SYNCHRONIZATION IN MICROMECHANICAL GYROSCOPES

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ABSTRACT

In this paper, we study the occurrence of synchronization between the two degenerate resonance modes of a micro disk resonator gyroscope. Recently, schemes involving the simultaneous actuation of the two vibration modes of the gyroscope have been implemented as a promising new method to increase their performance. However, this strategy might result in synchronization between the two modes, which would maintain frequency mode-matching but also may produce problems, such as degrading stability and sensitivity. Here, we demonstrate for the first time synchronization between the degenerate modes of a micro-gyroscope and show that synchronization dramatically reduces frequency instability at the cost of increased amplitude instability.

INTRODUCTION

Synchronization describes the tendency of weakly-coupled oscillators to track each other's frequency. This phenomenon has been observed for centuries in various fields in science, describing human's internal clock modulated by daylight periodicity [1], the behavior of grasshoppers singing in harmony [2] or fireflies' flashes emitted at once and with the same period [3]. Nowadays, synchronization is implemented in technologies, providing beat stability in dysfunctional hearts thanks to pacemakers [4] and enhancing telecommunication systems [5].

With the development of micro-technologies, synchronization has also been studied in Micro-ElectroMechanical Systems (MEMS), locking the micro-oscillator's frequency to an external generator [6] or to another MEMS [7] in order to reduce their frequency fluctuations, one of the main sources of noise in micro-sensors. More specifically in MEMS Gyroscopes, frequency fluctuations are intensively investigated since they directly affect the stability of the gyroscope [8]. Different methods have been suggested in order to reduce these fluctuations [9], but most of them rely on active feedback while synchronization passively fixes the resonance frequency.

This paper focuses on the implementation of synchronization in a Disk Resonator Gyroscope (DRG) and the resulting outcomes of this technique. In the first part, we will describe the device and the gyroscope's working regime. We will then explain the synchronization implementation between one mode of the gyroscope and an external oscillator. In the following part, we will present the implementation of mutual synchronization between the two modes of the gyroscope. In the last part, we will discuss the performance of the gyroscope operated with these techniques.

THE DISK RESONATOR GYROSCOPE

The micro-gyroscope is a silicon disk with a diameter of 600μm and a thickness of 40μm (Fig. 1), first time presented in [10], made through the *epi-seal* process [11]. The device is anchored at the center and is surrounded by electrodes. As we apply a voltage difference between the electrodes and the device, we can drive the resonant modes of the disk, tune the resonance frequency [10], control the nonlinearity [12] and detect the vibration amplitude. In our disk, the gyroscope operation is based on the actuation of the two in-plane 2θ wineglass modes (described in the following as X and Y modes), the dynamics of which are described by the following equations:

$$\ddot{A}_X + \Delta\omega_X \dot{A}_X + \omega_X^2 A_X + q_X A_Y + \frac{8\omega_X}{3} \alpha_X A_X^3 = F_X + 2\Omega k \dot{A}_Y \quad (1)$$

$$\ddot{A}_Y + \Delta\omega_Y \dot{A}_Y + \omega_Y^2 A_Y + q_Y A_X + \frac{8\omega_Y}{3} \alpha_Y A_Y^3 = F_Y - 2\Omega k \dot{A}_X \quad (2)$$

where $A_{X,Y}$, $\Delta\omega_{X,Y}$, $\omega_{X,Y}$, $q_{X,Y}$, $\alpha_{X,Y}$, k , and $F_{X,Y}$ represent the amplitude, bandwidth, resonance frequency, stiffness (or quadrature) coupling coefficient, Duffing coefficient, Coriolis coupling coefficient (or angular gain), and force applied for respectively the X and Y modes. Due to both the fabrication process and the symmetry of the device, the two modes present very similar parameters, especially bandwidth $\Delta f = \Delta\omega/2\pi$ and natural resonance frequency $f_0 = \omega/2\pi$ of respectively 3 Hz and 250 kHz. The coupling coefficient q , often introduced as quadrature, describes the linear coupling of one mode on the other and can be controlled by electrostatic means [13]. The Duffing coefficient α affects the nonlinearity of the system and mainly transforms the resonance frequency into an amplitude-dependent parameter [12].

Figure 1: Schematic of the DRG and the shapes of the two modes X and Y used in the synchronization process.

When the device is operated as a gyroscope, one mode is driven as an oscillator (the drive mode, X), while $F_Y = 0$. As a rotation Ω is applied to the device, energy is transferred to the second mode (the sense mode, Y) due to Coriolis coupling through the angular gain k . This energy transfer takes place at the resonance frequency of the driven mode, and in the simplest case where nonlinearity and mode coupling are negligible, and the resonance frequencies of the two modes are near matching, the amplitude of the sense mode is described by:

$$A_Y = -\frac{k \Omega F_X}{\omega_Y \Delta\omega} \frac{1}{\omega_Y - \omega_X + i\frac{\Delta\omega}{2}} \quad (3)$$

The amplitude of the sense mode directly provides a measurement of the rate Ω applied to the gyroscope, and maximum sensitivity is obtained when the two modes' resonance frequencies match ($\omega_Y = \omega_X$), a condition known as mode-matching. Neglecting the fluctuation of the dissipation $\Delta\omega$, the performance of the gyroscope relies then on the frequency stabilization of the modes.

As the gyroscope experiences temperature changes, its stiffness is altered as well, resulting in a temperature coefficient of frequency (TCF) for the X and Y modes of respectively 14.9

ppm/°C and 15.5 ppm/°C. Due to this asymmetry of 0.6 ppm/°C, temperature drifts lead to variations in the frequency difference between the two modes. According to Eq.(3), this variation results in a amplitude fluctuations of Y which directly affects the stability of the gyroscope's bias and scale factor. In the following, we present the effects of the synchronization of the gyroscope's modes on the frequency and amplitude stabilization with two different schemes.

SYNCHRONIZATION TO AN EXTERNAL SOURCE

When an oscillator experiences a perturbation at a frequency close enough to its resonance frequency, the oscillator will tend to match its own frequency to that of the perturbation [1], which is defined as the synchronization process. Investigating this phenomenon in our gyroscope requires a specific setup, presented in Fig. 2.

Figure 2: Setup for studying synchronization between the X mode and an external tone.

First, the resonance (here the X mode) needs to be driven as an oscillator, which is performed thanks to a Phase-Locked Loop (PLL), maintaining oscillation at the resonance frequency f_X . Note that the natural frequency $f_{0,X}$ of X does not always coincide with the resonance frequency f_X , a fact which will be at the core of the following parts.

In a second step, we send an external tone to the system at f_e , provided by a temperature-stabilized quartz oscillator, inducing an additional force F_e applied to the mode. As we sweep the frequency of the external tone within few bandwidths from the natural frequency of the X mode, we observe a region where synchronization takes place (Fig. 3), in which the mode tracks the frequency of the external tone.

Figure 3: Frequency of the X mode f_X as the frequency of the external tone f_e is swept upwards (orange) and downwards (green). f_X follows f_e within the synchronization range, which increases quadratically with the amplitude of X (inset), as predicted from the theory.

The range of this synchronization regime $\delta\omega_X$ is described by:

$$\delta\omega_X = \frac{F_e}{F_X} \sqrt{\Delta\omega^2 + 16 \alpha_X^2 A_X^4} \quad (4)$$

As long as the external tone's frequency f_e is not further than $\delta\omega_X/2$ from the natural frequency $f_{0,X}$ of the oscillator, the oscillator's frequency f_X is locked to f_e . Because of temperature variations, the natural frequency of the oscillator is fluctuating as well, such that steady synchronization can only occur for a synchronization range larger than the frequency fluctuations. Considering the DRG's TCF, $f_{0,X}$ fluctuates by 3.7 Hz/°C, which is larger than the bandwidth of our system at small amplitude.

From Eq.(4), in order to increase the synchronization range we can either increase the force of the external tone F_e or the nonlinearity of the system $\alpha_X A_X^2$. In order to have synchronization, the range of F_e is limited by F_X , restricting the synchronization range to be smaller than the bandwidth of the system. On the other hand, entering the nonlinear regime allows a larger synchronization range to be reached (inset of Fig. 3 shows ranges up to five bandwidths). Thanks to this nonlinear regime, we kept the mode X synchronized to the external tone over 4 °C of temperature change.

For a fixed frequency f_e , the synchronized oscillator's frequency f_X remains fixed and is locked to f_e even in the presence of fluctuations of the DRG's natural frequency $f_{0,X}$. In order to quantitatively observe the improvement of the frequency stability in the synchronization regime, we present in Fig. 4 the Allan deviation of the resonance frequency of the oscillator with and without the implementation of synchronization. The Allan deviation describes the precision of a parameter (here the resonance frequency) as we average it over time. Due to temperature fluctuations, the Allan deviation of the unsynchronized DRG's oscillation frequency increases for averaging times greater than 0.1s. On the other hand, in the synchronization regime, the DRG's frequency stability is determined by the stability of the external temperature-stabilized quartz oscillator. However, to improve mode matching in a gyroscope, one needs to reduce the fluctuations of the frequency difference between the drive and the sense modes, involving a more complex implementation of synchronization.

Figure 4: Allan deviation of f_X with and without synchronization. At 100s averaging time, the Allan deviation is reduced by 3 orders of magnitude in the synchronization regime.

MUTUAL SYNCHRONIZATION IN THE DRG

Starting from the previous section, one possible way to synchronize both modes of the gyroscope would be to duplicate the setup presented in Fig. 2 for both the X and Y modes. In such a scenario, each mode's frequency would be locked to the external tone's frequency, providing mode-matching at f_e . However, a less demanding implementation of synchronization can be performed, keeping the vibrating mode in the linear regime while enabling a passive frequency mode-match between the two modes that rejects temperature variations.

As discussed previously, synchronization only occurs with oscillators, thus both the X and Y modes need to be driven with separate PLLs (see Fig. 5). Then, instead of implementing an external tone to which the modes would be synchronized, we exploit the mechanical (quadrature) coupling existing between the two modes.

Figure 5. Setup for the mutual synchronization of the X and Y modes through natural mode coupling (curved arrows).

In gyroscopes, this coupling is usually cancelled out to reduce bias and bias instability, however for the purpose of this technique, we enable a small coupling to remain (typically 5% to 10% of the mode's amplitude) which we control by electrostatic means [13]. Due to this coupling, the actuation of one mode affects the other one, which induces a perturbation acting like the external tone in the previous section. As we drive both X and Y, each mode locks its frequency to the other one, entering in mutual synchronization. Thus, as one mode's frequency drifts, the other one follows, as long as the difference between their natural frequencies is sufficiently small. To first approximation, the maximum difference is determined by:

$$f_{0,X} - f_{0,Y} \leq \sqrt{\left(\frac{\Delta\omega}{2}\right)^2 \left(\frac{q_X A_Y}{F_X} - \frac{q_Y A_X}{F_Y}\right)^2 + 4 \left(\frac{q_X A_Y}{F_X} \alpha_X A_X^2 + \frac{q_Y A_X}{F_Y} \alpha_Y A_Y^2\right)^2} \quad (5)$$

Note that Eq.(5) is equivalent to Eq.(4) if one of the mode's coupling is negligible compared to the other one, in which case the synchronization acts only in one way. On the other hand, if both modes are operated at the same amplitude with the same coupling, the linear terms in Eq. (5) cancel and synchronization occurs solely due to nonlinearity.

As discussed previously, tuning the nonlinearity enables a larger synchronization range than linear coupling. However, in this particular scheme, the fluctuations that must be handled to perform synchronization are much smaller than in the case with only one mode. Because the TCFs of the two modes are closely matched, the frequency difference between the two modes remains small even though the absolute frequency of each mode shifts by 3.7 Hz/°C. Thus, the linear coupling by itself can handle small temperature variations (according to Eq.(5), up to 2 °C for a coupling of 10%). In this configuration, the Allan deviation of the resonance frequencies' difference (see Fig. 6) shows a stability similar as in Fig. 5, demonstrating mode-matched operation at long integration time. However, as presented in Eq. (3), the impact of the synchronization phenomenon on the performance of the gyroscope can only be determined by considering the amplitude stability of the sense mode.

Figure 6. Allan deviation of the frequency difference between X and Y ($f_X - f_Y$) with synchronization. Inset: fluctuations of the resonance frequencies of X and Y in mutual synchronization in the time domain.

DISCUSSION ON THE AMPLITUDE STABILITY

By using the linear coupling between the two modes, the gyroscope is operated in a perfect frequency mode-matched regime, the frequency fluctuations being cancelled through the synchronization process. However, these frequency fluctuations are converted into amplitude fluctuations (Fig. 7), the magnitude of which is directly related to the synchronization parameters.

In the linear limit of synchronization between X and Y, the coupling needed to compensate frequency fluctuations will induce amplitude fluctuations in the same proportion. If 10% coupling enables synchronization, amplitude fluctuations of up to 10% may be induced. In Fig. 7, we observe frequency fluctuations of 0.4 ppm between the X and Y modes without synchronization, which correspond to 0.1 Hz. Considering our bandwidth of 3 Hz, we need a linear coupling of at least 3.3% of the mode's amplitude, which should induce amplitude fluctuations of the same amount, very close to our measurement of 3%. Because of these induced amplitude fluctuations, the gyroscope performance decreases in the synchronization regime with the current configuration. This result is generic to any gyroscope wherein 1) the modes are operated in a frequency mode-matched regime, 2) the modes are coupled (which is induced at least by the Coriolis force), 3) the modes are driven simultaneously as separate oscillators.

Figure 7. Phase spaces resulting from the two synchronization setups presented (left: X synchronized with an external tone in the nonlinear regime, right: synchronization between X and Y in the linear limit, both graphs using the same scales) and compared with the non-synchronized scenario. These plots reveal that the frequency fluctuations vanish in the synchronization regime at the cost of greater amplitude noise.

In parallel, recent work has shown promising results by driving both modes of a gyroscope simultaneously [14,15]. The results presented here suggest that in such a configuration, care should be taken to avoid accidental synchronization, which could decrease gyroscope performance. To prevent synchronization, the mode-matched regime can be avoided, a phase difference may be fixed between the two modes [14] (which are no longer separate oscillators), only one PLL may be used for the two modes [15] (which are no longer driven as oscillators), or by any means which violates one of the three statements of the previous paragraph.

CONCLUSION

We demonstrated that the synchronization of a mechanical oscillator enables stabilization of its resonance frequency in the presence of temperature variations. By driving both modes of a DRG, we used the coupling between the modes to perform mutual synchronization, wherein each mode tracks the other one's frequency. This configuration enables a perfect passive frequency mode-match between the two modes, robust over temperature variation. However, we demonstrated that this technique converts the frequency fluctuations into amplitude fluctuations, which decreases the gyroscope performance.

This work highlights the importance of avoiding synchronization in gyro control schemes where both modes are simultaneously excited as the synchronization may induce amplitude instability.

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