RESEARCH ARTICLE

Orthotropic homogenized modeling of masonry wall panels

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Abstract

The behavior of masonry walls which consist of two different materials, brick and mortar, is quite complex. Therefore, the mechanical behavior of masonry walls can be simplified by adopting the homogenization technique, which considers the panel with average properties in the x - and y -directions. The wall is approximated as an orthotropic homogenized material with uniform properties in the bed direction, x -direction, and also in the direction perpendicular to the bed, y -direction. The nonlinear orthotropic mechanical behavior of masonry walls is developed by adopting an incremental finite element analysis approach for the prediction of the peak and post peak behavior. Based on continuum damage mechanics and a proposed failure criterion, the calculation of the damage evolution can be expressed by adopting the thermodynamics approach in the tension regime and a secant stiffness approach based on a proposed and modified constitutive law in the compression regime. Using the principles of continuum damage mechanics, the damage evolution can be then described and evaluated. The proposed model has been verified in compression and tension with very good correlation with tests. In addition, a lateral pushover analysis of a masonry panel has been performed and compared with experimental results, and the comparison show good correlation.

Keywords

Masonry wall; Orthotropic material; Homogenized approach; Failure criterion; Constitutive relations; Nonlinear behavior; Damage mechanics

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1. Introduction

Masonry is a composite anisotropic material with different directional properties as a result of its constituent components; e.g., bricks and mortar joints. For the nonlinear analysis of masonry walls with the aid of the Finite Element Method, there are two concepts; the homogenized and the discrete formulations, Fig. 1. In the latter, the bricks and joints, which represent lines of weakness in the

wall, are discretized and hence the mesh is complex. This approach is referred to as the micro modeling and it demands excessive computation effort. In order to overcome the disadvantages of a detailed discretized modeling a homogenization of the brick/mortar assemblage can be performed, obtaining a good correlation between experimental and numerical curves. In this approach, the Cartesian coordinates coincide with the bed joint

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and in perpendicular (in the direction of the head joint). The effectiveness of such a technique has been demonstrated by several authors such as [1-3]. This approach, referred to as the macro modeling, is adopted here, which means that the overall behavior of the masonry panel will be evaluated.

Suquet [4] defined the homogenization technique as: “the procedure of replacing a strongly heterogeneous material by a homogenous material similar to the previous one within the normal range”. Unlike a micro-model where bricks and mortar are separately modeled a homogenized model represents an efficient method to predict the overall nonlinear behavior of the masonry panel. Nevertheless, the different constitutive characteristics of the material in tension and compression regimes and the non-associative damage behavior in the orthotropic directions create additional computational complexity. In the homogenization technique a periodic unit cell is identified so that its repetition generates an entire masonry panel and its behavior is analyzed as an individual problem in such a way to know the average values of the masonry panel as a single homogeneous material based on the actual geometrical and mechanical characteristics of the constituent material [5-6]. In addition, a periodic unit cell could be used as well to perform a limit analysis for the prediction of collapse loads and homogenized failure mechanisms [7-9].

In order to perform numerical nonlinear analysis of masonry walls, it is required to define suitable damage variables, an appropriate damage threshold/failure criterion, derive the direction and amount of damage evolution in tension and compression regimes, and establish a constitutive relation, which describes, with enough accuracy,

the real stress-strain behavior in the pre- and post-cracking states.

For modeling masonry with its complex orthotropic behavior and physical nonlinearity an incremental finite element analysis approach is adopted. Based on the defined damage variables (state variables), effective and nominal stress fields, and the relevant proposed failure criterion, the calculation of the damage evolution (direction and magnitude) can be expressed by adopting the thermodynamics approach in tension regime and a secant stiffness approach, based on a proposed and modified constitutive law, in compression regime. Using the principles of continuum damaged mechanics, the damage evolution can be then described and evaluated. The effectiveness of the proposed model has been validated by experimental and numerical studies. The proposed model can adequately describe the highly nonlinear behavior of masonry panels.

2. Failure surface

2.1. Adopted model

The failure criterion is the key step in the nonlinear structural analysis. In this regard, for the purpose of simplifying the computation effort, the wall is treated as a homogenized orthotropic material, with uniform properties in the bed direction and in perpendicular to the bed. The wall is considered as a plane stress problem with uniform properties through its thickness.

The bed direction is considered as the x -axis, the normal to the bed as the y -axis and the out of plane as the z -axis; thus, the material representing the wall is assumed to be isotropic in the positive x - and y -directions.

Fig. 1. Masonry wall; (a) matrix, (b) micro-model, and (c) homogenous macro-model, [10].

Syrmakezis and Asteris [11] developed the following failure surface of wall panel, as a plane stress problem.

$$f(\sigma_x, \sigma_y, \tau_{xy}) = a_1\sigma_x + a_2\sigma_y + a_3\sigma_x^2 + a_4\sigma_y^2 + a_5\tau_{xy}^2 + a_6\sigma_x\sigma_y + a_7\sigma_x\sigma_y^2 + a_8\sigma_x^2\sigma_y + a_9\sigma_x\tau_{xy}^2 + a_{10}\sigma_y\tau_{xy}^2 - 1 = 0 \quad (1)$$

where σ_x and σ_y are the normal stresses and τ_{xy} is the shear stress. Eq. (1) was found by Syrmakezis and Asteris [11] to fit reasonably well Page tests [12], Fig. 2.

The ten parameter model failure criterion in Eq. (1) requires 10 tests at least in order to estimate the parameters. These tests should include all the possible cases of stress combinations; axial tension

(compression) in the two directions, pure shear and combinations of the three stress components. As a result of the variability of the strength of masonry walls ten tests will not reflect the average properties of the wall unit. Hence, the number of tests is expected to be much larger than 10 and hence a regression analysis will be an essential procedure for the evaluation of the model parameters.

Upon substitution with the results of every test, σ_x , σ_y and τ_{xy} , a number of equations equal to the number of tests can be obtained. Since the number of equations exceeds the number of unknowns, model parameters, the parameters, $a_1, a_2, \dots, a_{10}$, have to be derived from regression analysis.

Fig. 2. Failure of brickwork under biaxial compression by Page [12]

2.2. Regression based on prescribed boundary conditions

Regression analysis aiming to determine that many unknowns, ten in this case, with a large number of test data is unlikely to result in accurate estimates of these unknowns. Therefore, boundary conditions of the five mono-axial tests will be introduced separately leading to the determination of five unknowns; hence, the failure surface is enforced to go through these points. These mono-axial tests are: (i) pure tension in the x - and y - directions (ii) pure compression in the x - and y - directions; and (iii) pure shear.

Upon introducing the results of the five mono-axial tests, the five constants a_1, a_2, a_3, a_4 and a_5 can be calculated. The values of any test of the mono-axial tests, introduced into the calculations of these constants, represent the average of the measured values of this test. These average values from Page tests [12] are shown in Table 1; each of these values is the average of 4 sets of measurements. The other constants, a_6 to a_{10} , can be derived from regression analysis of the other test data.

In order to describe the orthotropic material behavior in continuum damage mechanics, with the adoption of homogenization, a closed failure surface is crucial. According to Tsai and Wu [13], this condition is satisfied if the total Gaussian curvature K is positive.

$$K = -\frac{1}{\left(\frac{\partial f}{\partial \sigma_x}\right)^2 + \left(\frac{\partial f}{\partial \sigma_y}\right)^2 + \left(\frac{\partial f}{\partial \tau_{xy}}\right)^2} D \quad (2)$$

Therefore,

$$D = \begin{vmatrix} \frac{\partial^2 f}{(\partial \sigma_x)^2} & \frac{\partial^2 f}{\partial \sigma_x \partial \sigma_y} & \frac{\partial^2 f}{\partial \sigma_x \partial \tau_{xy}} & \frac{\partial f}{\partial \sigma_x} \\ \frac{\partial^2 f}{\partial \sigma_x \partial \sigma_y} & \frac{\partial^2 f}{(\partial \sigma_y)^2} & \frac{\partial^2 f}{\partial \sigma_y \partial \tau_{xy}} & \frac{\partial f}{\partial \sigma_y} \\ \frac{\partial^2 f}{\partial \sigma_x \partial \tau_{xy}} & \frac{\partial^2 f}{\partial \sigma_y \partial \tau_{xy}} & \frac{\partial^2 f}{(\partial \tau_{xy})^2} & \frac{\partial f}{\partial \tau_{xy}} \\ \frac{\partial f}{\partial \sigma_x} & \frac{\partial f}{\partial \sigma_y} & \frac{\partial f}{\partial \tau_{xy}} & 0 \end{vmatrix} < 0 \quad (3)$$

The constant a_6 has to fulfill the condition $(-\sqrt{a_3 a_4} < \frac{a_6}{2} < \sqrt{a_3 a_4})$ in order to ensure convexity, and in the same time results in the minimum sum of squares. Hence, the value a_6 should accordingly be selected within the mentioned range.

Table 1. Boundary conditions for Page [12] mono-axial tests

Test	Constant				
	a_1	a_2	a_3	a_4	a_5
σ_x	-4.3	0.4	0	0	0
σ_y	0	0	-7.55	0.1	0
τ_{xy}	0	-7.55	0	0	0.4

2.3. Evaluation of the model parameters of Page tests

2.3.1. Transformation of the test data to the Cartesian coordinates

Page [12] tests were performed in the principal axes directions, Fig. 2. However, the weak lines in a masonry panel are the vertical and horizontal mortar joints. This explains the staggered type failure in biaxial loading and in general the failure in these lines. As stated before, the bed direction is assigned the x -axis, the normal to the bed is assigned the y -axis and the out of plane is the z -axis. Damage defined in the x - and y -directions incorporates tensile and compression failure in addition to sliding shear failure, which represents the most common types of failure in masonry. In the following, a methodology for the calculation of the failure surface using the data released by Page [12] is illustrated.

In order to develop an expression in the σ_x - σ_y - τ_{xy} space, all tests performed by Page [12] are first transferred into the Cartesian coordinates; the x - y plane. It is noted that the points on the curves with $\theta = 22.5^\circ, 45^\circ, 67.5^\circ$ do not have the same τ_{xy} ; hence, they are not in the same plane.

2.3.2. Evaluation of the equation constants

In order to determine the constant terms a_1 to a_5 the boundary conditions from mono-axial tests are used. The boundary conditions for Page mono-axial tests, calculated as the average of the measured results, are given in Table 1. It is to be noted that, due to the lack of tension tests for the angles $\theta = 22.5^\circ, 45^\circ$ and 67.5° the tensile behavior is approximated according to Samarasinghe et al. [14]. The tension tests are taken according to Page at $\theta = 0$ and the other angles are approximated according to the variation of the tensile failure criterion for each angle as shown in Fig. 3.

Fig. 3. Tension failure criterion according to Samarasinghe et al. [14]

From the boundary conditions in Table 1 of Page's mono-axial tests the first five constants a_1 to a_5 are calculated as $a_1 = 2.271$, $a_2 = 9.868$, $a_3 = 0.573$, $a_4 = 1.324$ and $a_5 = 6.25$. Then, the constant a_6 should be assumed as explained in the previous section. For the failure surface to be closed and convex, the constant a_6 is selected as $a_6 = -0.6$, which results in the least sum of squares. Afterwards, regression on Page's test data is performed in order to determine the other constants.

Regression analysis is sensitive to the choice of data; therefore, three approaches have been followed in order to determine the appropriate procedure for performing regression. In the first approach, regression was carried out using the 186 tests of Page at once. In the second approach, for each set of tests (load combination and inclination angle) the average values were calculated making 52 data points on the surface representing the average of all sets of experiments, and these data points have then been used. This second approach was found to give better fit of the tests than the first approach since numerical errors are significantly reduced. In the third approach, it was found efficient to separately perform regression on the curve for the results of each inclination angle θ of Page's experimental tests. Then, from each curve of such angle 9 points are generated and then transformed into the σ_x - σ_y - τ_{xy} space. From the

regression of the points of the five angles, making 45 points, the remaining four constants of the global equation are evaluated.

Regression is performed as stated by the third approach, the 45 points, leading to the following equation, [15].

$$2.271\sigma_x + 9.868\sigma_y + 0.573\sigma_x^2 + 1.324\sigma_y^2 + 6.25\tau_{xy}^2 - 0.6\sigma_x\sigma_y - 0.0136\sigma_x\sigma_y^2 - 0.0077\sigma_x^2\sigma_y + 0.268\sigma_x\tau_{xy}^2 + 0.328\sigma_y\tau_{xy}^2 - 1 = 0 \quad (4)$$

The results of different approaches of failure surface representing Page's tests are shown in Fig. 4 for the case of $(\sigma_x \text{ and } \sigma_y \text{ plane } (\tau_{xy} = 0))$ and in Fig. 5 for the case of $((\sigma_x = \sigma_y) - \tau_{xy})$. The equation obtained from the 45 points represents the test data better than that obtained from the 52 points and the Symakezis and Asteris equation [11], Figs. 4 and 5.

3. Material damage

3.1. Damage variables, effective and nominal stress and strain fields

In the present work, the damage factor for elastic strain energy equivalency is calculated through the square root ratio of the Young's moduli as, $d = 1 - \sqrt{E/\tilde{E}}$, where d is the damage ratio, $\tilde{E}$ represents the initial young's modulus and E represents the modulus after damage, in the respective direction.

Fig. 4. Failure surface for $\theta = 0$ from different equations.

Fig. 5. $(\sigma_x=\sigma_y)$ - τ_{xy} plane from different equations.

For a plane stress problem, the two-dimensional damage tensor D and its related integrity tensor are given in Eqs. (5) and (6).

$$[D_{ij}] = \begin{bmatrix} d_x & 0 \\ 0 & d_y \end{bmatrix} \quad (5)$$

$$[\Omega_{ij}] = \begin{bmatrix} \Omega_x & 0 \\ 0 & \Omega_y \end{bmatrix} = \begin{bmatrix} \sqrt{1-d_x} & 0 \\ 0 & \sqrt{1-d_y} \end{bmatrix} \quad (6)$$

The damage tensor D is a diagonal tensor and d_x and d_y are the eigenvalues of the damage tensor

which represent the damage ratio along each direction. The damaged stiffness tensor C for an orthotropic material as a function of the undamaged stiffness, $\bar{C}$, is

$$[C] = \begin{bmatrix} \bar{C}_{xx}\Omega_x^4 & \bar{C}_{xy}\Omega_x^2\Omega_y^2 & 0 \\ \bar{C}_{xy}\Omega_x^2\Omega_y^2 & \bar{C}_{yy}\Omega_y^4 & 0 \\ 0 & 0 & 2G_{xy}\Omega_x\Omega_y \end{bmatrix} \quad (7)$$

where $\bar{C}_{xx}$, $\bar{C}_{yy}$ and $\bar{C}_{xy}$ are the components of the undamaged stiffness matrix and G_{xy} is the shear modulus. Ω_x and Ω_y are components of integrity in the x - and y -directions, which represents the complementary to the damage.

The relation between the effective and actual (nominal stress) as well as between the effective and actual strain components

$$\begin{aligned} \bar{\sigma}_x &= \sigma_x \Omega_x^{-2} & \bar{\epsilon}_x &= \epsilon_x \Omega_x^2 \\ \bar{\sigma}_y &= \sigma_y \Omega_y^{-2} & \bar{\epsilon}_y &= \epsilon_y \Omega_y^2 \\ \bar{\sigma}_{xy} &= \tau_{xy} \Omega_x^{-1} \Omega_y^{-1} & \bar{\epsilon}_{xy} &= \epsilon_{xy} \Omega_x \Omega_y \end{aligned} \quad (8)$$

3.2. Derivation of direction and amount of damage evolution

Based on the defined damage variables (state variables), effective and nominal stress fields, and the relevant failure criterion, the calculation of the damage evolution (direction and magnitude) can be expressed by adopting one of the two approaches given next.

3.2.1. Fundamentals of the thermodynamics approach

The damage variables (D , state variables) represent the history of what happened to the material. For each damage variable, there is a conjugate thermodynamic force (Y). A kinetic equation is needed to predict the evolution of damage in terms of the thermodynamic forces.

It is convenient to derive the kinetic law from a potential function (Gibbs potential function), similar to the flow potential used in plasticity theory. The damage potential controls the direction of damage evolution.

A damage surface (g) and a convex damage potential (f), as functions of the thermodynamic

forces and hardening material characteristics, are needed.

The damage surface delimits a region in the space of thermodynamic forces Y , where damage/incremental damage does not occur, kinetic equation is used to determine the magnitude of the rate of damage $\dot{D} = \frac{\partial D}{\partial Y} = \dot{\lambda} * \frac{\partial f}{\partial Y}$ where $\dot{\lambda}$ is the damage multiplier, which yields the magnitude of the damage increment and $\frac{\partial f}{\partial Y}$ is a direction in the Y space. $\dot{\lambda}$ is also needed to calculate the ratio of change of the hardening variable; $\dot{\delta} = \dot{\lambda} * \frac{\partial g}{\partial Y}$. Then, the degraded stiffness matrix and hence the constitutive relation can be formed in terms of continuum damage variables [16].

3.2.2. Nonlinear behavior of masonry walls under tension regime

The thermodynamic forces Y can be derived from the Gibbs free energy function, which represent the onset and accumulation of the in-plane matrix cracks of the orthotropic masonry material as follows:

$$\chi = \frac{1}{2\rho} \left[\frac{\sigma_x^2}{(1-d_x)^2 \bar{E}_x} + \frac{\sigma_y^2}{(1-d_y)^2 \bar{E}_y} + \frac{\tau_{xy}^2}{(1-d_x)(1-d_y)G_{xy}} - \frac{(\frac{\tilde{\nu}_{yx}}{\bar{E}_y} + \frac{\tilde{\nu}_{xy}}{\bar{E}_x}) \sigma_x \sigma_y}{(1-d_x)(1-d_y)} \right] \quad (9)$$

and hence, the thermodynamic forces can be derived from Eq. (10),

$$Y = \rho \frac{\partial \chi}{\partial D} \quad (10)$$

where ρ is the density. The damage activation function represents the relation between the progressed damaged space and the corresponding thermodynamic forces. It can be formulated as [16]:

$$g = \hat{g} - \hat{\gamma} = \sqrt{\left(1 - \frac{G_I}{G_{II}}\right) \frac{E_1}{F_{1t}^2} Y_1^2 + \frac{G_{Ic}}{G_{IIc}} \left(\frac{E_1}{F_{1t}^2}\right)^2 Y_1^2 + \left(1 - \frac{G_I}{G_{II}}\right) \frac{E_2}{F_{2t}^2} Y_2^2 + \frac{G_{Ic}}{G_{IIc}} \left(\frac{E_2}{F_{2t}^2}\right)^2 Y_2^2} - \hat{\gamma} \quad (11)$$

where E_1 and E_2 are young's moduli in the x - and y -directions, F_1 and F_2 are the maximum tensile strengths in the x - and y -directions, G_I and G_{II} are the fracture energies, in each of the orthotropic direction.

The softening behavior of masonry in tension may be modelled by the isotropic softening model proposed by Barbero [16]. In tension the stress strain relation until peak can be assumed as linear elastic and after peak a simple exponential decreasing curve can be used to model the softening behavior, $\gamma = c_1 \left[e^{\left(\frac{\sigma}{c_2}\right)} - 1 \right]$, in which c_1 and c_2 are constants depending on the mechanical properties of the masonry material, to be determined experimentally from the uniaxial tensile test of masonry panel.

Based on the defined damage variables, its conjugate thermodynamic forces, the damage activation function (at the same time failure criterion) and the softening model used, the incremental change of the state variables can be calculated.

As the constitutive law is non-associative due to the orthotropic character of masonry, a return mapping algorithm is needed to interpolate approximately the magnitude and direction of the state variables D and plastic strains ε_p at the end of each increment.

Beside the extensive computational effort and the convergence difficulties, one of the major shortcomings of this approach is still the need for additional experiments to determine parameters that are particular to each model.

3.2.3. Fundamentals of the secant stiffness incremental approach

Based on experimentally verified constitutive law for masonry and the relevant failure surface, the direction and amount of damage evolution can be easily derived through the incremental secant stiffness degradation. This approach avoids some of the shortcomings of the thermodynamics approach discussed before.

Fig. 6 describes the stress strain characteristics of materials with hardening behavior in (a) and with

Fig. 6. Secant stiffness [16]; (a) hardening, and (b) softening behavior

softening behavior in (b). The elastic domain is defined by the initial threshold values, $\sigma \leq \sigma_0$ and $\varepsilon \leq \varepsilon_0$, while the load is inside this region no damage can occur. A stress threshold increases for materials with hardening behavior and decreases for materials with softening behavior.

The secant stiffness $(1 - D) * \bar{E}$ can represent the constitutive law for the material at any incremental level of damage D , where $\bar{E}$ is the initial elastic modulus. While the stress strain relation for masonry in tension regime can be assumed as linear elastic until the peak stress and after peak a simple exponential decreasing curve can be used to model the softening behavior, the constitutive behavior of masonry in compression regime is more demanding and may be compared to concrete material behavior.

The following constitutive law was suggested for high strength concrete by Collins and Porasz [17], based on the constitutive law of concrete proposed by Popovics [18].

$$\sigma_{ci} = f_{cp} \left(\frac{\varepsilon_{ci}}{\varepsilon_{cp}} \right) * \frac{n}{n-1 + \left(\frac{\varepsilon_{ci}}{\varepsilon_{cp}} \right)^{nk}} \quad (12)$$

The building block in Popovics [18] material model for describing concrete in compression regime is the peak stress f_{cp} and the strain at peak stress ε_{cp} , where n and k are parameters describing the hardening/ softening and cracking behavior of concrete (must be determined and verified experimentally). In the following a brief comparison between the material behavior of concrete and masonry is illustrated:

- Degradation of material properties induced by the creation or propagation of defects,
- Anisotropic behavior because of damage characteristics,

- Dissymmetry in behavior between tension and compression,
- The values of the compressive strength and the tensile strength in both materials differ of at least one order of magnitude.
- The crack pattern of masonry in the damaged state having mostly stepped form (in the mortar bed direction and perpendicular to it). while in concrete the crack pattern takes the directions of the principal tensile stresses
- The compressive strength of concrete is mostly 4 to 6 times the compressive strength of masonry
- According to Hoffman and Schubert [19], the ratio between the uniaxial compressive strength parallel and normal to the bed joints ranges from 0.2 to 0.8 [20].

3.2.4. The proposed constitutive law of masonry in compression regime

It is based on the constitutive law of concrete by Popovics [18] and is modified and simplified in this paper to suite masonry material. This leads to the following $k=1$ for ascending and descending part of the constitutive relation).

$$\sigma_{ci} = f_{cp} \left(\frac{\varepsilon_{ci}}{\varepsilon_{cp}} \right) * \frac{n}{n-1 + \left(\frac{\varepsilon_{ci}}{\varepsilon_{cp}} \right)^n} \quad (13)$$

In Eq. (13) n is a function of $(f_{cp}/\varepsilon_{cp})$. The effect of the value of the variable n on the peak stress f_{cp} in case of constant strain at peak stress ε_{cp} is shown in Figs. 7 and 8, where f_{ci} and ε_{ci} are the current stress and strain values.

Fig. 7. Effect of n on masonry stress strain relation for $\varepsilon_{cp} = 0.003$.

Fig. 8. n -value and relation to f_{cp}/ε_{cp} ratio.

The optimal value of n to reach the desired peak stress f_{cp} and its conjugate strain ε_{cp} can be determined through the equations illustrated in Fig. 8. To avoid numerical difficulties minimum value of $n = 1.25$ and maximum value of $n = 7$ must limit the use of the proposed relation. The strain corresponding to the peak stress f_{cp} is not known and can be experimentally determined. Typical values of different mortar types vary between $\varepsilon_{cp} = 0.0012$ and maximum value of $\varepsilon_{cp} = 0.0041$ according to Kaushik [21].

3.2.5. Calculation of the incremental damage

The procedure is based on the principle of energy equivalence and the proposed modified constitutive relation for masonry panels, as described in section 3.2.3, using the incremental secant stiffness degradation.

$$\frac{\partial \sigma_{ci}}{\partial \varepsilon_{ci}} = -\frac{f_{cp}}{\varepsilon_{cp}} * \frac{n}{n-1+\left(\frac{\varepsilon_{ci}}{\varepsilon_{cp}}\right)^n} + \frac{f_{cp}}{\varepsilon_{cp}} * \frac{n^2 * \left(\frac{\varepsilon_{ci}}{\varepsilon_{cp}}\right)^n}{[n-1+\left(\frac{\varepsilon_{ci}}{\varepsilon_{cp}}\right)^n]^2} \quad (14)$$

For $\frac{\partial \sigma_{ci}}{\partial \varepsilon_{ci}}$ at $\varepsilon_{ci} = 0$, Eq. (14) yields to

$$\tilde{E} = \frac{f_{cp}}{\varepsilon_{cp}} * n/(n-1) \quad (15)$$

The secant stiffness E_{sec} is used to derive the damage factor shown in Eq. (17);

$$D = 1 - \sqrt{E_{sec}/\tilde{E}} = 1 - \sqrt{(\sigma/\varepsilon)/\tilde{E}} \quad (16)$$

$$D = 1 - \sqrt{\frac{n-1}{(n-1)+\left(\frac{\varepsilon_{ci}}{\varepsilon_c}\right)^n}} \quad (17)$$

The incremental damage can be then calculated from the derivation of the damage tensor $\mathbf{D}$ with respect to the incremental strain.

$$\dot{D} = \frac{dD}{d\varepsilon} = \frac{n*(n-1)*\left(\frac{\varepsilon_{ci}}{\varepsilon_c}\right)^{n-1}}{2*\sqrt{((n-1)*[n-1+\left(\frac{\varepsilon_{ci}}{\varepsilon_c}\right)^n])^3}} * \frac{1}{\varepsilon_c} \quad (18)$$

3.2.6. Damage threshold and the proposed failure criteria

The failure criteria of a masonry wall in the compression regime including the effects of damage and the biaxial stress state on the peak stress is proposed as shown in Eq. (19).

$$\begin{aligned} \hat{g} = & a_1 \frac{\sigma_x}{(1-d_x)} + a_2 \frac{\sigma_y}{(1-d_y)} + a_3 \frac{\sigma_x^2}{(1-d_x)^2} \\ & + a_4 \frac{\sigma_y^2}{(1-d_y)^2} + a_5 \frac{\tau_{xy}^2}{(1-d_x)(1-d_y)} \\ & + a_6 \frac{\sigma_x \sigma_y}{(1-d_x)(1-d_y)} \\ & + a_7 \frac{\sigma_x \sigma_y^2}{(1-d_x)(1-d_y)^2} + a_8 \frac{\sigma_x^2 \sigma_y}{(1-d_x)^2(1-d_y)} \\ & + a_9 \frac{\sigma_x \tau_{xy}^2}{(1-d_x)^2(1-d_y)} + a_{10} \frac{\sigma_y \tau_{xy}^2}{(1-d_x)(1-d_y)^2} - 1 = 0 \end{aligned} \quad (19)$$

The peak stress value must continuously be updated according to the stress-state and damage level for every incremental secant degradation.

4. Numerical analysis and verification of masonry walls

In order to demonstrate and verify the proposed mechanical model four different cases are considered. Since the adopted failure criteria, used in this analysis, has been developed in this study to represent a specific type of brick masonry [12], it is expected to have quite different results when compared with other test/numerical results, especially in the post-cracking/ damage state since the damage models and assumptions are not the same. The main goal of this verification is to test a masonry wall under basic compression and tension loading conditions, as well as under biaxial compression, tension and lateral loading conditions. For the nonlinear finite element analysis the ABAQUS [22] computer program has been used with a material subroutine written to introduce the material behavior, developed in this study, into the analysis.

4.1. Case 1 - Masonry wall under biaxial tension

A masonry panel is subject to biaxial tensile displacement-controlled loading condition of 0.4 mm in both x - and y -directions [20]. The material properties of the panel are given in table 2. Since the wall panel is symmetric in both directions, only one quarter has been analyzed with 4×4 mesh. The obtained results for the load-displacement relation of the panel along with Lourenco results [20] are given in Fig. 9. The trend of the results reflects the reliability of the proposed formulation. The elasticity moduli in the x - and y - directions, E_x and E_y , the passion's ratio, ν , tensile strength, f_t , and fracture energy, G_f , are given in Table 2.

4.2. Case 2 - Masonry wall under biaxial compression

A simple masonry panel is subjected to biaxial compressive displacement-controlled loading condition [20]. The loading displacement had been applied with a value of 3 mm in the x -direction and 6 mm in the y -direction. The material properties of the panel are listed in table 3. Since the wall panel is symmetric in both directions, only one quarter of the panel has been analyzed with a 4×4 mesh. The obtained results for biaxial compression are shown in Fig. 10.

Fig. 9. Load-displacement curve of case 1.

Table 2. Tensile material properties of case 1

$E_x = E_y$ [N/mm ²]	ν	f_t [N/mm ²]	G_f [N/mm]
10000	0.2	1.0	0.15

The results indicate that the peak stress values reached in each direction are higher than the uniaxial peak strength (refer to Fig. 4). This increase can be attributed to the consideration of the biaxial effect in both the uncracked and damage states. The elasticity moduli in the x - and y -directions, E_x and E_y , shear modulus, G_{xy} , and the poisons ratios, ν_{xy} and ν_{yx} , in the x - and y -directions, are given in Table 3.

Fig. 10. Stress-strain curve in compression of case 2, the direction 11 is the bed direction and 22 is the perpendicular direction.

Table 3. Properties of tested material of case 2

E_x [N/mm ²]	E_y [N/mm ²]	G_{xy} [N/mm ²]	ν_{xy}	ν_{yx}
10000	15000	1460	0.2	0.3

4.3. Case 3 - Masonry wall under lateral loading condition

The analysis is performed with the same material properties as the previous example and the boundary conditions shown in Fig. 11. The loading is applied in two steps, in the first, a pre-stressing compressive displacement loading of magnitude $U_y = -0.1\text{mm}$ in the vertical direction, while in the second step is loaded bi-axially as shown in Table 4. The results, Fig. 12, show the effectiveness of the proposed model under lateral loading condition, despite the difference in the developed failure criteria [20].

Fig. 11. Hinged-roller boundary condition and loading configuration of step 2 of case 3.

Table 4. Loading table of case 3

Step	U_x [mm]	U_y [mm]
1	0	- 0.1
2	0.54	0.27

4.4. Case 4 - Pushover of masonry panel (TU Eindhoven masonry shear wall)

Lourenco [20], analyzed a shear wall tested in TU Eindhoven (Vermeltfoort shear wall) to reproduce the results of the experiment. Table 5 shows the material properties of the masonry panel used in the analysis of Lourenco's macro-modeling [20]. Fig. 13 shows the dimensions and boundary conditions of the masonry panel. The shear wall is subjected in a first step to vertical compressive load of 30 kN, while in a second step an additional horizontal, monotonically increasing, load was applied under top displacement control.

E_x and E_y are the young's moduli in the x - and y -directions, G_{xy} is the shear modulus, F_{tx} and F_{ty} are the maximum tensile strengths in the x - and y -directions, G_{Ic} and G_{IIc} are the fracture energies, in each of the orthotropic direction, γ_o is the original damage threshold and c_1 and c_2 are constants describing the material in tension.

Fig. 12. Load displacement curves of case 3.

Fig. 13. Dimensions and boundary conditions of the wall of case 4.

Table 5. Properties of masonry panel of case 4 [20]

E_x , MPa	E_y , MPa	G_{xy} , MPa	ν_{xy}	F_{tx} , MPa	F_{ty} , MPa	G_{Ic} , N/mm	G_{IIc} , N/mm	γ_0	c_1	c_2
7520	3960	1460	0.09	0.35	0.25	0.05	0.018	1.0	0.5	-1.8

The obtained results based on the developed failure criteria and the mechanical and damage model used are shown in Fig. 14 along with Lourenco [20] and the test results. It is noted that the macro-model in this example over predicts the failure load due to the extremely localized failure type of the test specimen which can only be captured by a micro-model [20]. Nevertheless, the response up to 2.0mm horizontal displacement is adequately modeled. In addition, the overall response of the macro-model shows a similar behavior to both Lourenco's model and the experimental tests.

For further verification, the maximum and minimum in-plane principal stresses are plotted in Figs. 15-18 at $U_x = 3.6$ mm and 10 mm. They show a satisfactory correlation with the expected behavior of the masonry panel under maximum damage. Upon comparing the pattern of the principal stresses at $U_x = 3.6$ mm, which corresponds to the maximum loading, a clear trend can be noticed (refer to Figs. 15 and 16 for the panel minimum and maximum in plane stresses). The concentration of stresses in the middle of the panel

resembles a compression strut which varies in width and magnitude of stress during loading. While at target displacement, the stresses in the middle of the panel are much lower due to the crack and damage propagation and for that reason the compressive strut can be bundled in a smaller strut. The reduction in the width of the compression strut in the middle of the panel has two reasons, the first is that the compressive stress at the target displacement is considerably smaller than the peak stress and the second reason is that, due damage propagation and cracking, the resistance of the panel decreases.

Damage has been implemented in the analysis of the masonry panel, Figs 15-18. This effect can only be seen through the reduction of stresses in the plastic region from one step to the other. The principal stresses in the middle of the wall at displacement $U_x = 3.6$ mm are approximately 2.3N/mm² (tension) and -4.9 N/mm² (compression), while at displacement $U_x = 10$ mm they are approximately 0.6 N/mm² (tension) and -0.66 N/mm² (compression).

Fig. 14. Full response of Vermeltfoort shear wall of case 4.

Fig. 15. Maximum tensile principal stresses (damaged) at $U_x = 3.6$ mm.

Fig. 16. Minimum compressive principal stresses (damaged) at $U_x = 3.6$ mm.

Fig. 17. Maximum tensile principal stresses (damaged) at $U_x = 10$ mm.

Fig. 18. Minimum compressive principal stresses (damaged) at $U_x = 10$ mm.

5. Summary and conclusions

This paper aimed to propose a modeling procedure for the mechanical behavior of masonry panels, where the panel has been treated as a homogenized orthotropic material. The modeling procedure starts with an efficient technique for the development of a failure criterion that fits the type of masonry under consideration. The biaxial effect has been accounted for in the failure criteria. Then, the implementation of the material damage has been introduced incrementally via the thermodynamics approach for the tension regime and via the secant stiffness approach for the compression regime. The

constitutive law of the wall panel in both tension and compression has then proposed. Since the constitutive law is non-associative due to the orthotropic character of the masonry panel a return mapping algorithm has been developed in order to interpolate the magnitude and direction of damage at the end of every solution increment.

Different examples have been presented in order to demonstrate the applicability of the proposed modeling procedure and its reliability. The obtained numerical results demonstrated the effectiveness of the proposed model, which have been verified against experimental and other

numerical studies. The orthotropic behavior of the masonry panel could be modeled adequately. The model has shown numerical sensitivity due to the large difference (ratio of 1 to 10) between the tensile strength and the compression strength.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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