HETERODYNED ELECTROSTATIC TRANSDUCTION OSCILLATORS EVADE LOW FREQUENCY NOISE ALIASING

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ABSTRACT

We recently demonstrated a polarization technique that eliminates the frequency drift due to dielectric charge in MEMS oscillators [1]. In this work we show that these oscillators are also first-order insensitive to low-frequency voltage noise, with potential for significant positive impact on predicted close-to-carrier phase noise performance. Close-to-carrier noise also determines the limits of some resonant sensor applications [2, 3].

INTRODUCTION

Electrostatic MEMS resonators are typically polarized with dc voltages. By simply changing resonator polarization v_p to ac (at frequency ω_B , Fig. 1), it has been shown that frequency drift due to dielectric charging disappears [1]. Under this scheme, the system electrically oscillates at two frequencies simultaneously ($\omega_R \pm \omega_B$), an effect caused by heterodyning, i.e. frequency mixing, at both the input and output transducers of the device. The mixing of frequencies within the oscillator is summarized in Fig. 2. The polarization $v_p(t)$ can be generated by frequency division of the oscillator's output as discussed previously in [1].

Kajakari [4] identifies the inherent force nonlinearity and current nonlinearity to be the primary sources of the aliasing of low frequency noise (Fig. 3(a)) into close-to-carrier current noise in conventional electrostatic MEMS oscillators. This aliased low-frequency noise determines the phase noise performance of the oscillator. In this work we show that although these sources of aliasing still exist under the ac actuation scheme, they cause low-frequency noise (such as $1/f$ noise) to appear far from the carrier frequencies. Instead, it is the higher frequency noise near the polarization signal at ω_B (Fig. 3(b)) that aliases to the carrier sidebands.

Figure 1: General oscillator representation and test setup with artificial 'noise' injection. The polarization source is generalized to frequency ω_B and voltage amplitude v_B . For DC polarization we set $\omega_B = 0$.

Figure 2: A simplified model of circulating signals in a heterodyned electrostatic transduction oscillator. Here, ω_R is the device resonance frequency, and ω_B is the polarization frequency. Although the device continues to mechanically resonate at frequency ω_R , we can only observe $\omega_R \pm \omega_B$ in the electrical domain.

Figure 3: Dominant noise contributions from $v_p(t)$. (a) With conventional (DC) polarization, the dominant close-to-carrier noise source in the oscillator output is the aliased $1/f$ noise near 0 Hz (marked in blue). (b) With AC polarization, the dominant noise source in the oscillator is adjacent to the polarization frequency (marked in blue), and not low frequency noise.

THEORY

We propose an extended version of Leeson's phase noise model [5] for ac polarized MEMS oscillators.

$$v_{nj}^2(\Delta\omega) = \frac{R^2}{2} \left(1 + \left(\frac{\omega_R}{2Q\Delta\omega} \right)^2 \right) (i_{n1}^2(\Delta\omega) + i_{n2}^2(\Delta\omega))$$

for $j = 1, 2$, where the subscripts 1, 2 indicate the two carrier frequencies [1], Q is quality factor, R is the device motional impedance, ω_R is the device resonance frequency, and $\Delta\omega$ is the frequency offset. This shows that the total current noise injection in the two carrier sidebands (i_{n1}^2, i_{n2}^2 including all noise sources and aliasing effects) at the amplifier input determines the close-to-carrier noise (v_{n1}^2, v_{n2}^2) performance of these oscillators as well. However, since low frequency noise in $v_p(t)$ no longer aliases to close-to-carrier current

Figure 4: Conventional (DC polarized) oscillator showing injection of low frequency noise to the carrier sidebands. DC Polarization is $v_B = 7.07$ VDC with frequency $\omega_B = 0$ Hz (DC). Noise is $v_n = 100$ mVpp with frequency $\omega_n = 100$ Hz.

noise i_{n1}^2, i_{n2}^2 , the oscillator phase noise is no longer directly affected by low frequency electrical noise sources. As a result, no sidebands should appear adjacent to the carrier frequency when the noise is at low frequency.

EXPERIMENT

One can experimentally simulate low frequency noise injection through an additional voltage ‘noise’ source on the polarization terminal (Fig. 1). Noise sidebands appear adjacent to the carrier frequency output from the oscillator, consistent with Kaajakari’s noise aliasing theory [4], when v_p is dc polarization (Fig. 4). Under the ac polarization scheme, however, the same low frequency noise does not appear near the carriers (Fig. 5(a)), validating the prediction in the previous section. Instead, only close-to-polarization noise appears in the carrier sidebands (Fig. 5(b)).

CONCLUSION

It has been previously asserted that close-to-carrier phase noise in electrostatic MEMS oscillators is limited by a $1/f^3$ trend appearing from aliased $1/f$ voltage noise [4], which is unavoidable in dc sources. By moving the polarization away from dc, the oscillator phase noise is now limited by the quality of the ac source in use. This enables us to have additional engineering capability of the noise trends of the oscillator, as trends other than $1/f$ (such as $1/f^2$ or $1/f^3$) can be obtained with ac sources. However, it is unclear whether such ac sources can outperform the $1/f$ trend available with dc voltage supplies, when given the same resources.

In conclusion, although it has been previously shown that nonlinearities inherent in electrostatic resonant MEMS cause noise aliasing, this work demonstrates that the same nonlinearities can be exploited to evade the low frequency noise aliasing effect.

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(a) Noise $v_n = 200$ mVpp with frequency $\omega_n = 100$ Hz

(b) Noise $v_n = 100$ mVpp with frequency $\omega_n = 9.9$ kHz

Figure 5: Heterodyned oscillator with AC polarized resonator. Here, the $\omega_R + \omega_B$ carrier is shown. In both sub-figures the polarization is $v_B = 7.07$ Vrms with frequency $\omega_B = 10$ kHz. (a) Low frequency noise does not alias near the carriers. FM artifacts appear at very large noise magnitudes (not shown). (b) Only high frequency noise sources, i.e. near ω_B , appear near the carriers.

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