MEMS CANTILEVER BEAM ELECTROSTATIC PULL-IN MODEL

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ABSTRACT

This paper describes a cantilever beam pull-in voltage model in terms of beam length, thickness, initial dielectric gap, and beam material Young's modulus. A closed form beam deflection model is described for use with voltage actuated MEMS cantilever beams via an underlying mechanically fixed electrode. Electrostatic force is summed over the deflected cantilever beam using a function integrated over beam length evaluated from anchor to beam tip. The net electrostatic moment is applied normal to the cantilever beam tip as a function of the deformed beam displacement angle. The proposed model consistently predicted pull-in voltage with less error, when compared to empirical and simulated results, than previously reported theoretical models without the use of empirical correction factors. Pull-in voltage model prediction was improved by over 12% when compared to a previously described theoretical model using polysilicon cantilever beam latch measurements as a reference.

INTRODUCTION

Multiple electrostatic pull-in models have been previously developed over the past decade for use in high-end MEMS software simulation tools [1,2]. Typically, these software simulation tools are based on finite element analysis (FEA) techniques requiring multiple solution iterations distributed amongst a plethora of nodes before converging to desired solution resolution values. Finite element analysis simulation provides a critical step towards MEMS design optimization and verification prior to device fabrication and characterization phases. However, FEA based model solution is not easily implemented via manual calculation techniques and can be difficult to include in high-level system transfer functions generated during early project definition phases.

MEMS system designers typically desire a more generic pull-in relationship [3] between critical beam layout dimensions and operational device voltage ranges during early phases of project transfer function specification using compact manual equation calculations and techniques without the use of empirical correction factors [4,5]. This first pass design feasibility approach allows the MEMS designer to identify and estimate critical device operation parameters to be optimized via computer based simulation techniques during subsequent project phases. Additionally, these compact models are useful to demonstrate generic system input/output parameter trends while training junior designers and engineering students. This is analogous to manually derived high-level transfer function approximations typically used by CMOS integrated circuit designers to model analog circuit blocks prior to system optimization and verification using high-end SPICE software tools.

Numerous MEMS applications incorporate singly clamped cantilever and/or doubly clamped beams as an integral part of their design. A brief application list includes resonators [6-9], vapor/pressure sensors [10-12], accelerometers [13,14], high-Q electronic filters [15-16], and micro relay switches [5,17,18].

Surface micromachined polysilicon cantilever beams suspended above an isolated electrode are common throughout MEMS commercial applications [13,14].

ELECTROSTATIC PULL-IN MODEL

The cantilever beam is defined in term of length, width, and thickness as shown in Figure 1.

Figure 1. Polysilicon cantilever beam dimensions.

The electrostatic moment M_1 (1) applied at the beam tip is described in terms of the beam displacement v along the x-axis in the z-direction [19], where E is the Young's modulus (150GPa), and I is the moment of inertia (2), as shown in Figure 2.

$$M_1 = -EI \frac{d^2v}{dx^2} \quad (1)$$

$$I = \frac{WT^3}{12} \quad (2)$$

Figure 2. Cantilever Beam w/Moment Applied at Free End.

The beam displacement v along the x-axis in the z-direction [19] is given in (3).

$$v = \frac{M_1 x^2}{2EI} \quad (3)$$

The electrostatic moment M_1 applied at the beam tip (4) represents the summed electrostatic force F_M along the beam (5) multiplied by the resulting beam tip deflection Δ_z (6) using beam displacement v (3), where Δ_z and Z_{tip} are defined at $x=L$ as measured from the fixed electrode reference. Note that the magnitude of F_M also represents the beam mechanical restoring force magnitude.

$$M_1 = F_M L \quad (4)$$

$$|F_M| = K_z \Delta_z \quad (5)$$

$$\Delta_z = z_0 - z_{tip} \quad (6)$$

The z-axis cantilever beam mechanical spring constant for a tip applied moment (7) is given by (8).

$$K_z = \left| \frac{F_M}{v(x=L)} \right| \quad (7)$$

$$K_z = \frac{EWT^3}{6L^3} \quad (8)$$

The capacitance between the deflected beam and fixed electrode is defined by (9) with substitution shown in (10) and simplification shown in (11).

$$C_{beam} = \int_{r_0}^L \frac{\epsilon_o W}{z_o - v} dx \quad (9)$$

$$C_{beam} = \int_{r_0}^L \frac{\epsilon_o W}{z_o - \frac{K_z(z_0 - z_{tip})Lx^2}{2EI}} dx \quad (10)$$

$$C_{beam} = \int_{r_0}^L \frac{\epsilon_o W}{z_o - \Delta_z \frac{x^2}{L^2}} dx \quad (11)$$

The indefinite integral is shown for this function in (12).

$$C_{beam} = \frac{\epsilon_o LW}{\sqrt{z_o \Delta_z}} a \tanh \left(\frac{x \Delta_z}{L \sqrt{z_o \Delta_z}} \right) \quad (12)$$

Model Simplification is possible by setting the electrode length equal to the beam length such that $r_0=0$ in (11) and (12) as shown in (13).

$$C_{beam} = \frac{\epsilon_o LW}{\sqrt{z_o \Delta_z}} a \tanh \left(\frac{\Delta_z}{\sqrt{z_o \Delta_z}} \right) \quad (13)$$

The summed electrostatic force is defined by (13) for static electromechanical equilibrium where V_M represents the applied voltage between the beam and fixed electrode. Note that F_M represents the system mechanical restoring force (5) and Z_{tip} is the distance measured from the fixed electrode to the displaced beam defined by (6).

$$F_M = -\frac{1}{2} \frac{dC_{beam}}{d(z_{tip})} V_M^2 \quad (13)$$

The applied beam voltage parameter listed in (13) is rearranged in (14) with solution given in (15).

$$V_M = \sqrt{\frac{2K_z(z_o - z_{tip})}{-\frac{dC_{beam}}{d(z_{tip})}}} \quad (14)$$

$$V_M = \sqrt{\frac{2K_z(z_o - z_{tip})z_0 z_{tip}}{\epsilon_o LW \left(z_0^2 - z_0 z_{tip} + z_{tip} \left(\sqrt{z_0(z_0 - z_{tip})} \right) a \tanh \left(\frac{z_{tip} - z_0}{\sqrt{z_0(z_0 - z_{tip})}} \right) \right)}} \quad (15)$$

The pull-in voltage V_{MPI} [20] given by (16) represents the peak voltage (15) satisfying electromechanical static equilibrium swept over the beam tip displacement range $0 < z < z_0$ as shown in Figure 3.

Figure 3. Normalized actuation voltage versus normalized beam tip displacement.

$$V_{MPI} = 2 \sqrt{\frac{2EIz_0^3}{3\epsilon_o L^4 W}} \quad (16)$$

The moment of inertia (2) is substituted into (16) and simplified as given by (17) in terms of beam length L , beam thickness T , initial dielectric beam gap z_0 , and the material dependent Young's modulus E .

$$V_{MPI} = \frac{1}{3} \sqrt{\frac{2ET^3 z_0^3}{\epsilon_o L^4}} \quad (17)$$

THEORETICAL PULL-IN MODEL COMPARISON

Cantilever beam pull-in voltage has been previously described using a novel beam theory model [3] by parameter V_{th} and a parallel plate model [21] by parameter V_{PI} as shown in Figure 4. The parallel plate model parameter V_{PI} estimates the pull-in voltage to be approximately $1/2$ that predicted by V_{th} . Typically, V_{PI} underestimates pull-in voltage. Similarly, V_{th} typically overestimates pull-in voltage. As beams width is increased, V_{th} typically predicts pull-in voltage with less error than V_{PI} . Maximum beam tip displacement, which has been previously measured using a confocal microscope as $0.46\mu\text{m}$ [22], is more

accurately described by V_{th} when compared to V_{PI} regarding model predicted maximum tip displacement prior to pull-in.

Comparing pull-in voltage magnitudes and beam tip displacements we note that V_{th} and V_{MPI} predict similar maximum stable beam tip displacements as $0.45z_0$ and $0.463z_0$ respectively. This small difference in maximum stable displacement is attributed to the similar deflected beam shape functions utilized by these models as shown in Figure 5. Note that the previous model directs electrostatic force exclusively along the z-axis, while the proposed model applies electrostatic force normal to the beam tip. The tip moment applied electrostatic force presented in this paper is analogous to the electric field boundary condition regarding field lines terminating normal to electrically conductive surfaces, such as the deflected beam and fixed electrode surfaces.

Figure 4. Normalized actuation voltage versus normalized beam tip displacement.

Figure 5. Normalized beam displacement contour versus beam length normalized from anchor to (beam tip = $0.463z_0$)

EXPERIMENTAL RESULTS

Polysilicon cantilever beam arrays were fabricated on Si_3N_4 passivated silicon wafers as shown in Figure 6. The characterized polysilicon beam arrays were $160\mu\text{m}$ long and $2\mu\text{m}$ thick. Beam widths were varied at $2\mu\text{m}$, $4\mu\text{m}$, and $6\mu\text{m}$. The initial dielectric gap was fixed at $2\mu\text{m}$ for all beam arrays. A typical polysilicon beam array with $160\mu\text{m}$ by $6\mu\text{m}$ by $2\mu\text{m}$ dimensions is shown in Figure 7.

Capacitance-Voltage (C-V) measurements [22] were performed using an HP-4284A LCR meter controlled via LabView software. Electrostatic actuation was accomplished by increasing the LCR meter DC bias voltage between the cantilever beam and underlying electrode in discrete 50mV increments. The pull-in voltage models are compared to empirical results in Table 1.

Figure 6. Polysilicon cantilever beam arrays with substrate fixed actuation electrodes.

Figure 7. Typical polysilicon cantilever beam array ($160\mu\text{m}$ long, $6\mu\text{m}$ wide, and $2\mu\text{m}$ thick) with underlying actuation electrode $2\mu\text{m}$ dielectric gap.

Table 1. Polysilicon cantilever beam electrostatic pull-in voltage model prediction values compared to empirical results.

Beam Array ID #	L = 160u T = 2u Width [u]	Empirical Pull-in [V]	Fig. 5 Vth [V]	Fig. 5 Vpi [V]	Eq (17) Vmpt [V]	Fig. 5 Vth % Error	Fig. 5 Vpi % Error	Eq (17) Vmpt % Error
1	2	14.20	22.28	11.07	19.18	36.3	-28.3	26.0
2	2	14.35	22.28	11.07	19.18	35.6	-29.6	25.2
3	2	14.15	22.28	11.07	19.18	36.5	-27.8	26.2
4	4	15.80	22.28	11.07	19.18	29.1	-42.7	17.6
5	4	16.30	22.28	11.07	19.18	26.8	-47.2	15.0
6	4	15.90	22.28	11.07	19.18	28.6	-43.6	17.1
7	6	17.85	22.28	11.07	19.18	19.9	-61.2	6.9
8	6	17.35	22.28	11.07	19.18	22.1	-56.7	9.5
9	6	17.60	22.28	11.07	19.18	21.0	-59.0	8.2

Multiple polysilicon cantilever beams were observed to remain latched [23] to the underlying polysilicon electrode post pull-in voltage excitation as shown in Figure 8. Beams were easily released from the underlying electrode using micromanipulator probes with no visual electrical current damage or welding observed.

ANSYS finite element analysis (FEA) software was used to simulate cantilever beam pull-in voltage with simulation results compared to theoretical model prediction and empirical results in

Table 2. The 3D solid 122 Ansys element type prediction yielded results with less than 0.6% Error when compared to empirical pull-in data.

Figure 8. Post electrostatic voltage actuated polysilicon cantilever beam tip to fixed electrode stiction.

Table 2. Pull-in voltage simulation results compared to model prediction values and average empirical result (6 μ m wide beams).

ANSYS Element Dimension	Ansys Element Type	Simulated	Empirical	Fig. 5	Fig. 5	Eq. (17)
		Pull-In [V]	Pull-In Avg [V]	Vth [V]	Vpi [V]	Vmpi [V]
2D	Plane82 with plane strain	22.3	17.6	22.28	19.18	18.50
2D	Plane82 with plane stress	18.5	17.6	22.28	19.18	18.50
3D	Solid122, W=6 μ m	17.5	17.6	22.28	19.18	18.50

CONCLUSIONS

We have presented a closed form algebraic model describing MEMS cantilever beam electrostatic actuation and pull-in. Maximum displacement of the beam tip is predicted to occur at 46.3% of the initial dielectric gap just prior to electrostatic pull-in. This model accounts for beam deflection applicable to cantilever beam and micro-relay electromechanical systems where the underlying electrode fully extends to the cantilever beam tip. The proposed beam theory model %Error was compared to empirical pull-in voltage measurements for the 2 μ m, 4 μ m, and 6 μ m wide beams as 25.8% ($\sigma = 0.5\%$), 16.6% ($\sigma = 1.4\%$), and 8.2% ($\sigma = 1.3\%$), respectively. Pull-in voltage model prediction was improved by 12.8% when compared to a previously described model using polysilicon cantilever beam latch measurements as a reference. We attribute the observed improvement in model performance to the tip applied beam moment to represent electrostatic force distributed along the beam underside in this paper. Applying the summed electrostatic force under the deflected beam normal to the beam tip as a moment reduced the system defined spring constant K_z by 33.3%. Reduction of this spring constant was attributed to better pull-in model prediction.

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