ALN PIEZOELECTRIC PARAMETRIC OSCILLATORS WITH LOW PHASE NOISE

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ABSTRACT

In this work, integrated parametric oscillators with low phase noise have been developed using geometrically optimized contour mode aluminum nitride (AlN) resonators. These resonators feature dual resonances for orthogonal contour modes at 37.2 and 74.4 MHz respectively. Their comprising parametric oscillators harness the resonantly enhanced parametric amplification in the mechanical domain to perform noise squeezing. The phase noise reduction via the parametric amplification process is analyzed with a numerical method. The measured result has shown a phase noise of -110 dBc/Hz at 1kHz offset for a 37.2 MHz parametric oscillator. Benchmarked to a conventional oscillator based on the identical resonator, the parametric oscillator has demonstrated the anticipated 25 dB reduction in phase noise at 1 kHz offset.

INTRODUCTION

Chip-scale oscillators based on piezoelectric MEMS resonators offer a potential alternative to the quartz-based counterparts as high performance frequency references. Among the various technology platforms, AlN based MEMS resonators are particularly promising due to their integration compatibility with CMOS [1], scalability towards high frequencies [2], and high quality factor (Q) in dry air [3]. Nonetheless, the phase noise performance of the state of the art AlN-based oscillators is still moderate in comparison to the commercial components based on surface acoustic wave (SAW) or quartz crystal resonators. The excessive phase noise originates from the temporal fluctuation in the resonant frequency [4], which is intrinsic to the material and structure of the device, and is challenging to diminish.

Recently, the exploitation of degenerate parametric amplification has attracted significant research interests as it promises a wide range of MEMS-based new applications, ranging from noise squeezing [5] to low power computing [6]. Oscillators and frequency dividers that leverage the parametric amplification either external [7] or intrinsic [8] to a resonator have been demonstrated with improved phase noise performance. These prior works harness the passive and extremely narrowband nature in degenerate parametric amplification to remove or filter/squeeze noise in an oscillatory system. However, an integrated piezoelectric parametric oscillator, in which the fundamental oscillation and parametric amplification are concurrently coupled in the mechanical domain of a resonant device, has never been demonstrated. It is expected that such a microsystem implementation can have higher parametric modulation depth, larger gain, and narrower bandwidth noise filtering for producing stabilized oscillations with more diminished phase noise.

In this work, we have developed parametric oscillators that integrate degenerate parametric amplification in the mechanical domain by utilizing geometrically optimized contour mode AlN resonators [9], and establish the feedback via a frequency doubler and phase shifter. In comparison to a conventional oscillator in which the amplification in the feedback loop is exclusively obtained using solid state devices, the parametric oscillator attributes a fraction of the close loop gain to parametric amplification, and thus benefits from noise squeezing and reduction inherent to a high Q passive amplification. The measured phase noise performance of the 37.2 MHz AlN parametric oscillators has shown a 25 dB improvement at 1 KHz offset over the conventional oscillators.

Figure 1: (a) Mocked up view of the 37 MHz parametric resonator. (b) SEM image of the fabricated device. The dimensions of the resonator are labeled in the figure.

Figure 2: (a) FEM simulated admittance response of both signal and parametric ports. (b) Longitudinal and transverse modes of the parametric resonator.

PARAMETRIC RESONATOR

The heart of our parametric oscillator is a geometrically optimized AlN parametric resonator. The structure and SEM image of a fabricated resonator are presented in Fig. 1. The length of the resonator is set to be 4 times the width so that the resonance of the first order transverse mode [Fig. 2(c)] is twice the resonance of the second order longitudinal mode [Fig. 2(b)]. The orthogonal modes are chosen because their resonances can be independently tuned via geometrically setting the length and width of the resonator. Thus, dual resonances at f and $2f$ can be readily achieved in fabricated devices. The dual resonances allow for efficient modulation of the stiffness of the parametric resonator with a pump signal at frequency $2f$, and resonant coupling of the parametrically amplified vibration at f simultaneously.

The admittance response of a typical parametric resonator is modeled with COMSOL based finite element analysis (FEA). The simulated admittance response of a typical parametric resonator [Fig. 2(a)] confirms the desired dual resonances for the orthogonal modes, which are set to 37.2 and 74.4 MHz respectively. The fabricated parametric resonators were first measured with an Agilent N5230A vector network analyzer (VNA). The measured admittance of a typical parametric resonator is shown in Fig. 3(a). The device dimensions and key parameters are listed in Table I. One of the ports of the resonator is used as the signal port of the resonator, while the other one is used as the parametric port. The parametric amplification in the fabricated resonators was first confirmed by

Figure 3: (a) Measured admittance response of a typical parametric resonator utilized in the oscillator. (b) and (c) Measured Q enhancement excited by a parametric pump on multiple devices.

TABLE I. DIMENSIONS AND KEY CHARACTERISTICS OF THE MEASURED PARAMETRIC RESONATORS

Design Parameter	Symbol	Device 1	Device 2
Resonator Width (μm)	W_R	200	200
Resonator Length (μm)	L_R	56.5	58.0
Electrode Length (μm)	L_E	46.5	48.0
Electrode Width (μm)	W_E	80	80
Anchor Length (μm)	L_A	20	30
Anchor Width (μm)	W_A	20	20
Top Electrode Thickness (nm)	T_{Top}	80	80
Bottom Electrode Thickness (nm)	T_{Bot}	80	80
AlN Thin Film Thickness (nm)	T_{Aln}	500	500
Longitudinal Resonance (MHz)	f	37.20	36.78
Transverse Resonance (MHz)	$2f$	74.42	73.60
Longitudinal Mode Quality Factor	Q_s	65	78
Transverse Mode Quality Factor	Q_p	610	570

* Without prior empirical data on frequency setting, resonators with the same length and slightly different widths were fabricated to ensure the experimental demonstration of devices that can resonantly support the parametric amplification.

measuring the admittance at the signal port while a 74.40 MHz signal is pumped into the parametric port. The pump signal amplifies the fundamental oscillation at 37.20 MHz by periodically modulating the stiffness constants of the AlN thin film in the longitudinal direction via Poisson's effect. As seen in Fig. 3(b, c), the amplification results in a larger admittance at the parametrically excited resonance and enhances the measured Q . Parametric amplification was only observed in devices where the dual resonances in orthogonal directions possess a two-times frequency relation.

PARAMETRIC OSCILLATOR

Using the AlN parametric resonator, a parametric oscillator has been designed as shown in Fig. 4(a). In addition to the AlN parametric resonator, the parts used in the feedback loop are all off-the-shelf components from either Mini-circuit or RF Lambda. In order to establish a self-sustainable oscillation, the 37.2MHz signal

Figure 4. Schematics of the (a) parametric and (b) conventional oscillators.

out of the signal port of the parametric resonator has to be doubled in frequency via a frequency doubler (FK-5-S) before being looped back to the parametric port of the resonator to provide amplification. Following the frequency doubler, a voltage controlled phase shifter (RVPT0117MBC) is inserted in the feedback to provide phase adjustment for satisfying the phase condition required for sustaining the parametric oscillation. The phase shift was fine-tuned during testing to achieve the best oscillator response. Several filters are used in feedback as well. A combination of a low pass (SLP-90+ 81MHz) and high pass filter (ZX75HP-65-S+) is utilized to ensure that only the 74.4 MHz pump signal is sent into the parametric port. An out-of-band rejection of 50 dB is obtained in experiment using these filters. A low pass filter (ZX75LP-40 40MHz) is used to provide 43 dB rejection to the parametric pump signal leaked to the signal port via the feedthrough capacitance of the resonator. It also ensures that only the 37.2 MHz signal is outputted from the signal port and sent to the input of the frequency doubler. Even though the parametric amplification produces a substantial gain (20 dB), it is still insufficient to overcome the conversion loss of the frequency doubler and the cascaded insertion loss of the other components in the loop. More importantly, because the conversion loss of the frequency doubler can be significantly improved by increasing the input signal power, additional solid state amplifiers ((ZKL-1R5+ and ZKL-2+)) have to be added to boost the signal strength to reach the highly nonlinear region of the frequency doubler. This, however, does not degrade phase noise performance of the parametric oscillator since the noise introduced by the solid state amplifiers will be filtered and reduced by the parametric amplification process. At last, in order to perform signal analysis at both ports of the parametric resonator in the closed-loop configuration, two directional couplers (ZX30-9-4+) are incorporated to form ports 1 and 2 of the oscillator.

To highlight the phase noise reduction in a parametric oscillator, we have also constructed a conventional oscillator with its schematics shown in Fig. 4(b) for performance comparison. The frequency doubler and high pass filters are removed from the feedback. Alternatively, the required closed loop gain is provided by the solid state amplifiers. These amplifiers operate in the highly linear region and thus cannot generate a sufficiently large second order harmonic that might accidentally start the parametric

Figure 5. (a) Amplification gain as a function of frequency deviation for various modulation depths. Maximum allowable frequency deviation as a function of modulation depth Γ with respect to (b) Q_s and (c) Q_p .

amplification. To ensure a fair comparison, the same parametric resonator is also used in the conventional oscillator. The measured phase noise of both the parametric and conventional oscillators is compared in the Results section.

NOISE SQUEEZING

The dynamics of the AlN parametric oscillator can be described by a simple one-dimensional model that represents the modulation of the stiffness in the mechanical domain [6]:

$$\ddot{x} + \frac{\omega_r}{Q_s} \dot{x} + \omega_r^2 (1 - \chi(n, Q_p, t, \delta)) x + \beta x^3 = A \cos(\omega_r(1+n)t + \varphi) \quad (1)$$

$$\chi(n, Q_p, t, \delta) = 2\gamma A(Q_p) h(n, Q_p) \cos(2\omega_r(1+n)t + \delta) \quad (2)$$

where x represents the longitudinal-direction displacement of the vibration. The damping in the oscillation is denoted by the second term in Eq. 1, for which ω_r is the signal resonant frequency (i.e. $2\pi f$) and Q_s is the quality factor for the resonance at f .

The third term in Eq. 1 captures the effect of the parametrically modulated stiffness. As formulated in Eq. 2, $\chi(n, Q_p, t, \delta)$ represents the pump modulation. γ is the modulation coefficient that measures how effectively the transverse mode displacement changes the stiffness of the AlN thin film in the longitudinal direction. $A(Q_p)$ is the amplitude of the transverse mode vibration excited by the parametric signal ($2f$). Q_p is the quality factor of the parametric resonance. Naturally, $A(Q_p)$ increases for a larger Q_p . $\gamma A(Q_p)$ is defined as the modulation depth, Γ . n is the frequency deviation off ω_r in percentage for a stabilized oscillation. Because the feedback in the parametric oscillator is comprised by a phase shifter and frequency doubler, the pump signal has a phase of δ and the same frequency deviation off $2\omega_r$ as the driving signal off ω_r . $h(n, Q)$ is the normalized displacement amplitude (with respect to the maximum displacement) for a mechanical resonant system with a quality factor of Q . Given a pump signal of frequency $2\omega_r(1+n)$ and coupled to a resonance of quality factor Q_p , h can be expressed as [10]:

$$h(n, Q_p) = 1 / \sqrt{1 + n + n^2(2 + n)^2 Q_p} \quad (3)$$

Figure 6. Typical temporal response of an established parametric amplification process. ($Q_s=65$, $\omega_r=37.2$ MHz, $\beta=4.85 \cdot 10^{12}$, $\Gamma=1.435 \cdot 10^{-2}$, $Q_p=610$, $\Lambda=1.435$ nm).

The fourth term in Eq. 1 entails the nonlinearity in the restoring force. β is the nonlinear elastic coefficient, which has a rather small value for AlN thin films. The term on the right side of Eq. 1 is the oscillator driving/output signal at f . A and φ are the amplitude and phase of the driving signal, which has a frequency of $\omega_r(1+n)$. To set up parametric amplification, the requirement below must be satisfied [11]:

$$\gamma A(Q_p) h(n, Q_p) = \Gamma h(n, Q_p) > 1/Q_s \quad (4)$$

where Γ is required to be larger than a threshold value for overcoming the loss in the system and sustaining the parametric amplification. Because $h(n, Q_p)$ maximizes at $n=0$, a driving signal with a non-zero frequency deviation would impose a high threshold value of Γ . Increasing n will eventually result in a cut-off of the parametric amplification as seen in Fig. 5(a). Consequently, the phase noise at offsets beyond a maximum allowable frequency deviation is significantly reduced in such a system. The dependence of the maximum allowable frequency deviation on both Q_p and Q_s is shown in Fig. 5(b) and (c) for device 1 in Table I. A Q_s of 65 and ω_r of 37.2 MHz are used in the simulation. It is worth noting that, in addition to the maximum allowable frequency deviation, the parametric gain [Fig. 5(a)] also influences the outcome of noise squeezing. It directly determines the magnitude of phase noise in contrast to the carrier for offsets beyond the maximum allowable frequency deviation. A higher Q_p leads to an increase in Γ and consequently a harder noise squeezing effect.

An established parametric amplification process can be numerically modeled by the steady state response of Eq. 1. As shown in Fig. 6, the input/driving oscillation is amplified by the parametric pump signal. The phase noise squeezing in the oscillator can be qualitatively explained as parametric amplification selectively enhances the signal around ω_r and thus reduces sideband phase noise. Different from prior work, this work harnesses the noise squeezing effect that collectively derives from the high Q_p resonant coupling of the pump signal and the intrinsic narrowband nature of parametric amplification.

EXPERIMENTAL RESULTS

Two parametric oscillators have been constructed using devices 1 and 2 in Table I. Their conventional counterparts as seen in Fig. 4 have also been built for performance comparison. The signals from ports 1 and 2 of the parametric oscillator based device 1 were first characterized in the time domain with an Agilent DSO54855A oscilloscope and then in frequency domain with an Agilent E445A FSA spectrum analyzer. The measured results are presented in Fig. 7. We can observe that an f oscillation is sustained by a $2f$ input signal coupled to the geometrically optimized AlN resonator. A 20 dB amplification of the f signal is induced by the $2f$ signal via parametric amplification. Additional amplification of 54 dB in the feedback loop is provided by the solid state amplifiers.

Figure 7. Parametrically excited oscillations measured (a) in time domain and (b) in frequency domain. The oscillation (port 2 signal) at f is amplified and established by pumping a $2f$ with an optimal phase into port 1.

Figure 8. Comparison of the measured phase noise performance between the parametric and conventional oscillators.

As discussed in the last section, the phase noise reduction will be more prominent for offsets beyond the maximum allowable frequency deviation. The maximum phase noise reduction is predicted to be more than 20 dB based on the noise squeezing model, material properties of AlN, and the experimentally Q values of the AlN parametric resonators.

To characterize the phase noise performance, the signals from ports 1 and 2 of both the parametric and conventional oscillators were measured with an Agilent E5052A signal source analyzer. The results are shown in Fig. 8. A phase noise of -110 dBc/Hz at 1 kHz offset has been demonstrated for the parametric oscillators, 25 dB lower than the conventional oscillators based on the same resonators. The measurements have confirmed the noise filtering effect predicted by the model for such parametric oscillators.

CONCLUSIONS

In this work, parametric oscillators with low phase noise have

been implemented with geometrically optimized contour mode AlN resonators. The noise squeezing effect induced by parametric amplification has been theoretically analyzed and predicted to enable low phase noise performance. The implemented parametric oscillators have consequently demonstrated a 25 dB lower phase noise at 1 kHz offset than that of the conventional oscillators, thus promising the use of integrated AlN parametric oscillators as low phase noise frequency references.

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