DESENSITIZING METHOD FOR MEASUREMENT OF THIRD-ORDER INTERMODULATION DISTORTION IN CMOS-MEMS MICROMECHANICAL RESONATORS

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ABSTRACT

A new desensitization method to characterize linearity of micromechanical resonator filters is introduced. The practical measuring range is larger than either the conventional 1 dB compression test or the two-tone test for RF frequencies. An analytic expression for a new 1 dB desensitizing point (DP_{1dB}) metric is directly related to the 1 dB compression point (CP_{1dB}). This method is verified on a CMOS-MEMS square-frame resonator (SFR) with resonant frequency of 5.849 MHz, quality factor of 1268 and insertion gain of -31.8 dB and -41.7 dB for input polarization voltage of 10 V and 30 V, respectively. The measured DP_{1dB} is 26.7 V ac corresponding to a CP_{1dB} of 32 dBm.

INTRODUCTION

Recent progress of microresonators as passive components in wireless communication applications achieves high quality factor (Q) and high operational frequency. For example, hollow-disk ring resonators have been demonstrated with $Q > 60,000$ at 24 MHz and $Q > 14,000$ at 1.2 GHz [1]. By mechanical or electrostatic coupling, MEMS resonators yield filtering and mixing functions with miniature size [2-4]. However, in order to fulfill strict wireless communication specifications, such as the minimum total third-order input intercept point (IIP3) of -18 dBm in the signal path for the European GSM standard [5], the measurement and the modeling of the nonlinear effects of micro resonators is critical.

Nonlinear effects of microresonators are presented, and two types of linearity measurement are discussed in part 1. In part 2, a new desensitization method is introduced, the measurement set-up is illustrated, and the mechanism of this method is examined. A series of experiments are conducted and the validation of this measurement method is analyzed in part 3.

Figure 1. Example of measured range for (A) a two-tone test and (B) a 1 dB compression test. Shaded areas are inaccessible for measurement.

NONLINEAR MICRORESONATORS

Nonlinearity of the communication systems, especially the third-order intermodulation distortion, is critical to quantify because it usually defines the upper bound of the power handling ability. Nonlinearity occurs when out-of-band tones are modulated and results in generation of in-band components. When the input signal, V_{in} , consists of a desired signal, $V_i \cos \omega_i t$, and two out-of-band interference tones, $V_{im-1} \cos \omega_{im-1} t$ and $V_{im-2} \cos \omega_{im-2} t$, the output components after transferring through a nonlinear system are given by [6]

$$V_o = (a_1 V_i + \frac{3a_3 V_i^3}{4} + \frac{3a_3 V_i V_{im-1}^2}{2} + \frac{3a_3 V_i V_{im-2}^2}{2}) \cos \omega_i t + \frac{3a_3 V_{im-1}^2 V_{im-2}}{4} \cos((2\omega_{im-1} - \omega_{im-2})t) + \frac{3a_3 V_{im-1} V_{im-2}^2}{4} \cos((2\omega_{im-2} - \omega_{im-1})t) + \dots \quad (1)$$

where component (a) is the fundamental tone and (e) and (f) are the intermodulation tone (IM). The nonlinear system in (1) is modeled as

$$V_o = a_1 V_{in} + a_2 V_{in}^2 + a_3 V_{in}^3 \dots \quad (2)$$

In order to quantify this phenomenon, the two-tone test has been applied by other groups [7-8]. This method measures the effect of component (e) and (f) in (1). The frequencies of the two tones are selected to maximize the IM tone, given by

$$2\omega_{im-1} - \omega_{im-2} = \omega_r \quad \text{or} \quad 2\omega_{im-2} - \omega_{im-1} = \omega_r \quad (3)$$

where ω_r is the resonant frequency. By plotting the output power versus input power for the fundamental tone and IM tone, the intercept of the extrapolation of these two lines is IIP3, shown in Figure 1(A). In addition, IIP3 is given by [6]

$$IIP3_{dBm} = \frac{\Delta P_{dB}}{2} + P_{in, dBm} \quad (4)$$

Where P_{in} is the input power and ΔP is the power difference between fundamental and IM tone. However, the mechanical filtering characteristic suppresses off-resonance output signals to some degree, which degrades the accuracy of extrapolating IIP3. This problem is magnified in the case of high quality factor resonators. The measurement range of this method is a function of dynamic range of measurement instruments, testing range, and the insertion loss of the measured device. For example, in the case that the maximum output power of signal generator is 20 dBm, the noise floor of spectrum analyzer is -100 dBm, the insertion loss of the measured device is 40 dB, and the testing range is

Figure 2. Desensitization test of intermodulation in a nonlinear resonator

15 dB, the intercept is point (A) shown in Figure 1(A), and the corresponding IIP3 is 37.5 dBm, which is approximately the upper bound of measurement range in the two-tone test. For a highly linear device with IIP3 in point (B), the IM tone is below the noise floor and IIP3 is not measurable.

Another method to determine the linearity is to measure the CP_{1dB} , which measures the effect of component (b) in (1). Nevertheless, if the nonlinear component is too weak relative to the component (a), then a high amplitude signal source is needed to cause measurable nonlinearity. This large input signal requirement limits the measurement range of this method. Given a maximum RF source power of around 20 dBm, the nonlinearity can be measured on microresonators with CP_{1dB} less than 20 dBm, as shown in Figure 1(B). The corresponding IIP3 is about 29.6 dBm, which is given by [6]

$$IIP3_{dBm} = CP_{1dB} + 9.6 \quad (5)$$

This relation is an approximation based on (2). Because of the effect of mechanical filtering, (5) might not hold for every situation, and a detailed IIP3 model for the tested device is needed.

DESENSITIZATION METHOD

As shown in Figure 2, this work introduces a desensitization method to measure the linearity of HF CMOS-MEMS square-frame resonator that consists of four free-free beams connected at the nodal points of the first flexural resonant mode [9]. While introducing a low-frequency strong desensitizing tone at ω_d with high-frequency desired signal within the filter passband at ω_i , the insertion gain will drop due to the third-order intermodulation effect, which is similar to component (c) or (d) in (1). The advantages of this method are that: first, it measures only the resonant output signal, which is not affected by the off-resonance suppression. Second, due to lower frequency, it is easy to obtain a large-magnitude signal source for the desensitizing tone. This extends the measuring range for testing highly linear devices. Third, fewer instruments are required than in the two-tone test. This method needs only a network analyzer and one low-frequency signal source, compared to two high-frequency signal sources and a spectrum analyzer for the two-tone test.

The schematic of the measurement setup is shown in Figure 3. The input signal, $V_i \cos \omega_i t$, the input polarization voltage, V_p , and the desensitizing tone, $V_d \cos \omega_d t$, are applied across the input electrodes. Fundamental force and nonlinear forces are given by

$$F = \frac{\partial C_i}{\partial x} (V_i \cos \omega_i t - V_d \cos \omega_d t - V_p)^2 + \frac{\partial C_o}{\partial x} (V_{po})^2 \quad (6)$$

where C_i and C_o are input and output capacitance, x is the mechanical displacement and V_{po} is the output polarization

voltage. With the assumption that mechanical displacement is much smaller than the electrode gap, (6) can be expanded into a Taylor series. With small electrode gaps, mechanical spring nonlinearities are negligible compared to the capacitive nonlinear effect [7]. The mechanical displacement is solved from the equation of motion with perturbation techniques [10]. The output voltage components at resonant frequency are

$$V_{O,fi} = \left[\left(i \frac{4C^2 Q V_p V_{po}}{g^2 K} - i \frac{32C^4 Q^3 V_d^2 V_p^3 V_{po}}{g^6 K^3} + \frac{8C^3 Q^2 V_d^2 V_p V_{po}}{g^4 K^2} \right) V_i \right. \\ \left. - i \frac{72C^4 Q^3 V_p^3 V_{po}}{g^6 K^3} V_i^3 + \dots \right] \omega_i Z \quad (7)$$

where C_i and C_o are replaced by C with the assumption of negligible mismatch, Q is the quality factor, g is the electrode gap, K is mechanical stiffness, Z is the transimpedance of the on-chip amplifier. The second and third terms of (7) are the first two substantial nonlinear output components for the tested device with the parameters listed in the Table 1. If the second term is the largest nonlinear component, the magnitude of output the voltage will decrease with increasing desensitizing voltage. However, if the third term is largest, the magnitude of the output voltage will increase with increasing desensitizing voltage.

EXPERIMENTAL RESULTS

The SEM picture of the tested resonator is shown in Figure 4. To model this resonator, results from a series of tests of sweeping input polarization voltage are shown in Figure 5. The resonant

Figure 3. The linearity measurement set-up for the desensitizing method and the perspective view of a resonator chip on a custom-designed test board.

Figure 4. The SEM picture of a SFR with self-assembling electrodes to provide small electrode gaps. [9]

$$CP_{1dB} = 20 \log \left(\frac{DP_{1dB}}{0.16} \right) - 12.19 \quad (9)$$

This superior linearity is due to a medium size electrical gap of 0.84 μm , compared to the results in other work [7-8]. A plot of DP_{1dB} , CP_{1dB} and S_{21} versus electrode gap size is shown in Figure 8, which shows a trade-off between linearity and insertion gain in reducing electrode gap size.

The desensitization method is eventually limited by the nonlinear effects in amplifiers and the network analyzer, and by electrical breakdown on the tested device at high values of V_d . While there are strong tones along with fundamental tone in the input of amplifiers and network analyzer, the nonlinear effect of those stages blocks the fundamental tone, the mechanism of which is similar to the phenomena shown in (1). Output spectrums with

Figure 5. (A) Measured response and analytic solution of SFR with V_p of 30 V, 20 mTorr of pressure, and V_{in} of 10 dBm for 1st flexural resonant mode. The parasitic effect is measured and subtracted. (B) and (C) are the extracted measured result and analytic solution of maximum S_{21} and resonant frequency versus V_p , respectively

frequency is 5.849 MHz, the quality factor is 1268, and the insertion gain is -31.8 dB and -41.7 dB for V_p of 10 V and 30 V, respectively. Figure 5(A) shows the measured frequency response with the analytic solution, which is calculated from the features listed in the Table 1. The effect of input polarization voltage in the S_{21} and resonant frequency is measured and calculated, shown in Figure 5(B) and (C). The measurement result is consistent with analytic model, showing that increasing the V_p will increase S_{21} and lower the resonant frequency. The discrepancy at low V_p in the Figure 5(C) might result from the measurement error with small magnitude signal and insufficient measurement resolution.

Frequency responses with different desensitizing voltages are measured, shown in Figure 6. The measured maximum S_{21} values are extracted and plotted with the analytic solution in Figure 7. The analytic solution is modeled by (7) with the parameters listed in the Table 1 where the second component is the dominant nonlinear effect and causes S_{21} drop while increasing desensitizing voltage. With 20 Vac of desensitizing voltage, the S_{21} drops to 0.55 dB; this is slightly larger than analytic result of 0.49 dB. This measurement validates the features of tested device listed in the Table 1 and the nonlinear model based on (7).

In order to compare the linearity performance of different devices, the DP_{1dB} is defined as the magnitude of desensitizing tone to cause 1dB drop in S_{21} . Neglecting other nonlinear components except the dominant second term in (7), the DP_{1dB} is

$$DP_{1dB} = 0.116 \frac{g^2 K}{CV_p Q} \quad (8)$$

For the tested devices, the DP_{1dB} is 26.7 Vac. The CP_{1dB} is calculated to be 32 dBm, caused by the forth term in (7), related to DP_{1dB} by

Figure 6. Measured response of the SFR for different desensitizing voltages, V_d , with $f_d = 1$ kHz, $V_p = 42.5$ V, $V_{po} = 45$ V, pressure = 11 mTorr, and $V_{in} = 10$ dBm.

Table 1. Features of tested microresonators

	Measured	Analytic
Beam Length(μm)	64.7	-
Beam Width(μm)	4.6	-
Beam Thickness(μm)	-	5.0
Input/Output Capacitor Width(μm)	-	20
Input/Output Capacitor Gap, g (μm)	-	0.84
Output Polarization Voltage, V_{po} (V) (for tests in the Figure 5)	20/-17.4	18.7
Input/output Polarization Voltage, V_p (V) (for tests in the Figure 6)	42.5/45	45.5/47
Mechanical Stiffness, K (N/m)	-	13942
Resonant Frequency(MHz)	5.849	5.849
Quality Factor, Q	1268	1268
S_{21} (dB) (for tests in the Figure 5, $V_p=30$ V)	-31.8	-31.8
Input Power, P_{in} (dBm)	10	10
On-Chip Amp. Transimpedance, Z (M Ω)	-	40.6

Figure 7. Measured and analytic S_{21} versus the desensitizing voltage, V_d . The measured data is extracted from Figure 6 and the features applied in analytic model are listed in the Table 1.

Figure 8. Analytic results of DP_{1dB} , CP_{1dB} and S_{21} versus the electrode gap with the assumption that all the features listed in the Table 1 are constant when adjusting the electrode gap.

different frequencies of the desensitizing tone and the magnitude difference between the fundamental tone, f_i , and the mixing tone, $(f_i - f_d)$, are shown in Figure 9. Since the magnitude of mixing tone is 12.5 dB smaller than the fundamental tone, the nonlinear effect of the amplifiers and network analyzer is insignificant.

Electrical breakdown on the tested device is the major limitation in this device. In one case, a combination of the V_d of 30 Vac at the f_d of 1 kHz with the V_p of 42.5 V has caused electrical breakdown of an input electrode on the microresonators. However, with improved layout to enhance the isolation of electrical wires, this range might be increased.

Another noticeable phenomenon is the magnitude drop with different f_d in the Figure 9, which results from the mechanical resonant shape. Even though this drop is not significant, it causes inaccuracy in the nonlinear models of (7) and the expression of DP_{1dB} of (8). A complete model is under development.

CONCLUSIONS

A new desensitization method of characterizing linearity of MEMS resonator filters is introduced. Verifications are performed on a CMOS-MEMS square-frame resonator, showing that the DP_{1dB} is 26.7 Vac and CP_{1dB} is calculated to be 32 dBm. This method is able to measure nonlinear effect of highly linear resonator filters, with a measuring range larger than that of a 1 dB compression test or a two-tone test. An analytical expression of 1 dB desensitization point was presented and verified, which can be used to balance the trade-off between linearity and signal gain in future resonator designs.

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Figure 9. Measured output spectrum of the SFR with input frequency of 5.8474 MHz along with desensitized tone of 15 Vac at different frequency, f_p . (A) 0.5 kHz (B) 1 kHz and (C) 1.5 kHz

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