generalized preopen set, but $f^{-1}(\{a\}, \{1\}) = \{r\}$ is not generalized open.

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