THERMO-ELECTRO-MECHANICAL MODELING OF HIGH FREQUENCY THERMALLY ACTUATED I²-BAR RESONATORS

Amir Rahafrooz and Siavash Pourkamali

Department of Electrical and Computer Engineering, University of Denver, Denver, CO, USA

ABSTRACT

This work presents development of a comprehensive electrical equivalent model for thermally actuated high frequency MEMS resonators with piezo-resistive readout and verification of the model with experimental results. We have previously demonstrated experimentally that as opposed to the general presumption, thermal actuation could be a very suitable mechanism for actuation of high frequency (VHF, UHF and higher) resonators and it becomes more powerful as the resonator dimensions are shrunk down [1]. Here, a combination of analytical, finite element and numerical analysis has been carried out to further justify this argument and allow prediction and optimization of the behavior of such resonators. It is demonstrated that thermal-piezoresistive nanomechanical resonators in the GHz frequency range, can provide motional conductance as high as 1mA/V with static power consumptions in the order of a few μ W.

INTRODUCTION

As a promising emerging technology for the next generation integrated frequency references, electronic filters, and resonant sensors, micromechanical resonators have been the center of significant attention over the past few years [2-6]. A wide variety of high frequency micro and nanoscale electromechanical resonators have been developed, most of which use piezoelectric [3,4] or electrostatic (capacitive) [5,6] electromechanical transduction. Piezoelectric micro-resonators require deposition of piezoelectric and metal thin films generally resulting in lower quality factors and frequency and quality control issues. In case of capacitive resonators, the weak electromechanical coupling leads to the need for deep submicron transduction gaps, associated with excessive squeezed film damping when operating in air (e.g. for sensory applications), fabrication challenges, and power handling limitations.

Thermal actuation is another well known mechanism that can be implemented at microscale without any fabrication challenges or the need for inconvenient material integration. It also has great properties such as large actuation force, low operating voltage and simplicity of design and integration. On the downside, power consumption and high body temperature of such actuators could limit their application in some cases. Furthermore, thermal actuators are usually considered and referred to as slow actuators only suitable for DC or very low-frequency applications. Consequently, although thermally actuated micromechanical resonant devices with frequencies in the hundreds of kHz have been utilized for chemical [7] or physical [8] sensory applications, there has been very little study on utilization of thermal actuation in high frequency resonant devices. In our previous work [1] we demonstrated thermally actuated resonators with frequencies as high as 61MHz (Fig. 1) using resonant structures usually referred to as I²-Bulk Acoustic wave Resonators (I²-BARs), or dog-bone resonators [5]. It was also briefly argued and shown experimentally that as opposed to the capacitive and piezoelectric actuators, for which actuator surface area is imperative for maintaining the required actuation force and output signal amplitude, thermal actuators become more powerful as the resonator dimensions are scaled down into the nanoscale to reach higher resonant frequencies.

In this work a comprehensive electrical equivalent model for thermally actuated high frequency MEMS resonators with piezo-resistive readout is developed and verified using the experimental results. The main goal is to study the transduction behavior of thermally-piezoresistive I²-BAR resonators with nanoscale dimensions operating in the GHz range. The model confirms that as the resonator dimensions are shrunk down the performance of thermally actuated resonators improves and higher or similar current gains can be achieved while consuming smaller DC power. Also this model paves the way to further optimize the resonator dimensions and structures.

Figure 1: a) Top view schematic diagram of a thermally actuated I²-BAR showing the current flow through the structure; b) SEM view of a fabricated 61MHz 15 μ m thick I²-BAR; c) Finite element modal analysis of the in-plane resonant mode of the structure (Red and blue show the maximum and minimum vibration amplitudes respectively); d) A measured resonance peak from the same device.

THERMO-ELECTRO-MECHANICAL MODELING

I²-BARs can be thermally actuated by applying a combination of DC and AC voltages between their two pads. This results in an alternating ohmic loss component with the same frequency as the input AC signal in the structure:

$$P_{ac} = \frac{2V_{dc}V_{ac}}{R_A} \quad (1)$$

where R_A is the electrical resistance of the thermal actuators and V_{dc} and V_{ac} are the DC and AC actuation voltages. The resulting temperature fluctuations in the thinner extensional bars in the middle of the structure actuate the structure in its in-plane resonant mode. The same pillars also act as piezoresistors modulating the DC current passing through them due to the alternating stress at resonance. Therefore, the overall input to output transfer function of the system can be divided into three sub-systems of thermal, mechanical and electrical nature as shown in Fig. 2a.

Figure 2b shows the equivalent electrical circuits of the thermal subsystem. The thermal equivalent circuit consists of a capacitance and a resistance representing the thermal capacitance and thermal resistance of the thermal actuators resulting in the following small signal electrical to thermal transfer function:

$$H_{th}(s) = \frac{T_{ac}}{v_{ac}} = \frac{2V_{dc}R_{th}}{R_A(1 + R_{th}C_{th}s)} \quad (2)$$

where T_{ac} is the fluctuating temperature amplitude, R_{th} and C_{th} are the effective thermal resistance and capacitance of the actuator, and $s=j\omega$ is the Laplace transform parameter.

The force generated by the resulting thermal stress can be calculated as:

$$F = 2\alpha T_{ac}AE \quad (3)$$

Where α is the thermal expansion coefficient of the thermal actuator structural material, A is the thermal actuator cross sectional area, and E is the Young's modulus of the thermal actuator along its length. The coefficient "2" has been added since there are two actuator beams in each resonator contributing to the actuation force.

Figure 2: a) Block-diagram demonstration of the thermo-electro-mechanical model of the resonator that can be divided into three sub-systems of thermal, mechanical and electrical nature; b) Equivalent electrical circuit for the thermal sub-systems; c) Equivalent electrical circuit for the mechanical sub-systems; d) Equivalent electrical circuit for the electrical sub-system.

In the equivalent electrical circuit of the mechanical subsystem shown in Fig. 2c, the voltage source represents the thermally generated mechanical force, and the inductor, capacitor, and resistor represent the effective mechanical mass, stiffness, and damping of the resonator respectively. Consequently, current represents velocity and charge ($Q = \int i dt$) represents displacement. Therefore:

$$X_{th}(s) = Q(s) = \frac{i(s)}{s} = \frac{F}{(Ms^2 + bs + K)} \quad (4)$$

where M , K and b are the effective mechanical mass, spring constant and damping coefficient of the structure, X_{th} is the

elongation amplitude of the thermal actuator. Replacing the actuation force from Eq. 3 into Eq. 4 results in the following transfer function for the mechanical subsystem:

$$H_m(s) = \frac{X_{th}(s)}{T_{ac}} = \frac{2\alpha AE}{(Ms^2 + bs + K)} \quad (5)$$

The undamped resonance frequency of the resonator is $\omega_0 = \sqrt{K/M}$ and its intrinsic quality factor is $Q = M\omega_0/b$. At the resonance frequency, the mechanical stiffness and mass in the mechanical transfer function cancel each other out and H_m is simplified to:

$$H_{m,res} = \frac{X_{th}(s)}{T_{ac}} \Big|_{s=j\omega_0} = \frac{2\alpha AEQ}{Mj\omega_0^2} = \frac{2\alpha AEQ}{jK} \quad (6)$$

where $j = \sqrt{-1}$ showing that the mechanical displacement is lagging behind the temperature (and force) fluctuations by 90°, which is typically the case for mechanical resonators at resonance. The displacement, which is the output of the mechanical subsystem, causes a change in the resistivity of the thermal actuators due to the piezoresistive effect:

$$r_{ac} = R_A \pi_l E \frac{X_{th}}{L} \quad (7)$$

where π_l is the longitudinal piezoresistive coefficient of the actuator beams, r_{ac} is the amplitude of the actuator resistance fluctuations, and L is the length of the thermal actuators. Since a DC bias voltage is applied to the thermal actuators, this change in resistance modulates the current in the actuators:

$$i_{ac} = \frac{r_{ac}}{R_A} I_{dc} \quad (8)$$

Therefore, the transfer function of the electrical subsystem (Fig. 2d) will be as follows:

$$H_e = \frac{i_{ac}}{X_{th}} = \frac{\pi_l EI_{dc}}{L} \quad (9)$$

Finally, the overall transfer function that relates the AC motional current to the AC input voltage is the product of the three transfer functions of the thermal, mechanical and electrical subsystems:

$$H_T = \frac{i_{ac}}{v_{ac}} = H_{th} H_m H_e = \frac{4\alpha AE^2 I_{dc}^2 \pi_l}{L(Ms^2 + bs + K)} \frac{R_{th}}{(1 + R_{th}C_{th}s)} \quad (10)$$

At resonance:

$$H_T \Big|_{s=j\omega_0} = \frac{4\alpha E^2 \pi_l Q A I_{dc}^2}{KLj} \frac{R_{th}}{(1 + R_{th}C_{th}s)} \quad (11)$$

If $\omega \gg \tau_{th}^{-1} = (R_{th}C_{th})^{-1}$, which is usually the case for the resonator dimensions in this work, H_T simplifies to:

$$H_T \Big|_{s=j\omega_0} = g_m = 4\alpha E^2 \pi_l Q \frac{A I_{dc}^2}{KLC_{th}\omega_m} \quad (12)$$

The transfer function in Eq. 12 can be referred to as small signal voltage to current gain, or motional conductance (g_m) of the resonator at its resonance frequency.

Resonator Equivalent Electrical Circuit: For the single-port I²-BAR resonators demonstrated in [1], where the same thermal actuators also act as piezoresistive sensors, the physical resistance of the resonator connects the input and output of the resonators. Therefore, the equivalent electrical circuit includes a resistance $R_A + R_s$ connected between the two terminals of the resonator, which is the overall resistance in the current path between the two pads of the resonator. R_s is the parasitic resistance in the current path which is not part of the actuators (mostly support beams). In parallel to the static resistance of the actuators, there needs to be a series RLC combination with the values shown in Fig. 3 that represents the mechanical resonant behavior of the structure.

Figure 3: overall equivalent electrical circuit for one-port thermally actuated resonators with piezo-resistive readout.

Calculation of Actuator Thermal Capacitance: To calculate the g_m of a thermal-piezoresistive I²-BAR, which is the most important parameter to be extracted from the model; all the parameters except the effective thermal capacitance of the actuators (C_{th}) are known for every set of resonator dimensions. Due to the distributed nature of the thermal parameters including thermal generator (resistance of the structure), thermal capacitance and thermal conductance, analytical derivation of the effective equivalent values for the thermal parameters could be quite complicated. Therefore, COMSOL finite element analysis was used to analyze the AC thermal behavior of the actuators and calculate the fluctuating temperature amplitude (T_{ac}) at different points along the length of the actuator beams (Fig. 4). The mean value of the small signal temperature amplitudes at different points along the actuators is the effective temperature fluctuation amplitude for the actuator.

If $\omega \gg \tau_{th}^{-1} = (R_{th} C_{th})^{-1}$, i.e. the thermal time constant of the actuator is much larger than the mechanical time constant of the resonator, which is generally the case for the resonator dimensions in this work, C_{th} can be calculated using the value of (T_{ac}) obtained from the finite element analysis and a rearranged and simplified version of Eq. 2:

$$C_{th} = \frac{I_{dc} I_{ac} R_A}{T_{ac} \omega_m} \quad (13)$$

RESULTS

In order to verify the validity of the derived models, several measurement results for I²-BAR resonators with different dimensions were compared to the values predicted by the model (Table 1). There is a good agreement between the measured and calculated g_m values for most of the data points. Considering the several possible sources of error the discrepancies between measurement and calculations seem to be quite justified. Some of the major sources of error include errors in calculation of the motional conductance from the logarithmic measured transmission data (from the network analyzer) and de-embedding the parasitic resistances, uncertainty in resistivity and other physical properties

of the structural material (e.g. piezoresistive coefficient) and dimensions, as well as changes in the material properties at elevated temperatures.

To further emphasize and demonstrate our hypothesis of thermal actuation becoming more suitable for higher frequency resonators, two scaled down versions of the resonator of Fig. 1 with resonant frequencies of 900MHz and 2.1GHz were analyzed using the developed model. As shown in Table 1, similar g_m as the 61MHz resonator can be achieved for the higher frequency resonators with significantly reduced DC bias current and power consumptions. Table 1 also includes examples of improved resonator dimensions for further minimization of resonator power consumptions in the GHz range.

It is desirable to maximize the resonator motional conductance (g_m) (for improved signal to noise ratio) with minimal power consumption. Therefore, a figure of merit for the resonators has been defined as the ratio of the resonator g_m to the overall DC power consumption and Q of the resonator:

$$F.M. = \frac{g_m}{Q \cdot P_{DC}} = 4\alpha E^2 \pi_l \frac{A}{K L C_{th} \omega_m (R_A + R_s)} \quad (14).$$

As clearly seen in Table 1, resonator figures of merit clearly increase for similar resonators (same structural aspect ratios) as the resonator dimensions are shrunk down.

Figure 4: a) COMSOL finite element thermal analysis of the resonator of Figure.1 by applying DC and AC currents of 60mA and 5mA respectively. The color code shows static temperature of the structure resulting from the DC bias current, and the numbers show temperature fluctuation amplitude (T_{ac}) of different points along the actuator; b) time domain temperature variations of a point in the middle of the thermal actuator showing T_{ac} of $\sim 0.01^\circ\text{C}$.

Table 1. Measurement and simulation results for thermal-piezoresistive I^2 -BARs with different dimensions.

Scale Factor	Resonator Dimensions (μm)						Measured/Assumed Parameters				Calculated Parameters				
							Current (mA)	Q. Factor	Freq. (MHz)	g _m (mA/V)	Power (mW)	R _A (Ω)	g _m (mA/V)	F.M. (10 ⁻⁴ /V ²)	Power (μW) @ g _m =1 (mA/V)
1X	274	30	67	31	4	10	16.8	29000	7.94	5.73	3.60	3.49	8.00	0.767	449
0.7X	193	21	47	22	2.7		40.5	37000	7.75	61.2	20.9	3.49	63.81	0.825	327
							26	24000	10.87	22.6	9.82	3.67	24.90	1.06	394
							30.5	16000	10.63	24.3	13.5	3.67	24.43	1.13	553
0.5X	144	16	35	16	2		21.3	12000	13.94	12.8	6.52	3.60	13.18	1.69	494
							30.2	7000	13.78	22.2	13.1	3.60	16.00	1.74	818
1X	80	53	16	64	17.4	10	55	28500	15.89	4.72	39.2	6.80	4.26	0.038	9191
							80	29000	15.65	17.5	82.8	6.80	9.60	0.040	8629
							50	10800	15.91	1.30	32.4	6.80	1.33	0.038	24346
							90	10900	15.73	9.30	105	6.80	4.50	0.039	23313
0.5X	39	25	8.5	32	8.5		20	29000	31.18	0.683	5.60	6.96	1.31	0.081	4276
							65	38500	30.72	24.2	59.2	6.96	19.21	0.084	3080
							30	11000	31.17	0.581	12.6	6.96	1.12	0.081	11264
							70	12000	30.9	6.17	68.6	6.96	6.82	0.083	10059
1X	22	15	4.4	18	5	15	60	14000	61.64	16.5	18.0	2.34	17.3	0.686	1041
							100	12000	60.85	62.3	50.0	2.34	42.8	0.713	1169
							60	7500	61.65	9.76	18.0	2.34	9.26	0.685	1945
							100	7700	61.11	37.5	50.0	2.34	27.1	0.704	1845
0.0723X	1.59	1.08	0.39	1.3	0.36	1.08	4.67	14000	900	-	1.57	32.6	17.3	7.87	90.7
0.5X Act.*				0.65	0.18		2.24	14000	904.5	-	0.378	32.6	17.3	32.7	21.8
0.25X Act.*				0.32	0.09		1.09	14000	905.7	-	0.0908	32.1	17.3	136	5.25
0.031X	0.68	0.46	0.14	0.56	0.15	0.46	1.99	14000	2100	-	0.667	79.1	17.3	18.5	38.6
0.5X Act.*				0.28	0.075		0.961	14000	2111	-	0.161	79.1	17.3	76.8	9.30
0.25X Act.*				0.14	0.037		0.475	14000	2113	-	0.0404	80.2	17.3	306	2.33
	= data obtained under atmospheric pressure														

* To improve the resonator figure of merit (lower power consumptions while maintaining the same g_m), the actuator beams have been shrunk down in these devices while keeping the same size for the rectangular masses on their two ends.

CONCLUSIONS AND FUTURE WORK

A comprehensive electrical equivalent model for thermally actuated high frequency I^2 -BARs was derived and verified using measurement results. A good agreement between the measurement data and theoretical calculations was demonstrated. As the model predicts and measurement data confirms, relatively high motional conductances can be maintained for such resonators by consuming orders of magnitude less power as their dimensions are shrunk down into the nanoscale. Based on the analysis in this work, thermal actuation of nanoscale resonators with frequencies in the GHz range is theoretically feasible and the power consumption for such devices will be in the few μW range.

The model also provides the capability to further optimize the thermal-piezoresistive resonators to achieve stronger electro-thermo-mechanical coupling (higher g_m), with lower power consumption.

ACKNOWLEDGEMENTS

This work was supported by National Science Foundation under grants #0839951 and #0800961.

REFERENCES

- [1] A.Rahafrooz, A.Hajjam, B.Tousifar, and S.Pourkamali, "Thermal actuation, a suitable mechanism for high frequency electromechanical resonators," IEEE MEMS 2010.

- [2] C. Nguyen, "MEMS technology for timing and frequency control," IEEE Trans. Ultrasonics, Ferroelectrics and Frequency Control, vol. 54, no. 2, pp. 251-270, Feb. 2007.
- [3] B. P. Harrington, and R. Abdolvand, "Q-enhancement through minimization of acoustic energy radiation in micromachined lateral-mode resonators," Transducers 2009, pp. 700-703.
- [4] M. Rinaldi, C. Zuniga, C. Zuo, and G. Piazza, "Ultra-thin Super High Frequency two-port ALN contour-mode resonators and filters," Transducers 2009, pp. 577-580.
- [5] G.K. Ho, K. Sundaresan, S. Pourkamali and F. Ayazi, "Low impedance, highly tunable, I2-resonators for temperature compensated reference oscillators," MEMS'05, pp. 116-120.
- [6] S. Pourkamali, et al, "Low-Impedance VHF and UHF Capacitive Silicon Bulk Acoustic-Wave Resonators—Part II," IEEE Transactions on Electron Devices, Volume 54, Issue 8, Aug. 2007, pp. 2024-2030.
- [7] J. H. Seo and O. Brand, "High Q-Factor In-Plane-Mode Resonant Microsensor Platform for Gaseous/Liquid Environment," JMEMS 2008, Vol. 17, issue 2, pp. 483-493.
- [8] A. Hajjam, J.C. Wilson, A. Rahafrooz, and S.Pourkamali, "Fabrication and characterization of resonant aerosol particle mass sensors," IEEE MEMS 2010.

CONTACT

Siavash Pourkamali, 2390 S. York St., Denver, Colorado, USA, Phone: +1-303-871-2471, Email: spourkam@du.edu