EIGENMODE OPTIMIZATION AND TOPOLOGICALLY PROTECTED STATES IN MAGNETO-MECHANICAL ULF TRANSMITTER ARRAYS

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ABSTRACT

Electromagnetic signals in the Ultra-Low Frequency (ULF) range 0.3-3 kHz are important for underground and underwater wireless communication systems, due to relatively low absorption through water, rock, and soil. However, the extremely large wavelength ($\sim 10^6$ m) causes man-portable ULF antennas to be inefficient to the point of impracticality. In this work, we present a new ULF transmitter approach through direct production of oscillating magnetic fields by an array of magneto-mechanical torsional resonators. We demonstrate transmitter power optimization through eigenmode engineering, and discuss the formation of topologically protected states. Our work illuminates a path to MEMS-based ULF transmitters capable of communicating through water and earth over km distances.

INTRODUCTION

Underground and underwater wireless communication is required for communication with divers, submarines, miners or trapped population in case of natural disaster, and also for employing sensor networks for monitoring the underground or underwater conditions [1]–[8]. Unlike wireless communication through air, underground and underwater communication presents a much greater challenge, due to high attenuation of the signals in rock, soil and water [2]–[4]. Ultra low Frequencies (ULF) which have very long wavelengths and thus larger penetration depth, are suitable for this type of communication [1], [5]. Traditional antennas structures are desired to be comparable in size to the operating wavelength for efficiency, in the ULF range this size is in the order of tens of kilometers, rendering them impractical [9], [10].

A different way for producing a low frequency signal is using a time varying magnetic field produced by an oscillating dipole, e.g. with a coil [1], [8], [11]. However, still large coils and large powers are required for transmission range more than a few meters [11]. Using a rotating permanent magnet to produce the time varying magnetic field, can reduce significantly the power requirements and thus the size of the transmitter. For example, to produce the same dipole, a coil occupying a volume of 0.1 ft^3 will require power in the order of 100 kW, while rotating a permanent magnet of that volume will be in the range of only 10 W. High field permanent magnets are readily available and can also be implemented in MEMS arrays [12].

Thus, here we focus on the later approach where the carrier signal is produced by mechanical oscillating motion of a permanent magnet. The magnetic field frequency is the resonance frequency of a dynamical system comprised of a mechanical spring, on which a permanent magnet is mounted, therefor it can be easily designed to be in the desired ULF range. We show that in order to decrease power consumption an array of many small magnets is desired, and therefore eigenmodes analysis is important for understanding the system behavior. Moreover, through eigenmode optimization the power consumption can be decreased.

DESIGN AND ANALYSIS

A permanent magnet rotating around its axis (Fig. 1a) produces a time varying magnetic field, proportional to the magnetic moment m , and the angular deflection θ , that can be sensed at some arbitrary receiver location $\vec{r}$. We mount a magnet on a mechanical

torsional spring, and produce a system oscillating at a frequency ω . The magnetic field generated by a point dipole at a location $\vec{r}$ at is given by:

$$\vec{B} = \frac{\mu_0 |\vec{m}|}{4\pi r^3} (3\hat{r}(\hat{m} \cdot \hat{r}) - \hat{m}) \quad (1)$$

where $\mu_0 = 4\pi \cdot 10^{-7} [N/A^2]$ is the magnetic permeability, $\vec{m}$ is the magnetic dipole, given by $\vec{m} = m(-\sin(\theta)\hat{x} + \cos(\theta)\hat{z})$ for a dipole pointing in $\hat{z}$ direction and oscillating about the $\hat{y}$ axis (as shown in Fig. 1a), and the deflection angle for an oscillating case is $\theta = \theta_{max} \cos(\omega t)$. $\hat{m}, \hat{r}$ are unit vectors in the direction of $\vec{m}, \vec{r}$ respectively. We start our analysis with determining the best point in space to position our receiver, assuming we wish to sense the ω component of the oscillating magnetic field $\vec{B}$ with the highest amplitude. To this end, we fit each component of $\vec{B}$ (i.e. $\tilde{B}_x, \tilde{B}_y, \tilde{B}_z$) to a polynomial in θ and by taking only the ω components from this fit we form a new vector $\vec{\tilde{B}}_\omega = \tilde{B}_{x,\omega}\hat{x} + \tilde{B}_{y,\omega}\hat{y} + \tilde{B}_{z,\omega}\hat{z}$, which is the best vector orientation for a sensor (receiver) at each receiver location $\vec{r}$. The simulated $\vec{\tilde{B}}_\omega$ at various receiver locations are presented in Fig. 1c, where vectors represent the greatest magnetic field for sensing, the best position for a receiver will be where the amplitude of $\vec{\tilde{B}}_\omega$ is maximum (red dot in Fig. 1c). From these simulations we can determine that this optimal sensing point for the ω component is along the $\hat{x}$ axis. Using the same analysis we can also obtain the best location for sensing the 2ω component which is found to be along the $\hat{z}$ axis, however this component has a much smaller amplitude.

For the spring-magnet resonator system, the dissipated mechanical power is given by:

$$P_{res} = \frac{\omega^3 I \theta^2}{2Q} \quad (2)$$

where I is the cross-sectional moment of inertia and Q is the mechanical quality factor for the torsional mode. A target value of the received oscillating magnetic field (for given $\vec{m}, \theta$) can be obtained either by a single large magnet, or by N smaller magnets with the same overall volume (Fig. 1b) undergoing synchronized motion. Assuming the cross section dimensions of a single large rectangular magnet are reduced by $1/\sqrt{N}$ to produce N smaller magnets. The moment of inertia I_{small} of a small magnet scales as $1/N^2$ relative to I_{large} of the single large magnet, and thus the power dissipated by N smaller magnets is reduced by a factor of N (see eq. 3), relative to the power dissipated by a single large magnet (P_{large}), promoting a MEMS approach.

$$I_{small} = \frac{M}{12N} \left(\frac{w^2}{N} + \frac{h^2}{N} \right) = \frac{I_{large}}{N^2} \quad (3)$$

$$P_{total} = NP_{small} = N \frac{\omega^3 \theta^2 I_{large}}{2Q N^2} = \frac{P_{large}}{N}$$

Our mechanical torsional spring is fabricated from a 0.4mm thick aluminum, we mount a neodymium N52 magnet 1/4 inch in diameter and 1/16 inch thick on the spring (Fig. 2a). We drive the torsional resonance mode (Fig. 2b) using a coil and sense local motion via magnetic field using a Hall sensor (Fig. 2a).

Figure 1: (a) Illustration of a magnetic dipole pointing in $\hat{z}$ and rotating around $\hat{y}$ with angular deflection θ . (b) Illustration of a single magnet divided to $N=4$ smaller magnets with the same overall volume. The use of smaller magnets may lead to lower dissipative power while maintaining the same amplitude of the sensed magnetic field. (c) Simulated magnetic field at different locations in space, the arrows represent the direction and magnitude of the greatest magnetic field for sensing. The middle blue arrow is the dipole orientation, and the two black arrows represent the angular deflection θ . The red dots mark the position of the maximal field.

Figure 2: (a) Photograph of the magneto-mechanical resonator fabricated from 0.4mm thick Aluminum. A small magnet is mounted in the center, and the system is mounted on an acrylic holder with plastic or aluminum screws (to prevent any interference from surrounding metals). (b) The resonator torsion resonance mode.

EXPERIMENTAL RESULTS

We produced a test array of eight coupled magneto-mechanical resonators. Figure 3 shows a picture of the array and the lumped

model used to characterize the dynamics of the system. The dynamics of the array are modeled using the coupled equations of motion, which for the i^{th} resonator in an array with symmetric coupling is given by

$$I\ddot{\theta}_i + c\dot{\theta}_i + (\kappa - \gamma)\theta_i + 2\gamma(\theta_{i-1} + \theta_{i+1}) = \tau. \quad (4)$$

Here c is a damping coefficient, κ is the torsional spring constant, τ a driving torque, and the parameter γ indicates the magnetic interaction that produces both coupling and a magnetic spring effect. For a linearized magnetic torque using small angles approximation, and for the array configuration in Fig. 3 (where all dipoles are the same and oriented in $\hat{z}$, rotating around $\hat{y}$, and are spaced only in $\hat{x}$) this magnetic parameter is given by

$$\gamma = \frac{3\mu_0 m^2}{4\pi d^3}. \quad (5)$$

Since γ is proportional to $1/d^3$, where d is the distance between magnets, the inter-magnet coupling can be tuned as needed.

Figure 3: Lumped model describing the resonator array, and a picture of an array with 8 resonators. In this array all the dipoles point in the $\hat{z}$ axis direction, and are equally spaced by distance d in the $\hat{x}$ direction. When driven in a torsional resonance mode the dipoles (magnets) rotate around the $\hat{y}$ axis.

To maximize the oscillating magnetic field produced by the array, a synchronized mode of vibrations is necessary. To that end, eigenmode analysis was used to characterize the system. We use $\tilde{\theta}_i$ to indicate the angular displacement for the i^{th} resonator (normalized such that $\sum \tilde{\theta}_i = 1$, corresponding to a fixed transmitted $|\tilde{B}|$) in a specific eigenmode. For an equidistant array of resonators, the synchronized eigenmode (Fig. 4b) is similar to a string mode where the end points have no deflection. However, since the dissipated power is $P \propto \sum \tilde{\theta}_i^2$ we find that this simplistic mode, while synchronized, is not power optimized. To better explain this, consider a target transmitted magnetic field $\tilde{B}^*$ which is obtained by an array of N magnets oscillating with a uniform angular deflection amplitude $\tilde{\theta}^*$. The magnetic field is proportional to $\sum \tilde{\theta}_i^* = N\tilde{\theta}_i^*$ and the dissipated power proportional to $\sum \tilde{\theta}_i^{*2} = N\tilde{\theta}_i^{*2}$. If however, the deflections are not uniform across the array but we wish to maintain the same amplitude of the magnetic field, it will result in some angular deflections being larger than $\tilde{\theta}^*$ and some smaller. This will maintain the same value of $\sum \tilde{\theta}_i^*$, however in terms of power the squared values results in a much larger dissipation. Meaning the more uniform the deflections across the array, the more power efficient the system is.

With this in mind, a more optimal configuration can be achieved by using a non-equal resonator spacing, in turn producing non-uniform inter resonator couplings, so that the eigenmode acquires a more evenly distributed amplitude. We analytically modeled and experimentally tested the optimized mode for improving power efficiency. We use an iterative approach in

Matlab, starting from equal distances between the resonators and changing them gradually until reaching the minimal power for a given array. We start the process with normalized equal distances of $r=1$ (Fig. 4c), then, each distance is assigned an index number symmetrically around the middle. For example for an eight resonator array the middle distance is the nominal $r=1$, and going outwards the following distances are r_1-r_3 (Fig. 5d). We first change the most outer distance (r_3 for the eight resonator example) until reaching a convergence criteria on the power, then we move to changing the next distance, and so on until the last distance (r_1). This process is then repeated until there is no more change to the power value, and we obtain the required ratio of distances for optimal power (Fig. 5b).

For our eight resonator array the simulations of this approach showed an improvement (reduction) in power dissipation of 14% (Fig. 4a). In our experiment setup however, there are discrete increments of spacing where we can position the resonators, and we are not able to meet the exact ratios of the distances needed. For the set of distances (closest to the optimal) possible to obtain in the experiment the reduction in power is about 10.5%. The simulated and experimental mode shapes for the initial equal distances and for the final modified distances are shown in Fig. 4c,d respectively, where circles represent deflection amplitude. In the experiment $\sum \theta_i^{*2}$ was reduced by 11%, which is very close to the simulation predicted value.

While adjusting the distances (topology) between resonators for power optimization, the system may form potentially undesired modal structures. Thus, we also experimentally tested how topological changes affect the eigenmodes and the overall frequency response of the array. Arrangements with different symmetries can results in very different frequency response characteristics, including having a bandgap or strong spatial localizations. Here we present the frequency response of three different configurations and demonstrate the formation of conductive, insulating, and topologically insulator phase having localized protected states (Fig. 5) [13]. A trivial insulator is formed by alternating the coupling between resonators from weak and strong, starting with a strong coupling (Fig. 5a), and as expected from an insulator we obtain a bandgap in the frequency response. A conductor is formed by an array with equally spaced resonators, having a conducting band and no bandgap (Fig. 5b). A topological insulator is a unique insulator having a band gap in the bulk, but strongly localized modes on the edges with frequencies in the middle of this band gap. This type of

response is achieved simply by having an arrangement with alternating coupling while starting with a weak coupling (Fig. 5c).

Figure 4: Power optimization process shown for an 8 resonator ULF transmitter array. (a) Simulated normalize power consumption per iteration, in each iteration the distances between resonators are modified. (b) The normalized distances received from the optimization process (for the initial case all distances are $r=1$), and the eigenmode shape for the most optimal case. Circles represent resonator amplitude, the red lines the direction of the phase, and the numbers indicate resonator spatial index. (c),(d) Simulated, and experimentally measured mode amplitudes, and a schematic illustration of the distances between resonators, for the starting point with equal distances (red dot in a) and the best modified case that we could obtain in experiments (green dot in a) respectively. We could not reach the final configuration because we are limited to discrete distances between resonators.

Figure 5: Experimentally measured band structure of different array arrangements. (a) A trivial insulator (b) A conductor (c) A topological protected state. In each experiment we individually drive and sense each resonator and obtain the system frequency response. For each case the plot shows the average susceptibility (amplitude/driving power) per frequency. For the topological case we separate the response of edge (pink) and bulk (blue) resonators. The insets shows each resonator average susceptibility across a range of frequencies, where circle sizes correspond to amplitude, and demonstrate the spatial distribution of the mode for each case.

SUMMARY

In this work we presented the equations of motions describing an array of magneto-mechanical resonators, where mechanical resonators are coupled through magnetic interaction that also produces a spring effect. We showed that through eigenmode optimization we can decrease the power dissipation of the system. This is achieved by changing the distances between resonators which in turn change the coupling strengths (effective spring stiffness), and deform the eigenmode shape. When deforming the eigenmodes, different and possibly undesired topologies can be formed in the system. We demonstrate three different arrangements of the coupling between resonators that result in very different frequency responses (trivial insulator, conductor and topological insulator). We also illustrate that a MEMS approach is recommended for further reduction of power. This work is a preliminary study opening a path towards MEMS based ULF transmitters.

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