RESEARCH ARTICLE

The strut-and-tie model and the finite element - good design companions

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Abstract

The strut-and-tie method (STM) can serve as a tool for a safe design of concrete structures or members. It aids to trace the flow of forces, appropriately lay-out the reinforcement, and safely predict the structure capacity. On the other hand, the linear elastic finite element can be utilized as an alternative in the development of the strut-and-tie models besides the load path method. In addition, the nonlinear finite element analysis assists in the optimization of the design results obtained from the STM. Hence, the two methods work well as companions in structural design. In order to demonstrate such understanding, different examples which include a deep beam with large opening and recess, continuous deep beams with and without openings, and beam ledges, have been utilized. In the STM solutions, the ACI 318-14 failure criteria have been adopted. In the nonlinear finite element analysis, material nonlinearity has been accounted for. The obtained solutions from the two methods, along with the experimental data of the selected examples of this study, revealed the reliability of the STM in obtaining a safe solution. Besides, the nonlinear finite element proved to be an efficient tool in obtaining an economic design.

Keywords

Reinforced concrete design; Discontinuity regions; Strut-and-tie model; Nonlinear finite element analysis; Deep beams; Beam ledge

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1. Introduction

The method of the strut-and-tie model, STM, has been well established in both academia and practice for the treatment of discontinuity (D) regions; e.g., deep beams, openings, corbels, etc. [1,2]. The STM, is a logical extension of the truss model by Ritter [3] and Mörsch [4], and the major difference between the two methods is that the strut-and-tie model is a set of forces in equilibrium but do not form a stable truss system. Thus, the STM is a generalization of the truss model by Schlaich et al.

[2] and Schlaich and Schäfer [5]. The method has been adopted by many codes of practice such as the ACI 318-14 Code [6], the Eurocode 2 [7], the Canadian Standards Association [8], and the CEB-FIP Guidelines [9,10].

On the other hand, the implementation of the nonlinear finite element analysis, NFEA, in practice is very limited. Due to its very intensive computation efforts, the method can only serve as a benchmark for other methods or solutions. This makes the method of STM appealing in practice for both strength assessment and design.

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In order to achieve a good design, the STM can serve as a reliable design tool that aids to trace the flow of forces and achieve successful detailing. For the development of the STM, the load path method can be used in most of the cases; however, in some cases, it becomes easier and a straightforward matter if a linear elastic finite element analysis is employed. Since the STM is a lower-bound solution, the design obtained from this method is always on the safe side. For a given design, if optimization is required, the NFEA can serve this purpose. Thus, both the STM and the finite element work together as good design companions. This understanding is demonstrated in this paper by many examples. These examples include deep beams with large openings and recess, continuous deep beams with and without large openings, and beams with ledges.

In the STM solutions presented herein, the failure criteria of the ACI 318-14 Code [6], are adopted. As for the linear/nonlinear finite element analyses, the computer software ABAQUS [11] is utilized. In the NFEA, material nonlinearity is accounted for. Most of the examples discussed in this paper had been tested experimentally. Comparisons between the experimental data and the predicted results by the STM and the NFEA are performed. The comparisons revealed the reliability of the STM in obtaining a safe solution. Also, the NFEA proved to be an efficient tool in obtaining an economic design.

2. The strut-and-tie method

An STM consists of three types of elements; concrete struts (with or without reinforcement), ties with or without reinforcement, nodes, and nodal zones, Fig. 1. Any STM may fail by one of the following modes: yield or anchorage failure of the tension tie, one of the struts could crush when the stress in the strut exceeds the effective compressive strength of concrete, or a nodal zone could fail by being stressed beyond its capacity. In this paper, the failure criteria of the ACI 318-14 Code [6] are employed in the presented applications.

Fig. 1. Components of a strut-and-tie model of a deep beam

2.1. Strength of struts

According to the ACI 318-14 [6] the nominal compressive strength of a strut without longitudinal reinforcement, F_{ns} , shall be taken as

$$F_{ns} = f_{ce}^s A_{cs} \quad (1)$$

where f_{ce}^s is the smaller of:

- The effective compressive strength of the concrete in the strut.
- The effective compressive strength of the concrete in the nodal zone.

and A_{cs} is the cross-sectional area at one end of the strut. In calculating A_{cs} , the strut width is measured perpendicular to the strut axis at its end.

The effective compressive strength of the concrete in a strut f_{ce}^s can be obtained from

$$f_{ce}^s = 0.85 f_c' \beta_s \quad (2)$$

where f_c' is the concrete cylinder strength and β_s is an effectiveness factor of concrete. Table 1 shows the values of β_s according to the ACI 318-14 Code [6], in which the influence of the stress conditions, strut geometry, and the angle of cracking surrounding the strut on strength are reflected.

2.2. Nodal zones

Smeared nodes do not need to be checked if the reinforcement is properly developed until the extremities of the stress field. A check of concrete stresses in the singular nodes is necessary because of the stress concentration at these nodal regions. The singular nodes are divided into four major types, C-C-C, C-C-T, C-T-T, and T-T-T nodes, where C and T stand for compression and tension, respectively.

Table 1. ACI 318-14 code values of coefficient β_s for struts [6]

Strut condition	β_s
<ul style="list-style-type: none"> A strut with constant cross-section along its length 	1.0
<ul style="list-style-type: none"> For struts located such that the width of the midsection of the strut is larger than the width at the nodes (bottle-shaped struts) <ul style="list-style-type: none"> a) With reinforcement normal to strut center-line to resist the transverse tension b) Without reinforcement normal to the center-line of the strut 	0.75 0.6 λ
<ul style="list-style-type: none"> For struts in tension members, or the tension flanges of members 	0.40
<ul style="list-style-type: none"> For all other cases 	0.6 λ

The effective compressive strength of the concrete in a nodal zone, f_{ce}^n , can be obtained from:

$$f_{ce}^n = 0.85f_c'\beta_n \quad (3)$$

where β_n is the effectiveness factor of a nodal zone. Table 2 shows the effectiveness factor, β_n , for nodal zones according to the ACI 318-14 Code [6].

2.3. Reinforced ties

The nominal strength of a tie, F_{nt} , shall be taken as $F_{nt} = A_{st}f_y$; where A_{st} is the area of steel and f_y is the yield stress. The width of a tie w_t , is to be determined to guarantee node safety, with an upper limit $w_{t,max}$.

- In case of using one row of bars without sufficient development length beyond the nodal zones, Fig. 2a:

$$w_t = 0.0 \quad (4a)$$

- In case of using one row of bars and providing sufficient development length beyond the nodal zones for a distance not less than $2c$, where c is the concrete cover, Fig. 2b:

$$w_t = \phi_{bar} + 2c \quad (4b)$$

where ϕ_{bars} is the diameter of the reinforcing bars.

- In case of using more than one row of bars and providing sufficient development length beyond the nodal zones for a distance not less than $s/2$, where s is the clear distance between bars, Fig. 2c:

$$w_t = n\phi_{bar} + 2c + (n-1)s \quad (4c)$$

where n is the number of rows of reinforcing bars. In the three cases in Fig. 2, the development length

according to the ACI 318-14 [6], l_{anc} , begins at the intersection of the centroid of the bars in the tie and the extensions of the outlines of either the strut or the bearing area.

For the extended nodal zone, Figs. 2b and 2c, the upper limit of the strut width,

$$w_{t,max} = \frac{F_{nt}}{b f_{ce}^n}$$

3. Linear/Nonlinear finite element analysis

In this study, 3D linear and nonlinear finite element analyses have been performed using the computer software ABAQUS [11]. The constitutive models adopted in the modeling process are briefly described here. The displacement control approach has been employed to accurately predict the capacity and post-cracking behavior of the studied examples.

3.1. Finite elements of concrete and steel reinforcement

Herein, the 3D solid continuum element C3D8R has been employed to model concrete. This element is an 8-noded linear brick having 3 translational degrees of freedom at each corner node. The reinforcing steel has been defined as individual truss elements with steel material properties and cross-sections using T3D2 element which is a (2-node linear displacement) 3D truss element.

3.2. Concrete material modeling

The ‘‘Concrete damage plasticity model’’ (CDP) is utilized for concrete modeling which assumes that the main two failure modes are tensile cracking and compressive crushing [11].

Table 2. ACI 318-14 Code values of coefficient β_n for nodes

Nodal zone	β_n
For nodal zones bounded by struts or bearing areas or both, C-C-C node	1.00
Nodal zones anchoring one tie, C-C-T node	0.80
Nodal zones anchoring two or more ties with the presence of one strut, C-T-T node	0.60
Nodal zones anchoring ties only, T-T-T node	0.40

Fig. 2. The width of the tie, w_t , used to determine the dimensions of the nodal zone

In this paper, the uniaxial compressive stress-strain relationship of the concrete model, as shown in Fig. 3a is proposed by Saenz [12] as follows:

$$\sigma_c = \frac{E_c \varepsilon_c}{1 + (R + R_E - 2) \left(\frac{\varepsilon_c}{\varepsilon_0} \right) - (2R - 1) \left(\frac{\varepsilon_c}{\varepsilon_0} \right)^2 + R \left(\frac{\varepsilon_c}{\varepsilon_0} \right)^3} \quad (5a)$$

where σ_c is the compression stress at any ε_c , and ε_0 the strain at the ultimate stress.

$$R = \frac{R_E (R_\sigma - 1)}{(R_\varepsilon - 1)^2} - \frac{1}{R_\varepsilon} \quad (5b)$$

$$R_E = \frac{\varepsilon_c}{\varepsilon_0} \quad (5c)$$

$$E_0 = \frac{f'_c}{\varepsilon_0} \quad (5d)$$

where R is the ratio relation; R_E is the modular ratio; $R_\varepsilon = 4$ = the strain ratio; $R_\sigma = 4$ = the stress ratio as reported in [13]; E_0 is the secant modulus. Poisson's ratio for concrete is assumed to be 0.20.

The softening curve of concrete under tension is shown in Fig. 3b, where f_{ct} is the tensile strength, and G_f is the fracture energy of concrete which is used to specify the post-peak tension failure behavior of concrete. According to the ACI-318 [14], E_c and f_{ct} were calculated by:

$$E_c = 4700 \sqrt{f'_c} \quad (6)$$

$$f_{ct} = 0.35 \sqrt{f'_c} \quad (7)$$

3.3. Reinforcing steel modeling

The steel reinforcement has been modeled as an elastic-perfectly plastic material in both tension and compression.

3.4. Bond between steel and concrete

A perfect bond between concrete and reinforcement has been assumed in this study, which was achieved by using a constrain called Embedded Element. The reinforcing bar is embedded into the concrete host element; thus, all embedded element nodes have the same translational degrees of freedom as the concrete host element's nodes [6].

4. Deep beam with large opening and recess

4.1. Description

The deep beam in Fig. 4a with an abrupt change of thickness and a large opening was designed to carry a factored concentrated load of 2000.0 kN (or a nominal load 2667.0 kN) [15]. From the beam geometry, it is entirely considered as a D-region.

Fig. 3. Concrete modeling

This beam is selected for comparison between the two-dimensional strut-and-tie modeling and the 3D nonlinear finite element analysis. Fig. 4 shows the beam geometry, loading, and reinforcement details of the beam.

The cylinder compressive strength of concrete, $f'_c = 31$ MPa and the reinforcement is high strength deformed steel bars, with yield stress, $f_y = 414$ MPa. The beam height is as shown in Fig. 4a, breadth, $b = 305$ mm, the width of the bearing plate $b_1 = 600$ mm, and $b_2 = b_3 = 400$ mm.

4.2. Strut-and-tie modeling

4.2.1. Geometrical parameters

The solution procedure starts with the development of the strut-and-tie model. With the aid of the stress pattern obtained from the linear elastic analysis, it is obvious how the struts go around the opening to transfer their forces to the supports. The proposed STM is shown in Fig. 5a, and for the next illustrations, the details of selected nodes are given in Fig. 5b.

Concerning Figs. 5 and 4b, the reinforcement 4T32 of tie T_1 is assumed to be well anchored at nodes 3 and 4. Hence, the width of this tie (height of nodes 3 and 4) is given by

$$w_{T_1} = 2(c + \phi_{stirrup}) + n\phi_{bar} + (n - 1)s = 202 \text{ mm.}$$

If tie T_1 yields, its nominal strength will be

$$T_{1n} = A_s \times f_y = (4 \times 804.25) \times 414 = 1332 \text{ kN.}$$

In the same manner, for the 4T25 reinforcement of each of the ties T_2 and T_3 , the height of each tie at nodes 6 and 7 is 181 mm and the nominal strength is 813.0 kN.

Regarding Figs. 5 and 4b, the reinforcement of each of the ties T_4 and T_5 is 5T13 (2-legs) vertical stirrups T13, spaced at 450 mm center to center. The width of each of these ties is

$$w_{T_4} = w_{T_5} = n(\phi_{bar} + s) = 2315 \text{ mm.}$$

Each of the ties T_4 and T_5 has a nominal strength

$$T_{4n} = T_{5n} = A_s \times f_y = (2 \times 5 \times 132.7) \times 414 = 549.5 \text{ kN.}$$

Also, T_6 is represented by 9T13 (2-legs) vertical stirrups, spaced at 230 mm center to center. The nominal strength of the tie T_6 is

$$T_{6n} = A_s \times f_y = (2 \times 9 \times 132.7) \times 414 = 989.12 \text{ kN.}$$

The width of this tie is

$$w_{T_6} = n(\phi_{bar} + s) = 2187 \text{ mm}$$

In the same manner, each of the ties T_7, T_8, T_9 , and T_{10} are represented by 6T32, the width of these ties or the height of nodes 10, 11, 12, 13, and 14, is 266 mm and the nominal strength is 1997.75 kN.

Fig. 4. Deep beam with large opening and recess

Fig. 5. Proposed STM of the deep beam with large opening and recess

4.2.2. Reactions

From equilibrium, $V_1 = 0.67P$ and $V_2 = 0.33P$.

4.2.3. Model geometry and forces

To obtain the forces in the model elements, the reinforcement of a selected tie was assumed to reach its yield stress. The solution to this problem was initiated by assuming that the reinforcement of tie T_2 reached its yield stress, $T_2 = 813.0\text{kN}$. Then, from simple truss analysis, the model forces were obtained, Fig. 6. This led to an applied load $P = F_1 + F_2 = 2560\text{ kN}$. It is noted that this load value is less than the required nominal strength.

4.2.4. Effective concrete strength of the struts

The effective concrete strength of the strut

$$f_{ce}^s = 0.85f'_c\beta_s.$$

For prismatic strut

$$f_{ce}^s = 0.85 \times 31 \times 1.0 = 26.35\text{ MPa}.$$

For bottle-shaped strut with sufficient reinforcement to resist the transverse tension

$$f_{ce}^s = 0.85 \times 31 \times 0.75 = 19.76\text{ MPa}$$

This value will be used throughout the example, and in the end, it will be verified.

4.2.5. Effective concrete strength of the nodes

The effective concrete strength of the different nodes, $f_{ce}^n = 0.85f'_c\beta_n$, are

$$\text{For C-C-C node, } f_{ce}^n = 0.85 \times 31 \times 1.0 = 26.35\text{ MPa}$$

$$\text{For C-C-T node, } f_{ce}^n = 0.85 \times 31 \times 0.80 = 21.08\text{ MPa}$$

$$\text{For C-T-T node, } f_{ce}^n = 0.85 \times 31 \times 0.60 = 15.81\text{ MPa}$$

4.2.6. Check the bearing of the nodes

For nodes 1 and 2, the nominal value of the load P , $P_n = 26.35 \times 600 \times 305 = 4822.03\text{ kN}$, which is greater than the applied load, $P = 2560\text{ kN}$.

For node 10, the nominal value of the reaction V_2 , $V_{2n} = 21.08 \times 400 \times 305 = 2571.75\text{ kN}$, which is greater than the reaction, $V_2 = 853.3\text{ kN}$.

For node 14, the nominal value of the reaction V_1 , $V_{1n} = 21.08 \times 400 \times 305 = 2571.75\text{ kN}$, which is greater than the reaction, $V_1 = 1706.7\text{ kN}$.

4.2.7. Check of stresses

Node 1:

Since the bearing stress has been checked before, there is no need to check it again. The width of the tie T_{10} , $w_{T_{10}} = 266\text{mm}$. Assuming that the force in the prismatic strut C_0 is equal to its nominal strength; then, $C_0 = 1762.7\text{ kN} = 26.35 \times w_{C_0} \times 305$, which gives, $w_{C_0} = 219.3\text{ mm}$. For strut C_1 , its angle of inclination $\theta_1 = 36.0^\circ$, which gives a width of the strut, $w_{C_1}^1 = 300 \sin \theta_1 + 219.3 \cos \theta_1 = 353.75\text{ mm}$. Then, the nominal strength of the strut $C_{1n} = 19.76 \times 353.75 \times 305 = 2132\text{kN}$ (# the smaller of the node strength and the strut strength), which is less than the strut force, $C_1 = 2166.7\text{ kN}$; i.e., $C_{1n} = 0.984 C_1$.

Fig. 6. Computed model forces (kN) assuming yielding of tie T_2 reinforcement

Node 2:

Similar to node 1, the bearing stress is safe and the nominal strength of the strut C_2 , $C_{2n} = 2132$ kN. Since $C_2 = 2166.7$ kN; hence, $C_{2n} = 0.984 C_2$.

Nodes 3 and 4:

The two nodes need not be checked, since they are smeared nodes, and "the reinforcing bars passing through the nodes are extended beyond the nodal zones and they have sufficient anchorage length." Hence, the strength of the struts connected with these nodes is governed by either the strength of the other connecting nodes or the strength of these struts.

Node 5:

The width of the tie T_2 , $w_{T_2} = 181$ mm, and assuming that the width of the prismatic strut C_7 is equal to the width of the bearing plate at node 14, $w_{C_7}^5 = w_{C_7}^{14} = 400$ mm, and the angle of inclination of the strut C_3 , $\theta_3 = 58.0^\circ$, the width of the strut C_3 , $w_{C_3}^5 = 400 \sin \theta_3 + 181 \cos \theta_3 = 435.13$ mm. Then, the nominal strength of the strut, $C_{3n} = 19.76\# \times 435.13 \times 305 = 2622.47$ kN (# the smaller of the node strength and the strut strength), which is greater than the strut force, $C_3 = 1512.22$ kN.

The width of the vertical strut C_7 , $w_{C_7}^4 = 400$ mm. Hence, its nominal strength, $C_{7n} = 21.08\# \times 400 \times 305 = 2571.76$ kN (# the smaller of the node strength and the strut strength), which is greater than the strut force, $C_7 = 1278.3$ kN.

Node 6:

The width of the tie T_2 , $w_{T_2} = 181$ mm and of tie T_4 , $w_{T_4} = 2315$ mm, and the angle of inclination of the strut C_8 , $\theta_8 = 34.0^\circ$; Hence; the width of the strut C_8 , $w_{C_8}^6 = 2315 \sin \theta_8 + 181 \cos \theta_8 = 1444.6$ mm. Then, the nominal strength of the strut $C_{8n} = 15.81\# \times 1444.6 \times 305 = 6965.93$ kN (# the smaller of the node strength and the strut strength), which is greater than the strut force, $C_8 = 761.8$ kN.

Node 7:

Similar to node 6, $C_{9n} = 6965.93$ kN, greater than $C_9 = 761.8$ kN. For strut C_5 , $w_{C_5}^7 = 181$ mm, the nominal strength of the strut $C_{5n} = f_{ce}^n \times w_{C_5}^7 \times b + A_s' f_y = (15.81 \times 181 \times 503) + (4 \times 490.87 \times 414) =$

2252.27 kN, which is greater than the strut force, $C_5 = 238.1$ kN.

Node 8:

This is a smeared node and hence there is no need to check it. Hence, the strength of the struts connected with this node is governed by either the strength of the other connecting nodes or the strength of these struts.

Node 9:

This is also a smeared node and hence there is no need to check it, provided that the reinforcement of the tie T_6 is adequately anchored.

Node 10:

Since the bearing stress has been checked before, there is no need to check it again. The width of the tie T_{10} , $w_{T_{10}} = 266$ mm. For strut C_{12} , its inclination angle $\theta_{12} = 42.0^\circ$, which gives a width of the strut $w_{C_{12}}^{10} = 400 \sin \theta_{12} + 266 \cos \theta_{12} = 465.3$ mm. Then, the nominal strength of the strut $C_{12n} = 19.76\# \times 465.3 \times 305 = 2804.4$ kN (# the smaller of the node strength and the strut strength), which is greater than the strut force, $C_{12} = 1273.2$ kN.

Nodes 11, 12, 13, and 14:

These nodes have been checked in the same manner and they have sufficient strength.

4.2.8. Results

From the obtained results the model elements are all safe except elements C_1 and C_2 (similar elements) $C_{1n} = C_{2n} = 0.984 C_1 = 0.984 C_2$. Therefore, the load P should be reduced to 98.4% of the calculated value; i.e., the predicted collapse load from the STM, $P_{STM} = 0.984 \times 2560 = 2519$ kN.

Upon checking the skin reinforcement, it is found that all the bottle-shaped struts have adequate transverse reinforcement to resist the transverse tension, as assumed.

4.3. Nonlinear finite element analysis

Both linear and nonlinear analyses have been conducted for the specimen under the applied concentrated load only. The right support is prevented from translation in the y- and x- directions, but the left support is prevented from

translation in the y-direction only. The objective of the linear analysis was to obtain the stress trajectories that guide to construct of the strut-and-tie model. On the other hand, the nonlinear analysis has been performed to obtain the displacement,

strains, and stresses at different load levels. The stress trajectories obtained from the linear elastic analysis are illustrated in Fig. 7a and the results obtained from the NFEA are presented in Figs. 7b to 7d.

Fig. 7. Finite element results of the deep beam with large opening and recess

As observed from the yielding stress zones of the reinforcement and the concrete plastic strain

distribution in Figs. 7b and 7c, respectively, the failure mode of the beam is a concrete crushing

failure (shear failure). The main cause of failure was the crushing of the struts below the loading plate and the yielding of the web reinforcement, which indicates that the part of the beam above the opening behaves as a deep beam. The maximum concrete compressive stress in the beam near failure was 30 MPa. Neither the beam bottom nor the top longitudinal bars below and above the opening reached their yield stress; they reached 300 MPa and 200 MPa, respectively. Fig. 7d shows the load-displacement curve, where the initial part of the curve is approximately linear, indicating the elastic range under a service load of 1334 kN. The curve revealed a gentle slope after the initial linear portion and exhibited some ductility at failure. These results comply with the prediction of the strut-and-tie model. The predicted failure load from the NFEA is 2437 kN.

4.4. Predicted capacity and required nominal strength

The strength predicted by the STM and the NFEA is given in Table 3 along with the nominal load. The prediction from the STM, as expected, is on the safe side. However, the results from the NFEA are closer to the nominal strength.

5. Continuous deep beams

5.1. Description

Three full-size normal strength reinforced concrete continuous deep beams (CDB1, CDB2, and CDB3)

subjected to concentrated loads at the middle of spans that had been tested by [16] have been selected for comparison between the 2D strut-and-tie modeling and the 3D nonlinear finite element analysis. Only the details of the strut-and-tie modeling and the finite element solutions of beam CDB2 are presented here and a summary of the results of the other two beams, CDB1 and CDB3, is given at the end of this section. The details of the reinforcement of the tested deep beams are shown in Table 4.

The beam height, $h = 625$ mm, depth, $d = 585$ mm, breadth, $b = 120$ mm, the width of bearing plates $b_1 = 120$ mm, and $b_2 = b_3 = 250$ mm. The shear span, $a = 660$ mm. The shear span-to-depth ratio, $a/d = 660/585 = 1.12$. The bottom tension steel, $A_{s1} = 452$ mm² and the top tension steel $A_{s2} = 609$ mm². The cylinder compressive strength of concrete, $f'_c = 33.7$ MPa, the yield stress of longitudinal steel, $f_y = 480$ MPa, the vertical web reinforcement is $A_{sv} = 15 \times 2R8$, and the horizontal web reinforcement is $A_{sh} = 2 \times 2R8$. The web steel yield stress of mild steel bar R8 is $f_{yv} = f_{yh} = 370$ MPa.

5.2. Strut-and-tie modeling of beam CDB2

Fig. 8 shows the geometry and reinforcement details of the deep beam CDB2, whereas Fig. 9 illustrates the details of its strut-and-model.

Table 3. Strength of the deep beam with large opening and recess

P_{STM} (kN)	P_{NFEA} (kN)	P_n (kN)	P_{STM} / P_n	P_{NFEA} / P_n
2519	2437	2667	0.94	0.91

Table 4. Details of considered continuous deep beams

Beam	f'_c (MPa)	Shear span-to-depth ratio (a/d)	Main longitudinal bars		Web reinforcement	
			Bottom	Top	Horizontal	Vertical
CDB1	30.6	1.12	4T12	4T12 + 2 T10	8R8	29R8
CDB2	33.7	1.12	4T12	4T12 + 2 T10	4R8	15R8
CDB3	22.4	1.12	4T12	4T12 + 2 T10	4R8	—

Fig. 8. Reinforcement details and geometry of the beam CDB2

Fig. 9. Details of STM for the continuous deep beam CDB2

5.2.1. Geometrical parameters

With reference to Fig. 9, the solution procedure starts with assuming some geometrical parameters. Since the reinforcement detailing allows the use of extended nodal zones, the height of node 1, w_{T1} , $w_{T1} = 2(t - d) = 2(625 - 585) = 80$ mm, or

$$w_{T1} = 2(c + \phi_{stirrup}) + n\phi_{bar} + (n - 1)s$$

The height of the tie in the upper node, node 2, w_{T2} ,

$$w_{T2} = 2(c + \phi_{stirrup}) + n\phi_{bar} + (n - 1)s = 90 \text{ mm}$$

The lever arm L_d is

$$L_d = d - 0.5 \times w_{T2} = 585 - 0.5 \times 90 = 540 \text{ mm}$$

The angle

$$\alpha_1 = \tan^{-1} \frac{L_d}{a - 0.25 b_2} = \tan^{-1} \frac{540}{400 - 0.25 \times 100} = 42.17^\circ$$

The angle

$$\alpha_2 = \tan^{-1} \frac{L_d}{a - 0.25 (b_2 + b_3)} = \tan^{-1} \frac{540}{400 - 0.25 (100 + 100)} = 45.27^\circ$$

5.2.2. Reactions

From the linear elastic analysis of the beam, the reactions, $V_1 = 0.313P$ and $V_2 = 1.375P$.

5.2.3. Model geometry and forces

With reference to Fig. 9a and assuming that the tension reinforcement A_{s1} of tie T_1 and A_{s2} of tie T_2 reach their yield stress,

$$T_1 = A_s \times f_y = 480 \times (4 \times 113) = 217 \text{ kN}$$

$$T_2 = A_s \times f_y = 480 \times (4 \times 113 + 2 \times 78.5) = 292.4 \text{ kN}$$

$$\text{The strut force } C_1 = T_1 / \cos \alpha_1 = 292.5 \text{ kN}$$

$$\text{The reaction } V_1 = C_1 \sin \alpha_1 = 196.1 \text{ kN}$$

Since $V_1 = 0.3125P$ from linear elastic analysis, the load P is then $P = V_1 / 0.3125 = 627.7 \text{ kN}$. Then, $V_2 = 1.375P = 863 \text{ kN}$.

From the equilibrium of node 3, the strut force $C_2 = 0.5V_2 / \sin \alpha_2 = 607.4 \text{ kN}$.

From the equilibrium of node 2, the tie force $T_2 = C_2 \cos \alpha_2 - C_1 \cos \alpha_1 = 210.5 \text{ kN}$, which is less than the previously calculated value assuming yielding of the tie reinforcement.

5.2.4. Effective concrete strength of the struts

The effective concrete strength of strut, $f_{ce}^s = 0.85f'_c\beta_s$.

For a prismatic strut,

$$f_{ce}^s = 0.85 \times 33.7 \times 1.0 = 28.645 \text{ MPa.}$$

For a strut of bottle-shaped stress field,

$$f_{ce}^s = 0.85 \times 33.7 \times 0.6^* = 17.187 \text{ MPa, (* to be verified).}$$

5.2.5. Effective concrete strength of the struts

The effective concrete strength of a node,

$$f_{ce}^n = 0.85f'_c\beta_n.$$

For C–C–C node, $f_{ce}^n = 0.85 \times 33.7 \times 1.0 = 28.645 \text{ MPa}$

For C–C–T node, $f_{ce}^n = 0.85 \times 33.7 \times 0.80 = 22.916 \text{ MPa}$

5.2.6. Check the bearing of the nodes

For node 1, the nominal value of the reaction $V_{1n} = 22.916 \times 120 \times 120 = 330.0 \text{ kN}$, which is greater than the force V_1 , $V_1 = 196.1 \text{ kN}$.

For node 2, the nominal value of the load $P_n = 22.916 \times 120 \times 250 = 687.5 \text{ kN}$, which is greater than the force P , $P = 627.7 \text{ kN}$.

For node 3, the nominal value of the reaction $V_{2n} = 22.916 \times 120 \times 250 = 687.50 \text{ kN}$, less than the force V_2 , $V_2 = 863.0 \text{ kN}$. Therefore, the force V_2 should be reduced to the value of V_{2n} ; i.e., $V_2 = 687.5 \text{ kN}$.

Hence, the load P is reduced to $P = V_1 + 0.5V_2 = 539.85 \text{ kN}$ and the strut force C_2 is subsequently reduced to $C_2 = 483.9 \text{ kN}$. The force in the tie T_2 is further reduced to $T_2 = 123.6 \text{ kN}$.

5.2.7. Refinement of the model geometry and forces

Regarding Fig. 9b, the height of the tension tie T_2 , w_{T_2} at node 2 should not exceed a value $w_{T_2 \max} = T_2 / f_{ce}^n b = 123.6 \times 1000 / 22.916 \times 120 = 44.94 \text{ mm}$; therefore, consider $w_{T_2} \approx 45 \text{ mm}$. The width of the prismatic strut C^* , in Fig. 9c, w_{C^*} , can be obtained as $C^* = T_1 = 217 \text{ kN} = f_{ce}^s \times w_{C^*} \times b = 28.645 \times w_{C^*} \times 120$, giving $w_{C^*} = 63.13 \text{ mm} \approx 66 \text{ mm}$.

The strut C_2 is split into two components C_{21} and C_{22} . This will lead to the detailed STM shown in Fig. 9c, with geometrical relations and forces calculated upon applying equilibrium at the nodes.

First Trial STM:

Regarding Fig. 9c, the solution will start by assuming that the width of the force $N_{21} = V_1 \times 250 / P = 196.1 \times 250 / 539.85 \approx 92 \text{ mm}$ and the width of force $N_{22} = N_{23} = 79 \text{ mm}$. Also, the width of either force V_{21} or V_{22} , is $250 / 4 = 62.5 \text{ mm}$. Thus, the values of $\alpha_1 = 41.1^\circ$, $\alpha_{21} = 42.17^\circ$, and $\alpha_{22} = 42.63^\circ$ and the force $C^* = T_1 = 217 \text{ kN}$, then, from horizontal equilibrium at the nodes, $C_1 = 288 \text{ kN}$ and $C_{21} = 292.8 \text{ kN}$. Next, from vertical equilibrium $N_{21} = V_1 = 189.3 \text{ kN}$ and $N_{22} = V_{21} = 196.6 \text{ kN}$. The remaining value of the load is transferred to $N_{23} = V_{21} = 0.5V_2 - V_{21} = 147.15 \text{ kN}$, then, the force $C^{**} = T_2 = 159.9 \text{ kN}$ and $C_{22} = 217.3$

kN is obtained from horizontal equilibrium and $P = N_{21} + N_{22} + N_{23} = 533$ kN. Then, a better estimate of the height of the tension tie T_2 , w_{T_2} , at node 2 can be calculated, $w_{T_2 \max} = T_2 / f_{ce}^n b = (159.9 \times 1000) / (22.916 \times 120) = 58.1$ mm; therefore, consider $w_{T_2} \approx 62$ mm. The STM will be updated leading to the second trial STM.

Second Trial STM:

As illustrated before, the width of the force $N_{21} = 189.3 \times 250 / 533 \approx 90$ mm, the width of the force $N_{22} = 196.6 \times 250 / 533 \approx 94$ mm, and the width of force $N_{23} = 66$ mm. Also, the width of the force $V_{21} = 196.6 \times 250 / 2(196.6 + 147.15) \approx 72$ mm and the width of the force $V_{22} = 53$ mm. Finally, the values of $\alpha_1 = 40.2^\circ$, $\alpha_{21} = 41.48^\circ$, and $\alpha_{22} = 41.73^\circ$, are obtained and the force $C^* = T_1 = 217$ kN. Then, from horizontal equilibrium at the nodes, $C_1 = 284.1$ kN and $C_{21} = 289.6$ kN. Next, from vertical equilibrium, $N_{21} = V_1 = 183.4$ kN and $N_{22} = V_{21} = 191.9$ kN. The remaining part of the load is transferred to $N_{23} = V_{21} = 151.9$ kN, then, the force $C^{**} = T_2 = 170.3$ kN and $C_{22} = 228.2$ kN can be obtained from horizontal equilibrium and $P = N_{21} + N_{22} + N_{23} = 527.2$ kN.

5.2.8. Check of stresses

Node 1:

Since the bearing stress has been checked before, there is no need to check it again. For strut C_1 , the inclination angle $\alpha_1 = 40.2^\circ$, and $w_{C_1}^1 = 120 \sin \alpha_1 + 80 \cos \alpha_1 = 138.6$ mm. Then, the nominal strength of the strut is $C_{1n} = 17.187\# \times 120 \times 138.6 = 285.8$ kN (# the smaller of the node strength and the strut strength), which is greater than the strut force.

Node 2:

Since the bearing stress has been checked before, there is no need to check it again.

For sub-node 21, the width of the strut C_1 , $w_{C_1}^{21} = 90 \sin \alpha_1 + 66 \cos \alpha_1 = 108.5$ mm. Then, the nominal strength of the strut is $C_{1n} = 17.187\# \times 120 \times 108.5 = 223.7$ kN (# the smaller of the node strength and the strut strength), which is less than the strut force, $C_{1n} = 0.787C_1$. There is no need to check the prismatic strut C^* since this was

considered during the estimate of the height of this sub-node.

For sub-node 22, the width of the strut C_{21} , $w_{C_{21}}^{22} = 94 \sin \alpha_{21} + 66 \cos \alpha_{21} = 111.7$ mm. Then, the nominal strength of the strut is $C_{21n} = 17.187\# \times 120 \times 111.7 = 230.4$ kN (# the smaller of the node strength and the strut strength), which is less than the strut force, $C_{21n} = 0.795C_{21}$.

For sub-node 23, the width of the strut C_{22} , $w_{C_{22}}^{23} = 66 \sin \alpha_{22} + 62 \cos \alpha_{22} = 90.2$ mm. Then, the nominal strength of the strut is $C_{22n} = 17.187\# \times 120 \times 90.2 = 186$ kN (# the smaller of the node strength and the strut strength), which is less than the strut force, $C_{22n} = 0.815C_{22}$.

Node 3:

For sub-node 31, the width of the strut C_{21} , $w_{C_{21}}^{31} = 72 \sin \alpha_{21} + 80 \cos \alpha_{21} = 107.6$ mm. Then, the nominal strength of the strut is $C_{21n} = 17.187\# \times 120 \times 107.6 = 222$ kN (# the smaller of the node strength and the strut strength), which is less than the strut force, $C_{21n} = 0.766C_{21}$.

For sub-node 32, the width of the strut C_{22} , $w_{C_{22}}^{32} = 53 \sin \alpha_{22} + 62 \cos \alpha_{22} = 81.5$ mm. Then, the nominal strength of the strut is $C_{22n} = 17.187\# \times 120 \times 81.5 = 168.2$ kN (# the smaller of the node strength and the strut strength), which is less than the strut force, $C_{22n} = 0.737C_{22}$.

5.2.9. Results

The nominal strength of the strut C_2 that consists of C_{21n} and C_{22n} is $[(0.766 \times 289.6) + (0.737 \times 228.2)] = 390$ kN. According to the ACI 318-14 [6], the resultant force is assumed to spread at a slop 2 longitudinally and 1 transverse. Hence, the sum of the transverse forces of the strut is 195 kN. The skin reinforcement of the beam within the strut length is 2 horizontal bars per face, which can resist a force of $2 \times 2 \times 50.26 \times 370 \times \sin 41.48 = 49.3$ kN perpendicular to the crack and 3 vertical bars per face, which can resist a force of $2 \times 3 \times 50.26 \times 370 \times \sin 48.52 = 83.6$ kN perpendicular to the crack. This makes a total of $49.3 + 83.6 = 132.9$ kN, which is less than the required force, 195 kN, which indicates that the transverse reinforcement doesn't satisfy the strut requirement. Hence, the assumed

value $\beta_s = 0.60$ in the calculation procedure is correct.

The load $P = N_{21} + N_{22} + N_{23} = 527.2$ kN, which is less than the nominal value at the loading nodes, which is also safe from bearing, $P_n = 687.5$ kN. From the obtained results the critical members are $C_{1n} = 0.787C_{11}$, $C_{21n} = 0.766C_{21}$, and $C_{22n} = 0.737C_{22}$, while the other members attain nominal strength greater than their forces. Therefore, the load components should be reduced to a value of $P = 0.787 \times 183.4 + 0.766 \times 191.9 + 0.737 \times 151.9 = 403.3$ kN. Thus, the STM nominal strength, $2P = 806.6$ kN. Since the measured collapse load was $2P = 950$ kN; then, the STM solution is 85% of the value measured in the test.

5.3. Nonlinear finite element analysis of deep beam CDB2

Beam CDB2 has been analyzed under the two applied concentrated loads at mid-spans and only the middle support is prevented from translation in the y- and x-directions, but the external supports are prevented from translation in y-direction only. Both linear and nonlinear analyses have been performed for the beam. The stress trajectories obtained from the linear elastic analysis are illustrated in Fig. 10a and the results obtained from the nonlinear analysis are presented in Figs. 10b-d.

The results of the NFEA, Figs. 10b and 10c, illustrate a major diagonal crack in the intermediate shear span of the beam, as in the experimental observations, Fig. 10d. The failure occurred in the interior shear spans between the loading points and middle support. The maximum concrete compressive stresses in the beam CDB2 near failure were 33MPa. Neither the bottom nor the top longitudinal bars reached yield; they were 360MPa and 150MPa, respectively; but the vertical web reinforcement almost reached its yield strength. The predicted failure load from the 3D nonlinear finite element analysis was $2P = 860$ kN, which is about 91% of the measured value in the test.

5.4. Predicted capacity and test

The strength predicted by the STM and the NFEA is given in Table 5 along with the experimental values. The prediction from the STM, as expected, is on the safe side. On the other hand, the results from the NFEA are closer to the measured strength.

6. Continuous deep beams with large openings

6.1. Description

Two full-scale continuous reinforced concrete deep beams with large openings, specimens A and B, subject to two concentrated loads at mid-spans tested by [17] were selected for the comparison between the two-dimensional strut-and-tie modeling and the 3D nonlinear finite element analysis, Figs. 11 and 12. The two specimens have identical overall dimensions and loading arrangement, albeit, different sized openings. The opening is 200×200 mm in specimen A and 400×200 mm in specimen B. Only specimen B is presented here with details and the results of the other beam will be given at the end of this section.

With reference to Fig. 12, the beam height, $h = 750$ mm, depth, $d = 705$ mm, breadth, $b = 180$ mm, the width of bearing plates $b_1 = 150$ mm, and $b_2 = b_3 = 200$ mm. The shear span, $a = 1000$ mm. The shear span-to-depth ratio, $a/d = 1000/705 = 1.41$. The tension steel, $A_{s1} = A_{s2} = 628.3$ mm². The concrete cylinder strength, $f'_c = 27.7$ MPa. High strength deformed steel bars, with yield stress, $f_y = 460$ MPa, and mild steel bar with a nominal yield stress, $f_y = 250$ MPa. The vertical and horizontal web reinforcement is as shown in the figures.

In addition to the reinforcements required at the tie positions, confining reinforcement was incorporated to improve the strength of nodes where potentially high compressive stresses were encountered. Confining reinforcement was used below the concentrated loads, above the supports, and around the openings, Fig. 12.

Fig. 10. Finite element results of the continuous CDB2

Table 5. Strength of continuous deep beam examples

Specimen	f'_c (MPa)	P_{STM} (kN)	P_{NFEA} (kN)	P_{Test} (kN)	P_{STM} / P_{Test}	P_{NFEA} / P_{Test}
CDB1	30.6	455.8	500	550	0.83	0.91
CDB2	33.7	403.3	430	475	0.85	0.91
CDB3	22.4	260.3	260	285	0.91	0.91

To eliminate potentially the detrimental effects of apparently inadequate reinforcing details on load-carrying capacity, the reinforcing bars passing the nodes were extended beyond the nodal zones and the bottom longitudinal reinforcing bars were extended into the end supports and were anchored using standard 180-degree hooks.

6.2. Strut-and-tie modeling of specimen B

6.2.1. Geometrical parameters

From the stress pattern obtained from the linear elastic analysis, Fig. 15a, it is obvious how the struts go around the openings to transfer their forces to the supports. In addition, the connecting ties maintaining equilibrium are logical and adhere to the reinforcement details. Thus, developing an *STM* of the beam becomes a straightforward matter. For, the proposed *STM*, shown in Fig. 13, the numbering of ties and struts is illustrated on the left part of the model and the numbering of the nodes is illustrated on the right part.

With reference to Fig. 13, the reinforcement of each of the ties T_1 , T_2 , and T_{13} are 2T20. Since the reinforcement details allow the use of extended nodal zones, the width of each of these ties, the height of nodes 1, 2, 11 and 15, are as follows,

$$w_{T_1} = w_{T_2} = 2(t - d) = 2(750 - 705) = 90\text{mm},$$

or

$$w_{T_1} = w_{T_2} = 2(c + \phi_{\text{stirrup}}) + n\phi_{\text{bar}} + (n - 1)s$$

Each of the ties T_1 , T_2 , and T_{13} has a nominal strength, $T_{1n} = T_{2n} = T_{13n} = 460 \times 628.3 = 289 \text{ kN}$. Each of the ties T_3 , T_6 , T_8 and T_{11} is represented by 2T13 (2-legs) vertical stirrups, spaced 90 mm center to center. The width of each of these ties is

$$w_{T_3} = w_{T_6} = \dots = 2c + n\phi_{\text{bar}} + (n - 1)s = 166 \text{ mm}.$$

The nominal strength of each of the ties T_3 , T_6 , T_8 and T_{11} is

$$T_{3n} = T_{6n} = \dots = A_s \times f_y = (2 \times 2 \times 132.7) \times 460 = 144.4 \text{ kN}.$$

In the same manner, each of the ties T_4 , T_5 , T_7 , T_9 , T_{10} and T_{12} are represented by (2T13 + 2T10) horizontal bars with width 143 mm and nominal strength 194.2 kN.

6.2.2. Reactions

From linear elastic analysis, $V_1 = 0.3125P$ and $V_2 = 1.375P$.

6.2.3. Model geometry and forces

In order to obtain the forces in the model elements, the reinforcement of a selected tie is assumed to reach its yield stress. The choice of the tie to start with is up to the designer; however, some elements come to mind at the first glance; e.g., ties T_2 or T_{13} , Fig. 13. Nevertheless, the solution to this problem is initiated by assuming that the reinforcement of tie T_{10} reaches its yield stress, $T_{10} = 194.2 \text{ kN}$. Then, from simple truss analysis, the model forces are obtained, Fig. 14.

Fig. 13. Proposed strut-and-tie model for specimen B

Fig. 14. Computed model forces (kN) assuming yielding of T_{10} reinforcement

6.2.4. Effective concrete strength of the struts

The effective concrete strength of strut,

$$f_{ce}^s = 0.85 f_c' \beta_s$$

For prismatic strut,

$$f_{ce}^s = 0.85 \times 27.7 \times 1.0 = 23.55 \text{ MPa}$$

For bottle-shaped strut with sufficient reinforcement to resist the transverse tension,

$$f_{ce}^s = 0.85 \times 27.7 \times 0.75 = 17.66 \text{ MPa}$$

This value will be used throughout the example and it will be checked later.

6.2.5. Effective concrete strength of the nodes

The effective concrete strength of a node,

$$f_{ce}^n = 0.85 f_c' \beta_n.$$

For C – C – C node,

$$f_{ce}^n = 0.85 \times 27.7 \times 1.0 = 23.55 \text{ MPa.}$$

For C – C – T node,

$$f_{ce}^n = 0.85 \times 27.7 \times 0.80 = 18.84 \text{ MPa.}$$

For C – T – T node,

$$f_{ce}^n = 0.85 \times 27.7 \times 0.60 = 14.13 \text{ MPa}$$

6.2.6. Check the bearing of the nodes

For node 1, the nominal value of the reaction V_1 , $V_{1n} = 18.84 \times 150 \times 180 = 508.6 \text{ kN}$, which is greater than the reaction, $V_1 = 186.8 \text{ kN}$.

For nodes 8 and 9, the nominal value of the load P , $P_n = 23.55 \times 200 \times 180 = 847.6 \text{ kN}$, which is greater than the load, $P = 598 \text{ kN}$.

For node 16, the nominal value of the reaction V_2 , $V_{2n} = 23.55 \times 200 \times 180 = 847.6 \text{ kN}$, which is greater than the reaction, $V_2 = 822.4 \text{ kN}$.

6.2.7. Check of stresses

Node 1:

Since the bearing stress has been checked before, there is no need to check it again. The inclination angle of the strut C_1 , $\theta_1 = 70.3^\circ$, which gives a width of the strut $w_{C_1}^1 = 150 \sin \theta_1 + 90 \cos \theta_1 = 171.5 \text{ mm}$. Then, the nominal strength of the strut $C_{1n} = 17.66 \# \times 171.5 \times 180 = 545.1 \text{ kN}$ (# the smaller of the node strength and the strut strength), which is greater than the strut force, $C_1 = 198.4 \text{ kN}$.

Node 2:

The width of the tie T_2 , $w_{T_2} = 90 \text{ mm}$ and of tie T_3 , $w_{T_3} = 166 \text{ mm}$, and the angle of inclination of the strut C_4 , $\theta_4 = 25.0^\circ$. Hence, the width of the strut C_4 , $w_{C_4} = 166 \sin \theta_4 + 90 \cos \theta_4 = 151.7 \text{ mm}$. Then, the nominal strength of the strut $C_{4n} = 14.13 \# \times 151.7 \times 180 = 385.7 \text{ kN}$ (# the smaller of the node strength and the strut strength), which is greater than the strut force, $C_4 = 189.2 \text{ kN}$.

Nodes 3, 7, 10, and 14:

These nodes need not be checked, since they are smeared nodes, and "the reinforcing bars passing the nodes are extended beyond the nodal zones and they have sufficient anchorage length." Besides, the strength of the struts connected with these nodes is

governed by either the strength of the other connecting nodes or the strength of these struts.

Node 4:

The width of the tie T_3 , $w_{T_3} = 166$ mm, and of tie T_5 , $w_{T_5} = 143$ mm, and the angle of inclination of the strut C_2 , $\theta_2 = 50.2^\circ$. Hence, the width of the strut C_2 , $w_{C_2}^4 = 166 \sin \theta_2 + 143 \cos \theta_2 = 219$ mm. Then, the nominal strength of the strut, $C_{2n} = 14.13\# \times 219 \times 180 = 556.8$ kN (# the smaller of the node strength and the strut strength), which is greater than the strut force, $C_2 = 243.2$ kN.

For strut C_3 , the angle of inclination is $\theta_3 = 25.0^\circ$. Then, the width of the strut, $w_{C_3}^4 = 166 \sin \theta_3 + 143 \cos \theta_3 = 199.7$ mm. Then, the nominal strength of the strut, $C_{3n} = 14.13\# \times 199.7 \times 180 = 508$ kN (# the smaller of the node strength and the strut strength), which is greater than the strut force, $C_3 = 252.6$ kN.

Node 5:

As for nodes 1, 2, and 4, $w_{T_6} = 166$ mm, $w_{T_7} = 143$ mm, $\theta_4 = 25.0^\circ$, $w_{C_4}^5 = 199.7$ mm, $C_{4n} = 508$ kN, greater than $C_4 = 189.2$ kN. Also, $\theta_5 = 54.0^\circ$, $w_{C_5}^5 = 218.4$ mm, $C_{5n} = 555.2$ kN, greater than $C_5 = 231.2$ kN.

Node 6:

For this node, assuming that the force in the prismatic strut C_7 is equal to its nominal strength; then, $C_7 = 228.9$ kN $= 23.55 \times w_{C_7} \times 180$, which gives $w_{C_7} = 54$ mm. The width of the tie T_6 , $w_{T_6} = 166$ mm and the angle of inclination of the strut C_3 , $\theta_3 = 25.0^\circ$. Hence, the width of the strut C_3 , $w_{C_3}^6 = 166 \sin \theta_3 + 54 \cos \theta_3 = 119.1$ mm. Then, the nominal strength of the strut, $C_{3n} = 17.66\# \times 119.1 \times 180 = 378.6$ kN (# the smaller of the node strength and the strut strength), which is greater than the strut force, $C_3 = 252.6$ kN.

Node 8:

Since the bearing stress has been checked before, there is no need to check it again. The width of the prismatic strut C_8 can be obtained upon assuming that the force in the strut is equal to its nominal strength; then, $C_8 = 273.3$ kN $= 23.55 \times w_{C_8} \times 180$,

which gives $w_{C_8} = 64.5$ mm. The angle of inclination of the strut C_6 is $\theta_6 = 76.6^\circ$. Hence, the width of the strut C_6 , $w_{C_6}^8 = 100 \sin \theta_6 + 64.5 \cos \theta_6 = 112.2$ mm. Then, the nominal strength of the strut, $C_{6n} = 17.66\# \times 112.2 \times 180 = 356.6$ kN (# the smaller of the node strength and the strut strength), which is greater than the strut force, $C_6 = 192$ kN.

Node 9:

Since the bearing stress has been checked before, there is no need to check it again. The width of the prismatic strut C_8 , $w_{C_8} = 64.5$ mm, and the angle of the inclination angle of the strut C_{10} is $\theta_{10} = 76.6^\circ$. Then, the width of the strut C_{10} , $w_{C_{10}}^9 = 100 \sin \theta_{10} + 64.5 \cos \theta_{10} = 112.2$ mm. The nominal strength of the strut C_{10} is then, $C_{10n} = 17.66\# \times 112.2 \times 180 = 356.6$ kN (# the smaller of the node strength and the strut strength), which is less than the strut force, $C_{10} = 422.7$ kN; i.e., $C_{10n} = 0.843 C_{10}$.

Node 11:

The width of the tie T_8 , $w_{T_8} = 166$ mm, and of tie T_{13} , $w_{T_{13}} = 90$ mm, and the angle of inclination of the strut C_{14} , $\theta_{14} = 25.0^\circ$. Then, the width of the strut, $w_{C_{14}}^{11} = 166 \sin \theta_{14} + 90 \cos \theta_{14} = 151.7$ mm. Then, the nominal strength of the strut, $C_{14n} = 14.13\# \times 151.7 \times 180 = 385.7$ kN (# the smaller of the node strength and the strut strength), which is less than the strut force, $C_{14} = 486.5$ kN; i.e., $C_{14n} = 0.793 C_{14}$.

Node 12:

As for nodes 1, 2, and 3, $w_{T_8} = 166$ mm, $w_{T_7} = 143$ mm, the angle of inclination of the strut C_{12} , $\theta_{12} = 54.0^\circ$, and $w_{C_{12}}^{12} = 218.4$ mm. The nominal strength of the strut C_{12} , $C_{12n} = 555.2$ kN, which is greater than the strut force, $C_{12} = 508.9$ kN. As for strut C_{13} , the angle of inclination, $\theta_{13} = 25.0^\circ$, and $w_{C_{13}}^{12} = 199.7$ mm, $C_{13n} = 508$ kN, which is greater than the strut force, $C_{13} = 485.9$ kN.

Node 13:

As for nodes 1, 2, and 3, $w_{T_{11}} = 166$ mm, $w_{T_{12}} = 143$ mm, the angle of inclination of the strut C_{14} , $\theta_{14} = 25.0^\circ$ and $w_{C_{14}}^{13} = 199.7$ mm. The nominal strength of the strut C_{14} , $C_{14n} = 508$ kN, which is

greater than the strut force, $C_{14} = 486.5$ kN. As for strut C_{15} , the angle of inclination, $\theta_{15} = 54.0^\circ$, and $w_{C_{15}}^{13} = 218.4$ mm; then, $C_{15n} = 555.2$ kN, which is greater than the strut force, $C_{15} = 508.9$ kN.

Node 15:

As for nodes 1, 2, and 3, $w_{T_{11}} = 166$ mm, $w_{T_2} = 90$ mm, the angle of inclination of the strut C_{13} , $\theta_{13} = 25.0^\circ$ and $w_{C_{13}}^{15} = 151.7$ mm. The nominal strength of the strut C_{13} , $C_{13n} = 385.7$ kN, which is less than the strut force, $C_{13} = 485.9$ kN; i.e., $C_{13n} = 0.794 C_{13}$.

Node 16:

Since the bearing stress has been checked before, there is no need to check it again. The width of the prismatic strut C_{18} can be obtained by assuming that the force in the strut is equal to its nominal strength; then, $C_{18} = 300.1$ kN $= 23.55 \times w_{C_8} \times 180$, which gives $w_{C_{18}} = 70.8$ mm. The angle of inclination of the strut C_{16} , $\theta_{16} = 76.6^\circ$; hence, its width, $w_{C_{16}}^{16} = 100 \sin \theta_{16} + 70.8 \cos \theta_{16} = 113.7$ mm. Then, the nominal strength of the strut, $C_{16n} = 17.66^\# \times 113.7 \times 180 = 361.4$ kN, which is less than the strut force, $C_{16} = 422.7$ kN; i.e., $C_{16n} = 0.855 C_{16}$.

6.2.8. Results

From the obtained results the model elements are all safe except elements C_{10} , C_{13} , C_{14} , and C_{16} , with C_{14} the most critical, $C_{14n} = 0.793 C_{14}$. Therefore, the load P should be reduced to 79.3% of the calculated value; i.e., the predicted collapse load from the *STM*, $P_{STM} = 0.793 \times 598 = 474.2$ kN. Since the measured collapse load was 555 kN; then, the predicted collapse load from the *STM* is 85.4% of the measured value in the test.

In the preceding calculations, it has been assumed that the bottle-shaped struts have adequate reinforcement to resist the transverse tension, which should be verified. However, with the obtained beam capacity from *STM*, the forces in all elements have been reduced to 79.3% of the calculated value, which makes the check unnecessary for all struts that attained nominal strength exceeding their original forces; (the ratio between the strength of

struts without transverse reinforcement to that with adequate transverse reinforcement is 0.8). To this, only struts C_{10} , C_{13} , C_{14} and C_{16} need to be checked. Upon checking these latter struts, it was found that they have adequate transverse reinforcement.

6.3. Nonlinear finite element analysis of specimen B

The specimen has been analyzed under the two concentrated loads at mid-spans and only the middle support is prevented from translation in the y- and x-directions, but the external supports are prevented from translation in the y-direction only. Both linear and nonlinear analyses have been performed for the specimen. The stress trajectories obtained from the linear elastic analysis are illustrated in Fig. 15a and the results obtained from the nonlinear analysis are presented in Figs. 15b to 15d.

As observed from the yielding stress zones and the plastic strain distribution, Figs. 15b and c, compression failure of one of the struts was the main cause of failure of the specimen (concrete crushing failure). The failure occurred in the interior shear spans between the loading points and middle support as detected in the experimental crack pattern, Fig 15d. The maximum concrete compressive stress in the specimen B near failure was 27 MPa. Neither the bottom nor the top longitudinal bars reached their yield strength; they reached 344 MPa and 270 MPa, respectively. As given in the experimental results, the horizontal reinforcement above and below the interior opening yielded. The predicted failure load from the 3D nonlinear finite element analysis of specimen B is 500 kN; about 90% of the measured values.

6.4. Predicted capacity and test

In Table 6, the strength predicted by the *STM* and the *NFEA* is given along with the measured values. The prediction from the *STM*, as expected, is on the safe side. However, as in the previous cases, the results from the *NFEA* are closer to the measured strength.

Fig. 15. Finite element results of specimen B

Table 6. Strength of specimens A and B of continuous deep beams with large openings

Specimen	f'_c (MPa)	$P_{STM}(kN)$	$P_{NFEA}(kN)$	P_{Test}	P_{STM}/P_{Test}	P_{NFEA}/P_{Test}
A	26.4	481.2	510	565	0.85	0.90
B	27.7	474.2	500	555	0.85	0.90

7. Ledges of inverted-T beams

7.1. Description

Four full-size normal strength reinforced concrete Inverted-T beams (SS3-42-2.5-06, C1-42-1.85-06,

DC1-42-2.5-03, and SC1-42-1.85-03) with various details of web reinforcement, ledge geometries, number of point loads, and shear-span-to-depth ratio (a/d) that had been tested by [18] were selected for comparison between the 3D strut-and-tie

modeling and the 3D nonlinear finite element analysis. Only the details of the strut-and-tie model and the finite element solutions of beam SC1-42-1.85-03 are presented here and a summary of the results of the other three beams is given at the end of this section.

Tables 7 and 8 present details and material properties of the selected beams, where f'_c is the concrete compressive strength, f_{yl} is the yield stress of the longitudinal reinforcement, f_{yv} is the yield stress of the transverse reinforcement, f_{yh} is the yield stress of the skin reinforcement, and f_{yha} is the yield stress of the hanger steel. Besides, Table 9 gives the distance from the center of the load to the end of the ledge for three ledge types. Fig. 16 illustrates a summary of the experimental program. More details of the test and material properties can be found in [18, 19].

Constant web width of 533 mm was used for the beams of the experimental program. The width of the ledge was also the same, 267 mm, on each side of the beams. All other dimensions varied between the beams. Figs. 17 and 18 show typical specimen geometries and reinforcing details of the studied Inverted-T beams [18].

7.2. Strut-and-tie modeling of specimen SC1-42-1.85-03

Imperative modifications for both the ACI 318 [14] and AASHTO LRFD Codes [20] particularly for the strength of the nodes and the strength of the strut-to-node interfaces of Inverted-T beams have been recommended by [18, 21, 22]. A brief overview of the *STM* method and failure criteria, adopted in this study, is presented in this section.

Table 7. Summary of beam details

Specimen	Web Height h , (mm)	Ledge Depth (mm)	Ledge Length	Load Point	b_w , (mm)	d , (mm)	a/d	Support Plate (mm)	Load Plate (mm)
SS3-42-2.50-06	1067	356	Short	3	533	956	2.50	406×508	457×229
C1-42-1.85-06	1067	----	Cut-off	1	533	956	1.85	406×508	762×508
DC1-42-2.50-03	1067	533	Cut-off	1	533	956	2.50	406×508	457×229
SC1-42-1.85-03	1067	356	Cut-off	1	533	956	1.85	762×508	660×229

Plate dimensions: [direction of span] × [perpendicular to span direction]

Table 8. Summary of specimen material strengths

Specimen	f'_c (MPa)	f_{yl} (MPa)	f_{yv} (MPa)	f_{yh} (MPa)	f_{yha} (MPa)	ρ_v	ρ_h
SS3-42-2.50-06	43.13	480	420	420	426.3	0.6%	0.6%
C1-42-1.85-06	25.7	480	420	420	440	0.6%	0.6%
DC1-42-2.50-03	27.8	480	430	430	440	0.3%	0.3%
SC1-42-1.85-03	30	460	460	460	440	0.3%	0.3%

Table 9. Distance from center of the load to the end of the ledge for three ledge types

Ledge Type	$C_{1.85}$	$C_{2.50}$
Cut-off	(76.2 mm) past bearing end	(76.2 mm) past bearing end
Short	(901.7 mm)	(863.6 mm)
Long	(1768 mm)	(2390 mm)

Specimen Depths	
Ledge Depths	
Ledge Length	
Shear Reinforcement Ratio	
Shear Span-to-Depth Ratio	
Number of Point Loads	

Fig. 16. Summary of experimental program

Fig. 17. Typical geometry for the 42-inch (1067 mm) shallow specimens

(a) 42-inch (1067 mm) deep ledge, half-depth ledge specimens (b) 42-inch (1067 mm) shallow ledge, third-depth ledge specimens (c) 42-inch (1067 mm) rectangular cross-Section

Fig. 18. Typical sections

The value of the limiting compressive stress at the face of the node, f_{cu} , can be obtained from:

$$f_{cu} = m v f'_c$$

where m is the triaxial confinement factor, v is the concrete efficiency factor, and f'_c is the specified compressive strength of the concrete. The triaxial confinement factor, m , is calculated using the following equation: $m = \sqrt{A_2/A_1}$, where A_1 is the loaded area and A_2 is measured on the plane defined by the location at which a line with a 2:1 slope extended from the loading area meets the edge of the member as shown in Fig. 19.

The concrete efficiency factors, v , are used to reduce the compressive strength of the concrete in the node depending on the type of node (CCC, CCT, or CTT) and face (bearing face, back face, strut-to-node interface) under consideration. These factors are summarized in Table 10.

A three-dimensional strut-and-tie model as shown in Fig. 20 has to be considered, because the loads in Inverted-T beams are transferred from the ledges to the web, from the tension- to the

compression-chord, and from the loading, points to support. However, for simplification, a three-dimensional strut-and-tie model is decomposed into the two-dimensional longitudinal model and two-dimensional cross-sectional model.

Input data:

The beam height, $h = 1067$ mm, depth, $d = 956$ mm, breadth, $b_w = 533$ mm, ledge height, $h_1 = 356$ mm, ledge depth, $d_1 = 306$ mm, the load plate width $w_l = 229$ mm, and the load plate length $l_l = 660$ mm, the support plate width $w_s = 508$ mm, and the support plate length $l_s = 762$ mm. The shear span, $L_1 = 1768$ mm. The shear span-to-depth ratio, $a/d = 1768/956 = 1.85$. The tension steel, $A_s = 12T36 = 12077.4 \text{ mm}^2$, and the compression steel, $A'_s = 6T36 = 6038.7 \text{ mm}^2$. The cylinder compressive strength of concrete, $f'_c = 30 \text{ MPa} = 4330 \text{ psi}$. High strength deformed steel bars, with yield strength, $f_{y-T36} = 460 \text{ MPa}$, $f_{y-T19} = 440 \text{ MPa}$, and $f_{y-T13} = 460 \text{ MPa}$ and the vertical and horizontal web reinforcement are as shown in Fig. 21.

Fig. 19. Determination of A_2 for stepped or sloped supports [6]

Table 10. Concrete efficiency factors, v [18]

Face	Node Type		
	CCC	CCT	CTT
Bearing	0.85	0.70	$0.45 \leq 0.85 - \frac{f'_c}{20\text{ksi}} \geq 0.65$
Back			
Strut-to-node interface*	$0.45 \leq 0.85 - \frac{f'_c}{20\text{ksi}} \geq 0.65$	$0.45 \leq 0.85 - \frac{f'_c}{20\text{ksi}} \geq 0.65$	

* If crack control reinforcement requirement of AASHTO Art. 5.6.3.5 is not satisfied, use $v = 0.45$ for the strut-to-node interface.

Fig. 20. STM of Inverted-T beam [23] (a) 3D STM; (b) 2D cross-sectional model; and (c) 2D longitudinal model

Fig. 21. Elevation and cross-sectional details of SC1-42-1.85-03

7.2.1. Longitudinal STM model

7.2.1.1. Geometrical parameters

The solution procedure starts with determining the geometrical layout for the longitudinal *STM* shown in Fig. 22. The 12T36 reinforcement of each of the ties T_1 , T_2 , T_3 , and T_4 , give a nominal strength,

$$T_{1n} = T_{2n} = T_{3n} = T_{4n} = A_s \times f_y = 5618.64 \text{ kN.}$$

Since the reinforcement details allow the use of an extended nodal zone, the width of each of these ties (height of nodes A, C, E, G, and H) are

$$w_{T_1} = w_{T_2} = 2(t - d) = 2(1067 - 956) = 222 \text{ mm.}$$

Fig. 22. Longitudinal strut-and-tie model for SC1-42-1.85-03

With reference to Fig. 22, the hanger ties, T_5 and T_6 are represented by (14T19) and (8T19), respectively, so their nominal strengths are

$$T_{5n} = A_s \times f_y = (14 \times 2 \times 283.5) \times 440 = 3492.7 \text{ kN.}$$

$$T_{6n} = A_s \times f_y = (8 \times 2 \times 283.5) \times 440 = 1995.8 \text{ kN.}$$

In the same manner, T_7 is represented by (7T13 + 3T19), giving a nominal strength,

$$T_{7n} = A_s f_y = [(7 \times 2 \times 132.7)(460) + (3 \times 2 \times 283.5)(440)] = 1603 \text{ kN.}$$

7.2.1.2. Reactions

From linear elastic analysis, $V_1 = 0.73P$ and $V_2 = 0.27P$

7.2.1.3. Model geometry and forces

In order to obtain the forces in the model elements, the reinforcement of a selected tie is assumed to reach its yield stress. The solution to this problem is initiated by assuming that the reinforcement of tie T_5 reaches its yield stress, $T_5 = 3492.7 \text{ kN}$. Then, from simple truss analysis, the model forces are obtained, Fig. 23. The size of the nodes has to be specified, to determine the locations of the struts and ties. The length of Node B in the longitudinal direction is determined by the spread length of the hanger tie T_5 , l_{sp} . It is proportionally divided into l_e and l_a according to the amount of the applied load that acts on the left and right support [22].

The load spread

$$(l_{sp}) = l_t + d_1 + 76.2 \text{ mm} = 1042.2 \text{ mm}$$

Portion of the load spread length, l_{sp} , on the right support (l_a) = $l_{sp} \times (L_2/L) = 758 \text{ mm}$

Portion of the load spread length, l_{sp} , on the left support (l_e) = $l_{sp} \times (1 - L_2/L) = 284.2 \text{ mm}$

Distance from center of length l_a to the location of tie

$$T_5(l_{ab}) = 0.50 l_a + l_e - 0.50 l_{sp} = 142.1 \text{ mm}$$

Distance from center of length l_e to the location of tie

$$T_5(l_{be}) = 0.50 l_e + l_a - 0.50 l_{sp} = 379 \text{ mm}$$

Distance from the end of the load spread l_{sp} , to the left support

$$(l_{2.50}) = (L_2 - l_{be})/3 = 1445.3 \text{ mm.}$$

The depth of the node is taken as the depth of the compression stress block, a ,

$$a = \frac{A_s f_y - A'_s f'_y}{0.85 b_w f'_c} = 206.7 \text{ mm}$$

Although a traditional flexural analysis is not valid in a D-region due to the nonlinear distribution of strains, defining a using the equation above is conservative according to [22]. The moment arm, $y_{ct} = d - 0.5a = 852.7 \text{ mm}$.

7.2.1.4. The limiting compressive stress at the face of each node

The limiting compressive stress at the face of the node, $f_{cu} = m \nu f'_c$

A concrete efficiency factor, ν , is applied to the concrete strength, f'_c , to limit the compressive stress at the face of each node:

Bearing and back face of C – C – C node,

$$\nu_{CCC-b} = 0.85$$

Bearing and back face of C – C – T node,

$$\nu_{CCT-b} = 0.70$$

Fig. 23. Computed model forces (kN) assuming yielding of T_s reinforcement

CCC and CCT strut-to-node interface,

$$v_{\text{CCT-stn}} = 0.85 - \frac{f'_c}{20 \text{ ksi}} = 0.85 - \frac{4.33}{20} = 0.634$$

The triaxial confinement factor, m , is used to account for the effects of confinement provided by the concrete when the loaded area, A_l , is smaller than the supported area, A_2 . In this case, the support plate is only 508 mm wide, whereas the beam is 533 mm wide.

$$A_l = l_s \times w_s = 762 \times 508 = 387096 \text{ mm}^2$$

$$A_2 = (l_s + 25 \text{ mm}) \times (w_s + 25 \text{ mm}) = 787 \times 533 = 419471 \text{ mm}^2$$

$$m = \sqrt{A_2 / A_l} = 1.041$$

Bearing and back face of C – C – C node,

$$f_{cu}^{\text{CCC-b}} = 1.041 \times 0.85 \times 30 = 26.55 \text{ MPa.}$$

Bearing and back face of C – C – T node,

$$f_{cu}^{\text{CCT-b}} = 1.041 \times 0.70 \times 30 = 21.86 \text{ MPa.}$$

CCC and CCT strut-to-node interface,

$$f_{cu}^{\text{CCT-stn}} = 1.041 \times 0.634 \times 30 = 19.80 \text{ MPa.}$$

7.2.1.5. Check the bearing of the nodes

For node A, the nominal value of the reaction V_1 ,

$$V_{1n} = f_{cu}^{\text{CCT-b}} \times l_s \times w_s = 21.86 \times 762 \times 508 = 8461.9 \text{ kN,}$$

which is greater than the reaction, $V_1 = 2540.3 \text{ kN}$.

The nominal value of the load P ,

$$P_n = 2f_{cu}^{\text{CCT-b}} \times l_t \times w_l = 2 \times 21.86 \times 229 \times 660 = 6607.8 \text{ kN}$$

which is greater than the load, $P = 3492.7 \text{ kN}$.

For node H, the nominal value of the reaction V_2 ,

$$V_{2n} = f_{cu}^{\text{CCT-b}} \times l_s \times w_s = 21.86 \times 762 \times 508 = 8461.9 \text{ kN}$$

which is greater than the reaction, $V_2 = 952.4 \text{ kN}$.

7.2.1.6. Check of stresses

Only the critical nodes, the support nodes A and H, and the hanger node B are checked in the longitudinal model. The other nodes, such as the nodes D, E, F, and G in Fig. 22, are not critical and therefore they are not checked here.

Node A:

Since the bearing stress has been checked before, there is no need to check it again. The inclination angle of the strut C_1 , $\theta_1 = 28.0^\circ$, which gives a width of the strut $w_{C_1}^A = 762 \sin \theta_1 + 222 \cos \theta_1 = 553.75 \text{ mm}$. Then, the nominal strength of the strut $C_{1n} = f_{cu}^{\text{CCT-stn}} \times w_{C_1}^A \times w_s = 19.8 \times 553.75 \times 508 = 5569.8 \text{ kN}$, which is greater than the strut force, $C_1 = 5463.88 \text{ kN}$.

Node B:

For strut C_2 , its inclination angle $\theta_2 = 31.0^\circ$, which gives a width of the strut $w_{C_2}^B = [(l_e + a \tan \theta_2) \sin \theta_2 - (\frac{a}{\cos \theta_2})] = -30.80 \text{ mm}$. The contributions ratios of hanger tie width l_a and l_e have to be changed. Trials have been conducted until convergence takes place. Finally, the geometric parameters are obtained, $l_a = 442.2 \text{ mm}$, $l_e = 600 \text{ mm}$.

The inclination angle of the strut C_1 , $\theta_1 = 28.0^\circ$, which gives a width of the strut $w_{C_1}^B = 442.2 \sin \theta_1 + 206.7 \cos \theta_1 = 390.10 \text{ mm}$. Then, the nominal strength of the strut $C_{1n} = f_{cu}^{\text{CCT-stn}} \times w_{C_1}^B \times b_w = 19.8 \times 390.10 \times 533 = 4116.9 \text{ kN}$, which is less than the strut force, $C_1 = 5463.88 \text{ kN}$; i.e., $C_{1n} = 0.75 C_1$.

For strut C_2 , the inclination angle, $\theta_2 = 31.0^\circ$, which gives a width of the strut, $w_{C_2}^B = [(l_e + a \tan \theta_2) \sin \theta_2 - (\frac{a}{\cos \theta_2})] = 131.85 \text{ mm}$. Then, the nominal strength of the strut, $C_{2n} = f_{cu}^{\text{CCT-stn}} \times w_{C_2}^B \times b_w = 19.8 \times 131.85 \times 533 = 1391.50 \text{ kN}$, which is

less than the strut force, $C_2 = 1872.70$ kN; i.e., $C_{2n} = 0.74 C_2$.

For strut C_3 , the nominal strength of the strut $C_{3n} = f_{cu}^{CCT-stn} \times a \times b_w + A'_s f_y = 19.8 \times 206.7 \times 533 + 6107.25 \times 460 = 4990.72$ kN, which is greater than the strut force, $C_3 = 3224.9$ kN.

Node H:

Since the bearing stress has been checked before, there is no need to check it again. The inclination angle of the strut C_6 , $\theta_6 = 31.0^\circ$, which gives a width of the strut, $w_{C_6}^H = 762 \sin \theta_6 + 222 \cos \theta_6 = 582.75$ mm. Then, the nominal strength of the strut $C_{1n} = f_{cu}^{CCT-stn} \times w_{C_6}^H \times w_s = 19.8 \times 582.75 \times 508 = 5861.53$ kN, which is greater than the strut force, $C_6 = 1872.70$ kN.

From the obtained results the longitudinal *STM* model elements are all safe except elements C_1 and C_2 , with C_2 is the most critical, $C_{2n} = 0.74 C_2$. Therefore, the load P should be reduced to 74% of the calculated value; i.e., the predicted collapse load from the *STM*, $P_{STM} = 0.74 \times 3492.70 = 2584.60$ kN.

7.2.2. Cross-Sectional *STM* model

7.2.2.1. Geometrical parameters

The geometrical layout for the cross-sectional *STM* model is shown in Fig. 24a, where the numbering of ties, struts, and nodes are illustrated. Based on the last obtained load capacity from the longitudinal *STM* model, $P_{STM} = 2584.60$ kN, the forces of the cross-sectional *STM* model can be obtained, Fig. 24b.

Truss geometry:

$$l_4 = 45.7 \text{ mm} \quad l_1 = l_4 + 25.4 \text{ mm} + 0.50 w_l = 185.6 \text{ mm}$$

$$l_2 = b_w - 2l_4 = 441.6 \text{ mm} \quad l_3 = h_1 + 0.50 w_{T_1} + l_s = 186.3 \text{ mm}$$

$$l_5 = 58.7 \text{ mm} \quad l_n = l_{sp} - w_{T_1} = 820.2 \text{ mm}$$

With reference to Fig. 24, the reinforcement of tie T_{AD} is represented by (12T19). The nominal strength of the tie is

$$T_{ADn} = A_s \times f_y = 12 \times 283.5 \times 440 = 1497 \text{ kN},$$

which is greater than the tie force, $T_{AD} = 1292.3$ kN.

7.2.2.2. Check of stresses

Since the cross-section and loading are symmetric, only one side needs to be checked.

Node A:

Since the bearing stress has been checked before, there is no need to check it again. For strut C_{AB} , the inclination angle, $\theta_{AB} = 45.0^\circ$, which gives a width of the strut, $w_{C_{AB}}^A = w_l \sin \theta_{AB} + 2l_5 \cos \theta_{AB} = 244.9$ mm. Then, the nominal strength of the strut, $C_{ABn} = f_{cu}^{CCT-stn} \times w_{C_{AB}}^A \times l_l = 19.8 \times 244.9 \times 660 = 3200.9$ kN, which is greater than the strut force, $C_{AB} = 1827.6$ kN.

Node B:

For strut C_{AB} , the inclination angle, $\theta_{AB} = 45.0^\circ$, which gives a width of the strut, $w_{C_{AB}}^B = 2l_4 \sin \theta_{AB} + w_{T_1} \cos \theta_{AB} = 221.6$ mm. Then, the nominal strength of the strut, $C_{ABn} = f_{cu}^{CCT-stn} \times w_{C_{AB}}^B \times l_{sp} = 19.8 \times 221.6 \times 1042.2 = 4573$ kN, which is greater than the strut force, $C_{AB} = 1827.60$ kN. The width of the prismatic strut, C_{BC} is equal to $w_{C_{BC}} = w_{T_1} = 222$ mm. Then, the nominal strength of the strut, $C_{BCn} = f_{cu}^{CCT-stn} \times w_{C_{BC}} \times l_n = 19.8 \times 222 \times 820.2 = 3605.3$ kN, which is greater than the strut force, $C_{BC} = 1292.3$ kN.

Since the cross-sectional *STM* model elements are all safe, the *STM* capacity is governed by the longitudinal *STM* model, $P_{STM} = 0.74 \times 3492.70 = 2584.60$ kN.

Fig. 24. Specimen SC1-42-1.85-03

The shear capacity calculated using *STM* provisions as implemented for Inverted-T beams, $V_{STM} = 0.73 \times P_{STM} = 0.73 \times 2584.6 = 1886.7$ kN. The maximum shear carried in the critical section of the test region, including self-weight of the specimen and test setup is, $V_{test} = 2060$ kN. The critical section was considered to be the point halfway between the support and the nearest load. Then, the predicted collapse load from the *STM* is about 91.6% of the value measured in the test.

7.3. Nonlinear finite element analysis of specimen SC1-42-1.85-03

The specimen has been analyzed under the applied loads, with the right support is prevented from translation in the y-direction only, whereas the left support is prevented from translation in the y- and x-directions. Both linear and nonlinear analyses have been performed for the specimen as in the previous examples. The stress trajectories obtained from the linear elastic analysis are illustrated in Fig. 25a and the results obtained from the nonlinear analysis are presented in Figs. 25b to 25d.

As observed from the yielding stress zones in steel and the concrete plastic strain distribution, Figs. 25b and 25c, a local failure in the ledge due to tie yielding associated with web shear failure was the main cause of failure, which goes along with the experimental model of failure Fig. 25d. As observed from the experimental results, the horizontal ledge stirrups reinforcement yielded, whereas the stresses in vertical web stirrups were near yield, they reached 430 MPa. The maximum concrete compressive stress in the specimen near failure was 30 MPa. Neither the beam bottom nor the top longitudinal bars reached their yield stress;

they reached 350 MPa and 250 MPa, respectively. The predicted maximum shear carried in the critical section from the 3D nonlinear finite element analysis is 1920 kN, which is about 93.2% of the measured values.

7.4. Predicted capacity and test

The strength predicted by the *STM* and *NFEA* for the four specimens is given in Table 11, along with the measured values. The prediction from the *STM*, as expected, is on the safe side. On the other hand, as in the previous cases, the results from the *NFEA* are closer to the measured strength.

8. Conclusions

The focus of this study was to investigate the applicability and reliability of the strut-and-tie modeling and the 3D nonlinear finite element analysis for the prediction of the capacity and behavior of discontinuity regions. The study aimed to demonstrate the complementary role of the two methods as design tools. The two methods have been successfully applied to different problems, simply supported deep beam with large opening and recess, continuous deep beams with and without large openings, and inverted beams with ledges. The solutions obtained by the two methods for the selected problems have been verified by experimental tests.

From the obtained results the following conclusions can be drawn:

1. Both the nonlinear finite element and strut-and-tie method are reliable as design tools for the prediction of the strength and behavior of

reinforced concrete structures with discontinuity regions.

2. The strut-and-tie method as a lower-bound solution always yields a safe solution. Also, the *STM* serves as a design tool where the designer can trace the flow of forces and hence can easily develop efficient reinforcement details.
3. The nonlinear finite element analysis provides a more accurate prediction of the ultimate

strength than the strut-and-tie method; however, there is no guarantee that it provides a safe solution in every case. Besides, the computation effort of the *NFEA* is tedious in co comparison with the *STM*. Clear that the *NFEA* provides a useful tool in understanding the behavior of reinforced concrete structures with discontinuity regions concerning stiffness and deformations.

(a) stress trajectories

(b) yielding stress zones (elevation and plan)

(c) concrete plastic strain distribution

(d) experimental cracking patterns at the ultimate test load

Fig. 25. Finite element results of specimen SC1-42-1.85-03

Table 11. Strength of ledge beam specimens

Specimen	f'_c (MPa)	V_{STM} (kN)	V_{NFEA} (kN)	V_{Test} (kN)	V_{STM}/V_{Test}	V_{FE}/V_{Test}
SS3-42-2.50-06	43.13	1846	2150	2295.3	0.80	0.93
C1-42-1.85-06	25.7	1904	2570	2833.5	0.67	0.91
DC1-42-2.50-03	27.8	1152	1500	1623.6	0.71	0.92
SC1-42-1.85-03	30	1886.7	1920	2060	0.92	0.93

Declaration of conflicting interests

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