PARALLEL-PLATE DRIVEN OSCILLATIONS AND RESONANT PULL-IN

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ABSTRACT

This work characterizes parallel-plate actuators for oscillations near the mechanical resonant frequency and amplitudes comparable to the actuator gap. Specifically, we show that at resonance, the structure can move beyond the well-known pull-in-limit but is instead limited to 56% of the gap by “resonant pull-in.” Above the resonant frequency, the structure is not limited by pull-in and can theoretically oscillate across the entire gap. We develop a describing function model, which includes an amplitude-dependent model for electrostatic spring tuning, to predict the steady-state frequency response. These results are verified experimentally.

INTRODUCTION

Many MEMS applications, such as gyroscopes, scanning mirrors, and filters, are based on vibrating mechanical structures. In many designs, there is a specific vibration frequency that is desired. However, in order to maximize the vibration amplitude, the mechanical structure must be vibrated at its resonant frequency. Because the resonant frequency varies due to fabrication tolerances, temperature, and age, it is desirable to tune the resonant frequency to match the desired frequency. Several techniques have been presented for tuning the resonant frequency of a mechanical structure, including thermal [1] and electrostatic comb designs [2,3,4].

Among those approaches, parallel-plate, electrostatic electrodes have been most widely used for frequency tuning as well as actuation. Because they are nonlinear, parallel-plate actuators generally are used in applications where the motion is much smaller than the gap. For example, it is well known that, at low frequencies, parallel-plate actuators are limited to deflections less than one-third of the gap before the actuator will snap [5]. We are interested in using parallel-plate electrodes to vibrate a gyroscope proof-mass (Figure 1) at its resonant frequency to achieve large-amplitude motion while tuning the resonant frequency to reduce drift. This work characterizes parallel-plate actuators for oscillations near the mechanical resonant frequency and amplitudes comparable to the gap. Specifically, we show that at resonance, the structure can move beyond the well-known pull-in-limit but is instead limited by “resonant pull-in.” Above the resonant frequency, the structure can move more than the resonant pull-in limit. We develop a describing-function model, which includes an amplitude-dependent model for electrostatic spring tuning, to predict the steady-state frequency response.

DEVICE MODEL

A parallel-plate driven structure is shown in Figure 1. It is modeled by a lumped mass, m , supported by a suspension with spring constant, k , and subject to viscous damping, b . The natural resonant frequency of the structure is defined as $\omega_n = \sqrt{k/m}$, and the quality factor is defined as $Q = m\omega_n/b$.

Figure 2 shows a schematic of the actuator including the digital drive electronics. The electrodes on either side of the

structure are switched between ground and voltage V_d . Using the parallel-plate approximation, the net electrostatic force applied to the proof-mass is

$$F_{el}(x, D) \approx \begin{cases} +\frac{C_0 V_d^2}{2d} \frac{1}{(1-x/d)^2}, & (D=1) \\ -\frac{C_0 V_d^2}{2d} \frac{1}{(1+x/d)^2}, & (D=0) \end{cases}, \quad (1)$$

where x is the proof-mass displacement, d is the nominal size of the gap, $C_0 = \epsilon A/d$ is the nominal parallel-plate capacitance of the actuator, A is the electrode area, ϵ is the permittivity of the fluid (air in our case) in the gap, and D is the digital signal that selects which electrode is activated.

RESONANT PULL-IN

At low excitation frequencies, pull-in occurs if the negative spring constant, due to the parallel-plate actuator, exceeds the mechanical spring constant. This occurs if the drive voltage exceeds the pull-in voltage. For drive voltages less than the pull-in voltage, the resonant frequency is lowered by the electrostatic spring but is not reduced to zero, as in the case of pull-in. Nevertheless, at resonance, there can be an instability, called

Figure 1. Parallel-plate driven, gyroscope proof-mass

Figure 2. Actuator schematic

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Figure 3. Drive voltage and position waveforms at resonance

Figure 4. Potential energy vs. deflection

resonant pull-in, that causes the structure amplitude to build up and then snap against one of the fixed electrodes. This instability depends on the energy provided by the parallel-plate actuator.

Figure 3 illustrates voltage and position waveforms at resonance. For half of the oscillation period, the drive voltage is constant. During this time, the potential energy can be written as the combination of spring potential energy and electrostatic potential energy:

$$PE(x, D) = \frac{1}{2} kx^2 - \begin{cases} \frac{C_0 V_d^2}{2} \frac{1}{(1-x/d)}, & (D=1) \\ \frac{C_0 V_d^2}{2} \frac{1}{(1+x/d)}, & (D=0) \end{cases} \quad (2)$$

This function is plotted in Figure 4. At resonance, the drive signal, D , switches at the peak deflection, $x = \pm A$. When D switches, the potential energy of the system increases. The change in potential energy represents the energy that is provided by the electrostatic actuator in one half-cycle:

$$E_{add}(A) = PE(A, 0) - PE(A, 1) = C_0 V_d^2 \frac{A/d}{1-(A/d)^2}. \quad (3)$$

As the structure moves, energy is removed by the mechanical damping and by the electrical resistance in the wiring and structure. The electrical resistance, R , can be ignored if it is much smaller than the equivalent resistance of the damper:

$$R \ll b/(C_0' V_d)^2, \quad (4)$$

where C_0' is dC/dx evaluated at $x=0$. In most designs, the wiring and structure resistance is significantly smaller than the equivalent

Figure 5. Energy added and energy removed vs. amplitude

Figure 6. Transient simulations. The lower plot illustrates resonant pull-in and the upper plot is stable.

resistance. Thus, it can be assumed that the energy lost is due to the mechanical damping. At resonance, the motion of the structure will be approximately sinusoidal with amplitude A , and the energy dissipated by the damper in one half-cycle will be

$$E_{lost} = \frac{\pi}{2} \omega_{tun} b A^2 = \frac{\pi}{2} \frac{k \omega_{tun}}{Q \omega_n} A^2, \quad (5)$$

where ω_{tun} is the tuned resonant frequency. The tuned resonant frequency, ω_{tun} , is lower than the natural resonant frequency, ω_n , because of the negative-spring effect of the parallel-plate actuator.

The energy added, Eq. (3), for three different voltage sources and the energy removed by a damper, Eq. (5), are illustrated in Figure 5. Amplitudes where the energy added equals the energy lost correspond to steady-state oscillations. For low voltages, there are two non-zero intersections. The largest amplitude solution is unstable. A slight increase in amplitude causes more energy to be added than is dissipated, which in turn causes the amplitude to increase further. The other solution is stable; a slight increase in amplitude causes more energy to be dissipated than is provided. For voltages greater than V_{rpi} there are no non-zero solutions. In this case, the oscillation amplitude builds up until the structure snaps against one of the electrodes. Figure 6 shows simulation results of resonant excitation for $V_d < V_{rpi}$ and for $V_d > V_{rpi}$.

The steady-state oscillation amplitude can be found from the condition, $E_{add} = E_{lost}$. Thus,

$$\frac{A}{d} \left(1 - \left(\frac{A}{d} \right)^2 \right) = \frac{2C_0 V_d^2}{\pi k d^2} \frac{Q \omega_n}{\omega_{tun}} \quad (6)$$

As will be shown in the next section, ω_{tun} has a slight dependence on A . However, if ω_{tun} is approximated as constant, it can be shown that the maximum stable amplitude, at the tuned resonant frequency, is

$$A_{\text{max}} \approx d/\sqrt{3} \approx .56d. \quad (7)$$

This amplitude occurs at the resonant-pull-in voltage

$$V_{\text{rpi}} = \sqrt{\frac{\pi}{Q\sqrt{27}} \frac{\omega_{\text{tun}}}{\omega_n} \frac{kd^2}{C_o}}, \quad (8)$$

which also depends on the tuned resonant frequency. If the frequency dependence is ignored, and the dc pull-in voltage is substituted into Eq. (8), the resonant-pull-in voltage can be expressed as

$$V_{\text{rpi}} \leq V_{\text{pi}} \sqrt{\frac{2}{Q}}, \quad (9)$$

where

$$V_{\text{pi}} = \sqrt{\frac{8}{27} \frac{kd^2}{C_o}}. \quad (10)$$

Thus, for $Q > 2$, resonant pull-in will occur at a lower voltage than DC pull-in.

DESCRIBING FUNCTION MODEL

In this section, a model is developed to predict steady-state oscillation amplitudes over a wide range of excitation frequencies. The preceding energy analysis could be easily extended to off-resonance excitation, but it does not account for the negative spring effect of the parallel-plate actuator. The negative spring effect motivates the describing function approach [6].

Figure 7 shows a nonlinear block diagram of a parallel-plate driven, vibrating structure. Because the electrostatic force has a nonlinear dependence on position, both the steady-state oscillation—if there is one—and the steady-state force will be periodic but not sinusoidal. In general, an exact solution requires a computer simulation. However, if Q is sufficiently high and the driving frequency is near or above the mechanical resonant frequency ($\omega \geq \omega_n/2$), the motion can be approximated by a single sinusoid because the harmonic components will be attenuated by the structure dynamics.

Assuming that $x = A \cos(\omega t)$ and that D lags the deflection by phase, ϕ , the electrostatic force, from Eq. (1), can be represented by its Fourier components:

$$F_{\text{el}}(\omega) = F_{\text{dc}} - k_{\text{el}} A \cos(\omega t) - b_{\text{el}} A \sin(\omega t) + K \quad (11)$$

where

$$k_{\text{el}}(A, \phi) = \frac{-1}{\pi A} \int_0^{2\pi} F(\tau) \cos(\tau) d\tau \quad (12)$$

$$b_{\text{el}}(A, \phi) = \frac{1}{\pi A} \int_0^{2\pi} F(\tau) \sin(\tau) d\tau$$

The average force, F_{dc} , on the structure is zero. The second term is the component that is in phase with position and represents the spring force. The third term is in phase with velocity and represents the energy added by the actuator. Assuming that the mass only responds to the dc and fundamental component, the harmonic terms are neglected.

At resonance ($\phi = -90^\circ$),

$$k_{\text{el}}(A, -90^\circ) = -\frac{C_0 V_d^2}{d^2} \frac{1}{(1 - (A/d)^2)^{1.5}} \quad (13)$$

Figure 7. Nonlinear block diagram

Figure 8. Block diagram including describing function elements

$$b_{\text{el}}(A, -90^\circ) = -\frac{C_0 V_d^2}{d^2} \frac{2}{\pi} \frac{1}{(A/d)} \frac{1}{1 - (A/d)^2}. \quad (14)$$

For small oscillation amplitudes ($A \approx 0$) the electrical spring constant is identical to the well-known parallel-plate spring constant. The energy provided, in one half-cycle, by the negative damping term ($E_{\text{add}} = -\pi b_{\text{el}} A^2/2$) matches Eq. (3).

Figure 8 shows the same block diagram from Figure 7 except that the nonlinear, electrostatic-force block has been replaced by the amplitude and phase dependent terms in Eq. (12). From the block diagram, two equations can be derived: the frequency equation,

$$\left(\frac{\omega}{\omega_n}\right)^2 = 1 + \frac{k_{\text{el}}(A, \phi)}{k}, \quad (15)$$

and the energy equation,

$$-\frac{b_{\text{el}}(A, \phi)}{k} = \frac{1}{Q} \frac{\omega}{\omega_n}. \quad (16)$$

Using a numerical method, equations (15) and (16) can be solved for amplitude and frequency as a function of phase. The results are discussed in the next section.

The spring and energy terms are plotted in Figure 9 and Figure 10. Resonant pull-in occurs if $db_{\text{el}}/dA < d\omega/dA$. At resonance, db_{el}/dA becomes negative at 56% of the gap. At $\omega = \omega_n$, $k_{\text{el}} = 0$, and $db_{\text{el}}/dA > 0$ for all amplitudes. Thus, the model predicts that vibration at the natural resonant frequency will be stable over the entire gap.

RESULTS AND DISCUSSION

The parallel-plate behavior was verified experimentally using the gyroscope structure in Figure 1. The structure is 2.25 μm thick. The resonant frequency was designed to be 10 kHz. The nominal drive capacitance is 70 fF with a 2 μm gap. The structure oscillates at atmospheric pressure.

Using the Computer Microvision system, designed at MIT [7], the frequency response of the structure was measured for different drive voltages. The results are shown in Figure 11 along with the results of the describing function model. The model data fit the measured results for a resonant frequency, $f_n = 9.3 \text{ kHz}$, $Q = 5.3$, and $C_0/(kd^2) = 0.0088$, or equivalently $V_{\text{pi}} = 5.8 \text{ V}$. According to equation (9), the resonant-pull-in voltage, $V_{\text{rpi}} < 3.56 \text{ V}$. Combining equations (8), (13), and (15) predicts $V_{\text{pi}} = 3.42 \text{ V}$ because the resonant frequency is tuned to $\omega_{\text{tun}} = 0.9\omega_n$.

In Figure 11, there are curves for three different drive voltages: 3.0 V, 3.4 V, and 4.5 V. For the curves corresponding to $V_d = 3 \text{ V}$ and $V_d = 3.4 \text{ V}$, the motion is stable for all frequencies, and

the amplitude is limited to less than ~56% of the gap. Notice that the resonant frequency is lower for the 3.4V drive voltage. As predicted in the resonant pull-in section, there are also unstable solutions, but these oscillations are unobservable without feedback.

For the curve corresponding to the largest drive voltage, there are three distinct frequency ranges: 1) In the lower frequency range, there are stable amplitudes that are limited to less than half of the gap. The solutions in the upper half of the curve are unstable and require feedback to be achieved. If the voltage is increased further, the maximum stable amplitude at low frequencies tends to the dc pull-in limit, one-third of the gap. 2) In the midband range, there are no solutions, stable or unstable. In this region, the energy provided always exceeds the energy removed by the damper, and the structure snaps. 3) In the high frequency range, there is one solution for every frequency. In this frequency range, there is no pull-in limit, and it is possible for the structure to achieve a stable amplitude greater than 56% of the gap. In fact, we recorded a 1.4 μ m amplitude vibration across the 2 μ m gap at 9kHz.

There are two non-idealities that might affect the parallel-plate actuator. The first is nonlinear damping due to squeeze-films [8]. However, for the test structure, in which the thickness and the gap are comparable, the data does not indicate that the damping depends on position. The second issue is electrostatic fringing fields. The fringing fields might introduce errors in the energy and spring terms, but they do not change the qualitative behavior.

CONCLUSION

Parallel-plate actuators that are driven near the resonant frequency can move more than the dc pull-in limit of one-third of the gap. However, actuators that are driven at the tuned resonant frequency, are instead limited to 56% of the gap due to resonant pull-in. Above the tuned resonant frequency, it is possible to oscillate more than 56% of the gap. If the structure is driven at the natural resonant frequency, the vibration theoretically should be able to span the entire gap. Using the describing-function model we can predict the steady-state amplitudes for excitation frequencies above $\omega_n/2$. If feedback is used, it should be possible to stabilize the normally unstable oscillation amplitudes to achieve a wide range of operating frequencies at low voltages.

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Figure 9. Spring component versus amplitude; $\gamma = (C_0 V_d^2)/(2kd^2)$.

Figure 10. Energy component versus amplitude; $\gamma = (C_0 V_d^2)/(2kd^2)$.

Figure 11. Describing function model and measured frequency response