A MESHLESS TECHNIQUE FOR ANALYSIS OF MICROELECTROMECHANICAL SWITCHES

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ABSTRACT

Many existing computer-aided design systems for MEMS require the generation of a three-dimensional mesh for computational analysis of the microdevice. Mesh generation requirements for microdevices are very complicated because of the presence of mixed-energy domains and the different numerical techniques implemented for each energy domain. For example, volume meshes are employed for finite-element analysis of structural deformation and surface meshes are employed for boundary-element analysis of electrostatic forces on the structure. This paper describes a meshless technique, using which the analysis of a complicated microdevice can be performed by sprinkling points instead of generating a mesh.

INTRODUCTION

The present approach to modeling and design of microelectromechanical systems involves the generation of a geometric model for the complicated three-dimensional microdevice, the generation of a mesh for the geometric model, a self-consistent mesh-based numerical analysis of the mixed-energy domains present in the microdevice, and postprocessing steps such as visualization. An example of a CAD system based on such an approach was presented by Senturia et. al. in [1]. The time consuming steps in such an approach are the generation of a mesh for a microdevice and the self-consistent analysis of the mixed-energy domains. Fast techniques for self-consistent analysis of mixed-energy domains is currently being pursued as a research area (see e.g. [2]).

In this paper we describe a new technique to develop computer-aided design systems for MEMS based on meshless techniques. In meshless techniques, the generation of a mesh for the complicated three-dimensional microdevice is eliminated, and instead points are sprinkled for the geometrical representation of the microdevice. Self-consistent analysis of the mixed-energy domains is then performed by new point-based techniques instead of the usual mesh-based techniques. The implementation of a meshless technique for analysis of microelectromechanical switches is described in this paper.

MESHLESS TECHNIQUE

The meshless technique presented in this work is based on the smooth particle hydrodynamics (SPH) technique presented by Monaghan in [3]. In SPH, an approximation $u^a(x)$ to $u(x)$ in a domain Ω is generated by a kernel approximation i.e.

$$u^a(x) = \int_{\Omega} w(x-s)u(s)ds \quad (1)$$

where $w(x-s)$ is the kernel or a weighting function. The kernel function is non-zero ($w(x-s) > 0$) in a subdomain Ω_I of Ω and zero outside the subdomain Ω_I . For a method to be convergent, it has to be stable and consistent. The numerical solutions with a non-convergent method can exhibit amplitude and phase errors and a deterioration in the accuracy of the solution. It can be shown that the discrete form of SPH is not consistent. The consistency of SPH can be established by correcting the approximation introduced in

equation (1). The concept of a correction function was introduced by Liu in [4] and a corrected kernel approximation of equation (1) is written as

$$u^a(x) = \int_{\Omega} \bar{w}_a(x; x-s)u(s)ds \quad (2)$$

where $\bar{w}_a(x; x-s)$ is the corrected kernel function which is given by

$$\bar{w}_a(x; x-s) = C(x, x-s)w_a(x-s) \quad (3)$$

$$w_a(x-s) = \frac{1}{a}w\left(\frac{x-s}{a}\right) \quad (4)$$

where $C(x, x-s)$ is called the correction function and a is the dilation parameter of the kernel function $w_a(x-s)$. The discrete approximation to equation (2) can be written as

$$u^a(x) = \sum_{I=1}^{N_p} \bar{w}_a(x; x-x_I)u(x_I)\Delta V_I \quad (5)$$

where the domain Ω is now represented by N_p points instead of a usual mesh and ΔV_I is a measure of the domain surrounding node I . Eventhough the summation in equation (5) is written over all the points in the domain, it is effectively over the points x_I where $\bar{w}_a(x; x-x_I) > 0$. Equation (5) can be rewritten as

$$u^a(x) = \sum_{I=1}^{N_p} N_I(x)u_I \quad (6)$$

$$N_I(x) = C(x; x-x_I)w_a(x-x_I)\Delta V_I \quad (7)$$

where $u_I = u(x_I)$ is the value of the function u at node x_I , and N_I is defined as the shape function for node I , and it is not the usual shape function referred to in the context of finite-element methods.

MESHLESS ANALYSIS OF SWITCHES

The static and dynamic behavior of many microelectromechanical switches can be characterized by studying the deformation of fixed-fixed and cantilever beams subjected to electrostatic voltages. The governing equation of a beam subjected to electrostatic forces is given by

$$\rho \frac{\partial^2 u}{\partial t^2} + EI \frac{\partial^4 u}{\partial x^4} = -\frac{w\epsilon_0 V^2}{2g^2} \left(1 + 0.65 \frac{g}{w}\right) \quad (8)$$

where ρ is the mass density per unit length of the beam, u is the displacement of the beam, E is the Young's modulus of the material, I is the moment of inertia, w is the width of the beam, ϵ_0 is the permittivity of free space, V is the applied voltage and g is the gap between the beam and ground electrode. Equation (8) is nonlinear because the gap, g , depends on the displacement of the beam i.e. $g = g(u)$.

A meshless approach to the solution of equation (8) comprises of the following steps:

1. A weak form to the strong form given in equation (8) is constructed by multiplying equation (8) by an arbitrary test function (say v) and integrating by parts.

2. The unknowns u and v are approximated by the shape functions constructed through a corrected kernel approach described above. From equation (6), it follows that

$$u = \sum_{I=1}^{N_p} N_I(x) u_I \quad v = \sum_{I=1}^{N_p} N_I(x) v_I \quad (9)$$

3. When the approximations to u and v are substituted in the weak form, a nonlinear system of equations is obtained, which is solved by a Newton-Raphson method.

In the current implementation, cubic splines are employed as the kernel functions and the correction function is obtained by writing consistency conditions to exactly represent second-degree polynomials.

RESULTS

The static and dynamic behavior of a fixed-fixed beam is modeled using the meshless technique. The beam considered here is $80\mu\text{m}$ long, $10\mu\text{m}$ wide, and $0.5\mu\text{m}$ thick. The initial gap between the beam and the bottom electrode is $0.7\mu\text{m}$. A Young's modulus of 169 GPa and a mass density of 2231 kg/m^3 are employed in the simulations shown here. The same device has been simulated in the work presented in [5] and the static and dynamic pull-in values obtained through experiments and simulations were reported.

Figure 1 shows the deflection of the beam as a function of a series of applied voltages. As the potential difference between the beam and the ground electrode increases, the gap between the beam and the bottom electrode decreases. At a certain voltage, the beam becomes unstable and collapses onto the ground electrode. For the beam under consideration, this voltage is computed to be 15.08 volts. The pull-in value reported in [5] is 15.17 volts and the error is less than 1%.

Figure 1. Deflection of a fixed-fixed beam for applied voltages

We have also done dynamic studies using the meshless technique. In particular, energy conserving dynamic algorithms have been implemented to study the fixed-fixed beam. The peak deflection of the beam, which is at the center of the beam, for an applied bias of 10 V is shown in Figure 2. The response in Figure 2 is obtained using a time step of $5 \times 10^{-4} \mu\text{s}$ and the simulation is performed for a total of $5\mu\text{s}$. The dynamic pull-in voltage is measured to be 13.7 volts and it matches with the value reported in [5]. The pull-in behavior of the beam for a bias of 13.7 volts is shown in Figure 3. Note that the dynamic pull-in voltage is smaller than the quasi-static pull-in voltage by about 9%.

Figure 2. Peak deflection (deflection at the center) of a fixed-fixed beam as a function of time

Figure 3. Pullin behavior of the beam for a voltage of 13.7 volts

CONCLUSIONS

A meshless technique for analysis of microelectromechanical devices is presented here for the first time. The approach is shown to be accurate by comparing pull-in voltages with previously reported data. Finally, it is anticipated that with meshless techniques in place, radically simpler and easy to use computer-aided design tools will be available for MEMS designers.

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