LUMPED-ELEMENT ANALYSIS OF AN ELECTROSTATIC SQUEEZE-FILM MEMS DROPLET EJECTOR

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ABSTRACT

We present a theoretical model of an electrostatically driven, squeeze-film dominated, MEMS droplet ejector. The ejector consists of a microfluidic chamber filled with fluid to be ejected, an orifice plate, and a piston positioned a few microns beneath the orifice. Drop ejection occurs when a high-frequency drive voltage is applied between the orifice plate and the piston. The lumped-element model of the ejector takes into account the coupled electrical-structural-fluidic interactions that occur during drop ejection. The model is used to study effect of fluid properties and drive waveform on key performance parameters such as piston displacement and ejected fluid volume.

INTRODUCTION

MEMS are finding increasing use for applications that require the controlled generation and delivery of picoliter-sized droplets. Common applications include biomedical and biochemical microdispensing and most notably, inkjet printing. The most common MEMS drop ejectors operate in a drop-on-demand (DOD) mode. In DOD devices, micro-droplets are produced as needed by generating a sharp, short-lived pressure pulse within a microfluidic chamber beneath an orifice plate. The pressure profile is tuned to eject a droplet with a desired volume and velocity. The most common methods for producing the drop ejection pressure involve piezoelectric actuation or the generation of a thermally induced vapor bubble (bubble-jet).

In this paper, we discuss an alternative method of drop generation that is based on electrostatic actuation [1-4]. Specifically, we study a MEMS drop ejector that consists of a microfluidic chamber with a piston that is suspended a few micrometers beneath and orifice plate (Fig. 1). The piston is supported by cantilevered polysilicon flexure members that act as restoring springs when the piston is displaced from its equilibrium position (Fig. 1a). To eject a drop, a potential difference is applied between the orifice plate and the piston, and this produces an electrostatic force that moves the piston toward the orifice. The moving piston generates a squeeze-film pressure distribution in the gap region above it that acts to eject the drop (Fig. 2). Specifically, a peak pressure (stagnation pressure) obtains at a specific radius (stagnation radius), which is greater than the orifice radius. Thus, the fluid within the stagnation radius is confined, and forced through the nozzle as the piston moves toward it. A portion of this fluid ultimately detaches from the ejector and forms into a droplet; the remainder retracts back into the ejector as the piston returns to its equilibrium position. A drop ejector based on this principle has been fabricated and characterized at Sandia National Laboratories (Fig. 1) [2-3].

In this paper, we discuss the basic operating physics of the ejector, and we present an analytical lumped-element model for predicting its performance. We use the model to study device performance. We compare the analytical predictions with CFD analysis that takes into account the coupled piston-fluid interactions as well as to the experimental data.

Figure 1: MEMS drop ejector (adapted from [2-3]): (a) schematic showing cantilevered piston and cut away view of nozzle plate; (b) close-up view of orifice and piston, and (c) SEM image of ejectors (cover removed).

THEORY

Equation of Motion

We model the drop ejector using a lumped-element axisymmetric analysis (Fig. 2). The motion of the piston is obtained from the equation for the force balance on the piston

$$\left(m_p + m_{eff}(t)\right) \frac{dv_p}{dt} = F_a(t) - kx_p(t) - 2\pi \int_0^{r_p} p(r, v_p, t) r dr + \sum F_f \quad (1)$$

where m_p , $x_p(t)$, and $v_p(t)$ are the mass, position, and velocity of the piston, $m_{eff}(t)$ is the effective mass of the fluid that it accelerates, $F_a(t)$ is the applied electrostatic force, k is a spring constant for the polysilicon support members, and $p(r, v_p, t)$ is the squeeze-film pressure distribution that develops between the piston and the nozzle, which acts to resist the piston motion. The term $\sum F_f$ represents other forces due to the fluid motion.

Pressure Distribution

Figure 2: Axisymmetric model of MEMS drop ejector.

The pressure distribution $p(r, v_p, t)$ developed by the moving piston is obtained by applying Reynolds lubrication theory to the axisymmetric geometry shown in Fig. 2. The pressure above the piston satisfies the following equation

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p(r, t)}{\partial r} \right) = - \frac{12\mu}{h^3(t)} v_p(t) \quad (r_o \leq r \leq r_p) \quad (2)$$

where μ is the fluid viscosity, v_p is the piston velocity, r_o and r_p are the radius of the orifice and the piston, respectively, and $h(t)$ is the distance from the piston to the nozzle plate. The general solution to this equation is of the form

$$p(r, t) = - \frac{3\mu v_p(t)}{h^3} r^2 + c_1 \ln(r) + c_2, \quad (3)$$

where C_1 and C_2 are constants determined from boundary conditions [2-3]. The pressure distribution (3) peaks at a value p_s called as stagnation pressure at the stagnation radius $r_s(t)$,

$$r_s(t) = \sqrt{\frac{h^3 c_1}{6\mu v_p(t)}}, \quad (4)$$

as shown in Fig. 2. We assume that fluid above the piston and within the stagnation radius ($r \leq r_s(t)$) flows toward the orifice, while fluid beyond this point ($r > r_s(t)$) flows into the reservoir. The boundary conditions for this problem are

$$\begin{aligned} p(r, t) &= p_B(t) & (r = r_o) \\ p(r, t) &= p_R(t) & (r = r_p) \end{aligned} \quad (5)$$

where $p_B(t)$ and $p_R(t)$ are the pressures beneath the orifice ($r \leq r_o$), and at the edge of the piston, respectively, which are related to the flow rates at those points. Analytical expression for $p(r, t)$, $r_s(t)$, $p_B(t)$, and $p_R(t)$ can be found in the literature [4].

Effective Mass

We take into account inertial effects by estimating the mass of fluid accelerated by the piston as it moves. As above, we assume that the fluid within the stagnation radius flows toward the orifice, while the fluid beyond this point flows through the gap into the reservoir. From our analysis we find that the total effective mass of the fluid is [4]

$$\begin{aligned} m_{eff}(t) &= \rho \pi \left[\frac{r_p^3 + r_o^3 + 4r_s^3}{3} - r_s^2 (r_p + r_o) \right] \\ &+ \rho \pi l_o r_s^2(t) + \rho \pi l_o r_s^2(t) + \rho \pi l_p (r_p^2 - r_s^2(t)) \end{aligned} \quad (6)$$

Electrostatic Force

Figure 3: Electrical equivalent circuit of the electrostatic actuator in conducting liquids [5].

An expression for frequency-dependent electrostatic force between two parallel passivated electrodes separated by a conductive medium is given by Panchawagh [5]. This force is computed from an electrical equivalent circuit of the device shown in Fig. 3. The average electrostatic force $F_a(t)$ is given by

$$F_a(t) = \frac{V_i^2(t)}{2} \frac{dC_l(x)}{dx_p} = \frac{1}{2} \frac{\epsilon_o \epsilon_l A V_i^2}{(h - x_p)^2}, \quad (7)$$

where $C_l = \epsilon_o \epsilon_l A / (h - x_p)$ and V_i are the capacitance and the effective actuation voltage across the liquid gap between the piston and the orifice plate. $C_d = \epsilon_o \epsilon_d A / (2t_d)$ is the total capacitance of the native oxide passivation on the piston and orifice plate. Here, A is the electrode area, t_d is the passivation film thickness on each electrode, and ϵ_l and ϵ_d are the relative permittivities of the fluid and passivation film. V_i is calculated from

$$\left(1 + \frac{C_l}{C_d} \right) \frac{dV_i}{dt} + \left(\frac{1}{C_d} \frac{dC_l}{dx_p} \frac{dx_p}{dt} + \frac{1}{C_d R_l} \right) V_i = \frac{dV}{dt}, \quad (8)$$

where $R_l = (h - x_p) / (\sigma_l A)$ is the bulk resistance of the liquid between electrodes and σ_l is the electrical conductivity of the fluid. The electrostatic force given by Eq. (7) depends on piston position, device geometry, fluid electrical properties, and actuation waveform.

Other Forces

The equation of motion, Eq. (1) contains an expression $\sum F_f$ that accounts for additional forces due to fluid flow. Two such forces arise from the flow across the top surface of the orifice, and boundary of the gap at the reservoir. These additional forces have the form,

$$F_o(t) = \rho \pi r_o^2 v_o^2(t), \quad (9)$$

$$\text{and} \quad F_g(t) = \rho 2\pi r_p g v_g^2(t), \quad (10)$$

where $v_g(t)$ is the average velocity across the gap/reservoir interface. Time-dependent piston displacement can be thus

calculated from Eq. (1) by collecting all of the relevant terms and solving it simultaneously with Eqs. (3) and (8). To perform device simulation, we integrate this nonlinear ODE using a fourth-order Runge-Kutta method.

Fluid Ejection

We use Eq. (1) to study the behavior of the drop ejector. We solve for the piston velocity, and use this to obtain the average velocity $v_o(t)$ and volume flow rate $Q_o(t)$ of the fluid ejected through the nozzle,

$$v_o(t) = \frac{r_s^2(t)v_p(t)}{r_o^2}, \quad (11)$$

$$\text{and} \quad Q_o(t) = \pi r_s^2(t)v_p(t). \quad (12)$$

It is important to note that this analysis does not take into account the complex free-surface dynamics that govern the fluid-nozzle interaction and the ultimate formation of the drop, i.e. pinch-off, satellites etc. To compensate for this, we estimate the actual observed flow rate $Q_{exp} = \beta Q_o$ using a fitting parameter β , which we determine using CFD analysis. Once determined, this parameter is fixed for all of the analysis. We also track the total volume of fluid V_{eject} ejected during actuation by integrating the flow rate through the orifice during the applied force,

$$V_{eject} = \int_0^{\tau} Q_{exp}(t) dt, \quad (13)$$

where τ is the duration of the applied voltage or electric field.

RESULTS

Important parameters used in analysis are listed in Table 1. The device geometry, fluid properties are based on experimentally tested devices from [2-3]. In experiments, high-frequency carrier frequencies of greater than 2.5 MHz were used to prevent electrode screening and electrolysis in conductive ink. In the analytical model results presented here, we used a 1 MHz sinusoidal carrier waveform. During the experimental studies, it was observed that if the carrier frequency (internal signal frequency) was amplitude modulated over a short time pulses, on the order of 4 to 5 μ s, the sudden piston motion created sufficient pressure to eject a drop. The drop ejection rate was on the order of 1-10 kHz. Here, we limit the analysis time to 4.4 μ s as per reference [3]. Additionally, in the experiments, a custom shape of drive waveform was developed that kept the total electric field across the liquid gap between the piston and orifice plate to a constant value that was below the breakdown limit [2] in order to create a constant electrostatic force. The predicted piston displacement for a constant actuation force is plotted in Figure 4. The analytical model predicted a maximum piston displacement just over 2 μ m when the initial gap between the piston and the nozzle is 3.5 μ m. This calculation was verified using a CFD model of the drop ejector that assumed an equivalent constant actuation force over the pulse time and accounted for the fluid-structural interactions.

From Figure 4, it is clear that the piston dynamic response is sufficiently fast to follow the drive waveform at the time scale of electrical pulse. The CFD analysis predicted the volume of ejected fluid during the pulse to be 5.6 pL. Note that only the primary drop volumes were recorded in CFD analysis, i.e. satellite drops were not included. This was used to calibrate the fitting parameter $\beta=0.75$ to determine ejected fluid volume using analytical model. These

predicted ejected fluid volumes are comparable to experimental data published in [2-3].

Table 1: Typical values of parameters used in analysis.

Piston radius	70 μ m
Piston thickness	2 μ m
Orifice diameter	20 μ m
Orifice-Piston Gap	3.5 μ m
Fluid density	1000 kg/m ³
Fluid viscosity	1 cp
Fluid conductivity	0.001 S/m
Fluid relative permittivity	70
Passivation film	2 nm thick native oxide
Native oxide relative permittivity	4.5
Spring stiffness	100 N/m
Carrier frequency	1 MHz
Electrical pulse time	4.4 μ s

Figure 4: Comparison of piston displacement during the actuation pulse for different drive waveforms with parameters as specified in Table 1. The constant field waveform analytical model assumes a constant field of 25 V/ μ m for the duration of the pulse. Other waveforms analyzed include constant amplitude waveform (75 V), a linear reduction in applied voltage from 137V at the start to zero, and a sinusoidal amplitude modulation of 110 V at the start of the pulse to zero at the end. The results are compared to a CFD model that includes fluid-structural interactions but assumes a constant electric field or actuation force during the actuation pulse.

Next, we illustrate effect of drive waveform on drop ejector performance. Generally, the carrier frequency, actuation pulse duration, and the modulation of the peak voltage amplitude during the pulse are important parameters that affect the piston displacement and ejected fluid volume. Results for three additional waveforms are plotted in Figure 4. Here, the magnitudes of peak

voltages in each of the waveforms were adjusted to get a maximum final piston displacement close to that for the constant field case. Required peak actuation potential, the piston trajectory, and therefore the ejected fluid volume depended on the waveform shape. A lower peak potential was required for the constant voltage waveform while piston displacement reached a plateau with amplitude modulated waveforms. The ejected fluid volumes are listed in Table 2 for these cases.

Table 2: Predicted ejected fluid volumes for different drive waveforms. $\beta = 0.75$.

Waveform	Model	Volume (pL)
Constant Field	CFD	5.60
Constant Field	Analytical	5.52
Constant peak potential	Analytical	5.95
Linearly modulation of potential	Analytical	7.16
Sinusoidal modulation of potential	Analytical	6.26

The predicted values of electric field in fluid for different waveforms are plotted Figure 5. The peak electric field determines the breakdown of the fluid and should be maintained below a threshold that is on the order of 30-100 V/ μ m and depends on the pulse width [3]. Other considerations in selection of the drive waveform include the maximum field across the passivation film (native oxide) and joule heating caused by the power dissipation due to conductivity of liquid between the electrodes. The model presented here can be used to determine these limits on the actuation waveforms.

Figure 5: Instantaneous electric field across the liquid gap between the orifice plate and piston for different actuation waveforms.

In addition to the waveform optimization, the model is also useful in studying effect of device geometry, passivation film and fluid properties. For example effect of fluid electrical conductivity and fluid viscosity are plotted in Figure 6. As expected the piston displacement was lower for higher electrical conductivity of the fluid because of the electrode screening [5] while the higher fluid viscosity resulted in increased squeeze film force on the piston and therefore lower displacements.

CONCLUSIONS

An analytical model for electrostatic drop ejector is presented that accounts for the coupled interaction between the electrostatic

Figure 6: Piston displacement for different values of fluid conductivity and viscosity for linearly modulation of drive potential. Other parameters same as in Table 1.

actuation in electrically conductive viscous fluids. The model is useful to optimize device geometry and waveform as well as to study effect of important parameters on its performance. The modeling approach is applicable to a wide range of other MEMS devices including resonating structures for bio-sensing applications, integrated MEMS pumps and valves in microfluidic devices and other applications where electrostatic actuators are operated in viscous conductive fluids.

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