SENSE FINGER DYNAMICS IN A $\Sigma\Delta$ FORCE-FEEDBACK GYROSCOPE

Joseph I. Seeger, Xuesong Jiang, Michael Kraft*, and Bernhard E. Boser

Berkeley Sensor & Actuator Center
University of California, Berkeley, CA 94720-1774

*Department of Electronics and Computer Science, Southampton University
Southampton SO17 1BJ, United Kingdom

ABSTRACT

This paper presents a gyroscope that uses $\Sigma\Delta$ force-feedback for closed-loop Coriolis-acceleration sensing and 1-bit digital output. The noise floor is approximately $3^\circ/\text{s}/\sqrt{\text{Hz}}$ at atmospheric pressure. The effect of capacitive sense-finger resonance on the $\Sigma\Delta$ feedback loop is analyzed. In order to minimize quantization noise the resonant frequency of the sense fingers should be designed above $f_s/4$, if the sense and feedback electrodes are separate. If the same electrodes are used for sensing and feedback, the finger resonance should be designed near $f_s/6$, where f_s is the sampling frequency.

INTRODUCTION

A vibratory-rate gyroscope determines the angular velocity of an object by vibrating a proof-mass attached to the object and measuring its Coriolis acceleration. Previous micromachined gyroscopes have demonstrated both open-loop [1] and closed-loop [2] Coriolis-acceleration sensing with analog outputs. Closed-loop sensing has the potential to increase the sensor bandwidth, improve linearity, and reduce sensitivity to process and temperature variations. This is especially important if the frequency of the Coriolis acceleration matches the proof-mass resonant frequency. Sigma-delta ($\Sigma\Delta$) modulation is a technique that provides high-resolution analog-to-digital conversion [3]. An approach demonstrated in accelerometers [4], combines $\Sigma\Delta$ modulation and force-feedback to make a sensor in which acceleration signals are converted directly to digital output. Recently, a $\Sigma\Delta$ force-feedback accelerometer has reported 18-bit resolution [5].

Closed-loop sensing can also cause instability, and $\Sigma\Delta$ modulation tends to increase the sensor noise due to quantization error. In $\Sigma\Delta$ force-feedback, the mechanical design of the proof-mass affects both the stability and the quantization noise. With proper design, $\Sigma\Delta$ force-feedback can provide a digital output without degrading the sensor performance. This paper presents a gyroscope that uses $\Sigma\Delta$ force-feedback for closed-loop Coriolis acceleration sensing and digital output. Additionally, this paper investigates the effect of capacitive sense finger resonance on the $\Sigma\Delta$ force-feedback performance.

DESIGN SUMMARY

Figure 1 shows a monolithic, surface-micromachined, vibratory gyroscope that is sensitive to rotations about the Z-axis, the axis normal to the plane of the chip. The gyroscope was fabricated by Sandia National Labs in an integrated MEMS process with a $2.25\mu\text{m}$ thick mechanical polysilicon layer and 5V, $2\mu\text{m}$ minimum gate length CMOS transistors. The gyroscope operates at atmospheric pressure.

Figure 1. Photograph of the gyroscope die

Figure 2 shows a block diagram of the gyroscope. It can be divided roughly into three parts: the proof-mass, the electronics for vibrating the proof-mass, and the electronics for sensing Coriolis acceleration. The proof-mass is vibrated along the X-axis (the drive mode), and its acceleration is measured along the Y-axis (the sense mode). The proof-mass is represented by the drive and sense dynamics, which ideally are coupled only by the Coriolis acceleration. The resonant frequency and quality factor in air, before electrostatic spring tuning, are approximately 10.5kHz and 4 in the sense mode and 9.5kHz and 3 in the drive mode. The proof-mass is $0.3\mu\text{g}$. It is a single structure (Figure 3) that consists of three sets of gap-closing electrodes: electrostatic actuation in the X-direction (70fF, $2\mu\text{m}$ gap), capacitive sensing in the Y-direction (70fF, $1.2\mu\text{m}$ gap), and force feedback in the Y-direction (2fF, $1.2\mu\text{m}$ gap). A fourth set of electrodes cancel quadrature coupling between the drive and sense axes [1].

The drive electronics consist of an off-chip oscillator and a 1-bit quantizer on chip that switches a fixed voltage between the two fixed drive electrodes. The structure is driven approximately $\pm 1\mu\text{m}$ at its resonant frequency using 3.2V. In this prototype design, the drive mode is open-loop, without any electronic position sensing.

The sense electronics implement a $\Sigma\Delta$ feedback loop that is similar to [4] and is explained in detail in [6]. The loop consists of 1) a capacitance sensing amplifier for position measurement; 2) correlated double sampling (CDS) to cancel offset and flicker

Figure 2. $\Sigma\Delta$ Gyroscope block diagram

Figure 3. SEM of the proof-mass

noise; 3) a gain stage; 4) a discrete-time lead compensator; 5) a one-bit quantizer, and 6) a feedback pulse that drives one of the two fixed feedback electrodes. The sense dynamics act as a noise-shaping filter for the $\Sigma\Delta$ loop. The sampling frequency f_s is 1MHz.

SENSE DYNAMICS

Ideally, the proof-mass dynamics in the sense-direction can be modeled with one resonant mode by a lumped mass, spring, and damper. In reality, the proof-mass is a distributed element with many resonant modes. Figure 4 illustrates a model of the sense dynamics that includes a flexible beam, representing the electrodes used for position sensing, and a lumped mass, representing the rest of the structure. The flexible beam has length L and thickness Nh , where N is the number of sense electrodes, and h is the thickness of the proof-mass. The spring represents the suspension compliance in the sense direction. The model also includes damping on the lumped mass, distributed damping on the flexible beam, and a distributed spring to account for electrostatic tuning of the sense fingers. All elements are constrained to move in the Y-direction. The deflection of the beam $y(x,t)$ is a function of position and time. The deflection of the lumped mass is $y(0,t)$, and the deflection of

Figure 4a. Model of the non-collocated sense dynamics

Figure 4b. Model of the collocated sense dynamics

Figure 5. First three computed sense-finger mode shapes

the beam tip is $y(L,t)$. Neglecting fringing fields, the total capacitance from the moving, flexible electrode to the upper, fixed electrode is approximately:

$$C_{s1}(t) = \epsilon N h \int_0^L \frac{1}{d - y(x,t)} dx,$$

where ϵ is the permittivity of air, and d is the nominal gap. The position sense electronics measure the differential capacitance, $C_{s1} - C_{s2}$, which can be simplified, assuming small deflections, to

$$\Delta C_s(t) \approx \frac{2\epsilon h}{d} \int_0^L \frac{y(x,t)}{d} dx.$$

Figure 6. Non-collocated (thick) and second-order (thin) frequency response of the average beam position to the feedback force.

Figure 7. Collocated (thick) and second-order (thin) frequency response of the average beam position to the feedback force.

The feedback force can be applied to the lumped mass (Figure 4a) or distributed across the sense finger (Figure 4b). The former case, the non-collocated design, models an implementation in which the feedback electrodes are separate from the sense electrodes. The latter case, the collocated design, models an implementation, such as [4], in which the fixed electrodes are time multiplexed for sensing and force-feedback.

The resonant frequencies and mode shapes are independent of where the force is applied. Figure 5 shows the first three mode shapes and their associated resonant frequencies computed using the Rayleigh-Ritz method [7]. The lowest resonant mode corresponds to the lumped mass and the sense electrodes moving together. The resonant frequency is approximately $\omega_0 = \sqrt{(k - k_{el})/m}$, where k is the suspension spring constant, k_{el} is the negative spring constant due to electrostatic tuning, and m is the total mass of the structure. The higher resonant modes correspond to small motion of the lumped mass and significant bending of the capacitive comb finger. The finger resonant frequencies can be approximated by the resonant frequencies of a cantilevered beam with a rectangular cross-section

$$\omega_i = \alpha_i^2 \frac{W}{L^2} \sqrt{\frac{E}{12\rho}} \quad \alpha_1 = 1.875, \alpha_2 = 4.694$$

Figure 8. Phase versus frequency of second-order dynamics, discrete-time compensator, and delayed feedback pulse.

where i is the mode number, W is the finger width, E is Young's modulus, and ρ is the material density [7].

The Bode plot of differential capacitance response to a sinusoidal feedback force is shown in Figure 6 for the non-collocated design. As is typical of non-collocated dynamics, there is an additional 180° of phase lag at frequencies above the resonant mode of the finger. Figure 7 shows the frequency response for the collocated design. Collocated dynamics provide 180° of phase lead below the finger resonance.

ΣΔ LOOP ANALYSIS

Even though the 1-bit quantizer is a nonlinear element, the ΣΔ loop can be analyzed using linear system techniques. The quantizer can be modeled by a linear gain plus a quantization error source. Reference [8] calculates the effective gain and the quantization error assuming that the signal at the input of the quantizer is white noise. The effective gain of the quantizer varies as a function of the signal at the input of the quantizer.

The output of the ΣΔ modulator includes quantization noise that is shaped by the noise transfer function, $1/(1+T(j\omega))$, where $T(j\omega)$ represents the open-loop gain. In order to reduce the quantization noise contribution in the signal band, the open-loop gain should be maximized in the signal band. However, the open-loop gain must be limited to prevent instability.

With the ideal second-order sense dynamics, the open-loop gain and hence the ΣΔ modulator performance is limited by the phase lag due to correlated double sampling and due to the time delay between the position sense and the feedback pulse. To compensate for this lag, a lead network is included in the ΣΔ loop. Figure 8 shows a plot of the phase versus frequency for an open-loop system consisting of second order dynamics, a lead compensator, $C(z)=2-z^{-1}$, and a feedback pulse centered about $T_s/2$, one-half of the clock period. Below $f_s/6$, the net phase is greater than -180° due to the lead compensator. At higher frequencies, the phase drops below -180° due to the time delay of the feedback pulse. The -180° crossover frequency f_{180} represents the maximum unity-gain frequency for stable closed loop operation. This frequency sets a limit for the low-frequency gain. If f_s is increased, f_{180} is also increased and therefore the loop gain can be increased. Thus, in the ideal case, increasing the sampling frequency tends to reduce the quantization noise. The -180° crossover frequency varies depending on the feedback delay, CDS,

Figure 9. Measured bitstream output spectrum

and aliasing. For reasonable values, f_{-180} varies between $f_s/6$ and $f_s/4$.

The non-collocated dynamics introduce an additional 180° of phase lag at the first resonant frequency of the flexible finger (ω_1). The phase lag due to the finger resonance sets an upper limit on the -180° crossover frequency. If the resonant frequency of the finger is below $f_s/4$, then f_{-180} approximately equals the finger resonant frequency and the maximum loop gain is reduced from the ideal. In this case, the $\Sigma\Delta$ loop might oscillate at the finger resonant frequency. If the finger resonance is greater than $f_s/4$, the phase lag due to the finger resonance does not degrade the $\Sigma\Delta$ performance. In the case of non-collocated dynamics, increasing f_s does not reduce quantization as much as in the ideal case.

The collocated dynamics introduce 180° of phase lead below the finger resonance. Collocated dynamics are stable in feedback provided that there is sufficient phase at the resonant frequencies of the flexible beam. If the resonant frequency is above $f_s/6$, the compensator does not provide sufficient phase lead. The increased amplitude response near the resonant frequency of the beam significantly reduces the maximum low-frequency loop gain. In this case, the $\Sigma\Delta$ loop might oscillate at the beam resonant frequency. If the resonant frequency is below $f_s/6$, the compensator provides sufficient phase lead to maintain stability. If the resonant frequency is too low, a higher order resonant mode of the beam that is above $f_s/6$ can degrade the $\Sigma\Delta$ performance.

EXPERIMENTAL RESULTS

The sense mode was designed with non-collocated feedback. The first resonant mode of the sense electrodes is 130 kHz. A 150,000 point FFT of the 1-bit $\Sigma\Delta$ modulator output is shown in Figure 9. The spectrum matches simulations run in SIMULINK. The peak at 130 kHz is due to the resonant mode of the sense fingers. The peak at 22 kHz indicates the unity-gain frequency. The unity gain frequency is significantly less than the finger resonance. This results from increased electronic noise at the quantizer input which reduces the effective quantizer gain. Thus, the finger dynamics do not affect this implementation, but they would become significant if the unity gain frequency were increased either by increasing the feedback force or by decreasing the electronic noise.

Simulations indicate that the residual motion of the proof-mass is 2.2Å_{rms} and the sense fingers flex 1.3Å_{rms} . Figure 10 shows the narrow-band response to a 50%/s, 4 Hz sinusoidal

Figure 10. Measured response to a 50%/s, 4 Hz sinusoidal rotation rate (0.25 Hz/bin)

rotation. The noise floor is approximately $3^\circ/\text{s}/\sqrt{\text{Hz}}$. The performance is limited by quantization noise.

CONCLUSIONS

The $\Sigma\Delta$ force-feedback gyroscope is functional. The resonant mode of the long capacitive fingers used for position sensing is excited by feedback, and it has been measured. In order not to degrade the $\Sigma\Delta$ performance, if the feedback and sense electrodes are separate, the sense fingers should be short enough so that the resonant frequency is above $f_s/4$. If the sense electrodes are also used for feedback, the fingers should be made long enough so that the resonant frequency is about $f_s/6$. In theory, the finger dynamics can limit the $\Sigma\Delta$ performance, but in practice, they are not the only factor that limits the $\Sigma\Delta$ force-feedback performance.

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