RESONANT STRUCTURES FOR INTEGRATED SENSORS

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I. Abstract

This paper reviews mechanically resonant microsensors and the design issues for this class of sensors. We apply Rayleigh's Method to determine the resonant frequencies of common microstructures, and show that it provides insight into perturbing the resonance in sensing applications. Resonant microsensors that measure mass loading and force or pressure are studied. We conclude with a discussion of some of the important issues in resonant microsensor design.

II. Introduction

Mechanically resonant structures are attractive as transducing elements in microsensors. Resonant frequencies are made sensitive to chemical or physical signals by coating the structure with sensitive films or by coupling mechanical loads to the structure[1,2]. Since the sensor output is a shift in resonant frequency, it can be measured precisely and converted to digital form by simple counting, in contrast to voltage or current outputs, which require conventional analog-to-digital conversion. Furthermore, spurious drifts in the resonant frequency can be canceled by taking the difference in resonant frequencies between matched structures as the output.

Over the past two decades, these advantages have motivated work on quartz mechanical resonators[15-18], quartz bulk wave resonators and surface-acoustic-wave (SAW) oscillators using piezoelectric substrates or thin films[8-13,19-24], and micromachined mechanical resonators[7,14,25-29]. Resonant microsensors have been demonstrated that measure pressure, acceleration, applied force, temperature, deposited film thickness, and gas or vapor concentration.

We begin this review with a discussion of Rayleigh's Method for calculating the resonant frequencies of

The diagram shows a string of length L fixed at both ends (x=0 and x=L). The displacement is denoted by y(x,t). The tension in the string is T. The string is shown in its fundamental mode of vibration.

$$y(x,t) = Y(x)e^{j\omega t}$$

$$KE_{MAX} = \omega^2 \frac{\rho A}{2} \int_0^L v^2(x) dx \quad PE_{MAX} = \frac{T}{2} \int_0^L \left(\frac{dY(x)}{dx} \right)^2 dx$$

$$\omega^2 = \frac{\frac{T}{2} \int_0^L \left(\frac{dY(x)}{dx} \right)^2 dx}{\frac{\rho A}{2} \int_0^L v^2(x) dx}$$

Figure 1. Rayleigh's Method calculation of resonant frequency. ρA = mass / length of string.

mechanical structures[3-6], and its extension to acoustic systems. This energy method provides insight into the design of resonant microstructures and is used to compare the sensitivities of microbridge and SAW vapor sensors. Resonant microsensors for force and pressure are also examined. In conclusion, we discuss some of the important design considerations for the development of useful resonant microsensors.

III. Mechanical Resonance

A. Rayleigh's Method

Rayleigh's Method is a vibration analysis technique in which the maximum potential energy of vibration is equated to the maximum kinetic energy to solve for the resonant frequency[5]. An example of this method is the calculation of the first resonant frequency ω_r of a stretched string, shown in Figure 1. The maximum potential energy of stretching occurs at the maximum deflection, whereas the kinetic energy maximum occurs as the string moves through the center line. We equate these energy terms, for an assumed mode shape $Y(x)$, to find ω_r . The exact first mode shape of a vibrating string is $Y(x) = \sin(\pi x/L)$; substitution into the equation in Figure 1 yields the exact resonant frequency:

$$\omega_r = \pi \left[\frac{T}{\rho A} \right]^{\frac{1}{2}}$$

The power of Rayleigh's method is that any reasonable mode shape which satisfies the boundary conditions yields a good estimate of the resonant frequency. It can be shown that any estimate of the first resonant frequency by this method is greater than or equal to the true resonant frequency[3]. Thus, Ritz's iterative method of minimizing ω_r by varying $Y(x)$ can be used to improve the estimate.

B. Beams, Bridges, and Plates

Table 1 summarizes the resonant frequency calculations for three typical micromechanical structures: cantilever beams, bridges, and plates. The potential energy contains contributions due to bending for all three structures, represented as the second derivative of the mode shape. The bridge and plate may also contain potential energy terms due to axial forces, shown for the bridge only. This term is identical to the vibrating string potential energy and may be present due to applied loads or built-in stresses in the structure. Rotational inertia and shear energy can be added for any of these structures, although in most cases these are negligible[17]. A variety of boundary conditions are possible, but most micromechanical structures are best modelled with built-in (clamped) ends.

All of these results neglect the effect of the fixed ends on resonant frequency, based on the assumption that the deflections at the center of the structure are small compared to the thickness. When these deflections become significant, axial tension develops and introduces another energy term[4]. This energy is a non-linear function of both the amplitude of vibration as well as any static deflection due to applied loads.

<p style="text-align: center;">BEAM</p> <p style="text-align: center;">$y(x,t) = Y(x)e^{j\omega t}$</p>	$\omega^2 = \frac{\frac{EI}{2} \int_0^L \left[\frac{d^2 Y(x)}{dx^2} \right]^2 dx}{\frac{\rho A}{2} \int_0^L Y^2(x) dx}$
<p style="text-align: center;">BRIDGE</p> <p style="text-align: center;">$y(x,t) = Y(x)e^{j\omega t}$</p>	$\omega^2 = \frac{\frac{EI}{2} \int_0^L \left[\frac{d^2 Y(x)}{dx^2} \right]^2 dx + \frac{T}{2} \int_0^L \left[\frac{dY(x)}{dx} \right]^2 dx}{\frac{\rho A}{2} \int_0^L Y^2(x) dx}$
<p style="text-align: center;">PLATE</p> <p style="text-align: center;">$w(x,y,t) = W(x,y)e^{j\omega t}$</p>	$\omega^2 = \frac{\frac{D}{2} \iint \left[\left(\frac{d^2 W}{dx^2} + \frac{d^2 W}{dy^2} \right)^2 - 2(1-\mu) \left[\frac{d^2 W}{dx^2} \frac{d^2 W}{dy^2} - \left(\frac{d^2 W}{dxdy} \right)^2 \right] \right] dx dy}{\frac{\rho h}{2} \iint W^2 dx dy}$

EI = Flexural Rigidity of Beam/Bridge T = Axial Force on Bridge ρ = Density
 D = Flexural Rigidity of Plate μ = Poisson's Ratio A = Cross-Sectional Area of Beam

Table 1. Resonant frequency of cantilever beam, bridge, and plate.

C. Acoustic Waves

An analog to Rayleigh's Method exists in the case of acoustic wave resonators, where it is possible to represent the resonant frequency of a non-piezoelectric resonator using variational methods as:

$$\omega^2 = \frac{\int_V \nabla_s \mathbf{v} : \mathbf{c} : \nabla_s \mathbf{v} dV}{\int_V \rho \mathbf{v} \cdot \mathbf{v} dV}$$

where the numerator is the strain energy and the denominator is the kinetic energy[6]. The velocity field distribution $\mathbf{v}(x,y,z)$ is analogous to the mode shape in the tensioned string example and is chosen to satisfy boundary conditions. Piezoelectric effects may be included by adding electrical potential energy terms. Perturbation methods are generally more tractable for calculating the effects of parameter variation on ω ; however, we will see in the next section that the energy method does yield a simple result for the mass change sensor.

VI. Resonant Sensors

A. Perturbations

Having determined the resonant frequency for some common microstructures, we now consider the use of resonant structures as transducing elements. In the following sections we consider the cases where the kinetic energy is perturbed by mass loading and the potential energy is perturbed by applying loads. For both cases we define a sensitivity to a parameter f as follows:

$$S = \frac{1}{\omega_0} \frac{d\omega}{df}$$

We write the resonant frequency as

$$\omega = \omega_0 \left[1 + \frac{dPE}{PE_0} \right]^{\frac{1}{2}} \quad \text{or} \quad \omega_0 \left[1 + \frac{dKE}{KE_0} \right]^{-\frac{1}{2}}$$

for a small perturbation dPE or dKE , where KE_0 , PE_0 , and ω_0 are unperturbed quantities. Note that KE actually represents the kinetic energy divided by ω^2 . Substituting ω into the sensitivity expression, we obtain:

$$S = \frac{1}{2} \frac{1}{PE_0} \frac{dPE}{df} \quad \text{or} \quad -\frac{1}{2} \frac{1}{KE_0} \frac{dKE}{df}$$

The sensitivity to perturbation varies as the ratio of the change in energy (potential or kinetic) over the initial energy. We have assumed that the amplitude of vibration remains constant during perturbation. This assumption is superfluous for small amplitude linear vibrations such as the tensioned string or the bending beam since the resonant frequency is independent of amplitude. However, it is required for large deflections, in which stretching energy becomes important.

B. Mass Change Sensors

One class of resonant microsensors detects changes in kinetic energy due to mass loading by vapor sorption or material deposition. Examples include resonant-microbridge vapor sensors, bulk wave gas detectors, SAW vapor sensors, and acoustic-resonator film-thickness sensors.

We now evaluate the sensitivities of the resonant-microbridge[14] and SAW vapor sensors[9]. These devices detect the mass change that occurs when vapor is absorbed by a thin polymer film coating the resonating bridge or SAW. If we consider the sensitivity of the

resonant-microbridge to variations of film density ρ_0 due to changes in ambient vapor concentration c , we find that

$$S = -\frac{1}{2} \left[\frac{\frac{d\rho}{dc} h}{\rho_0 h + \rho_1 d} \right]$$

where h and ρ_0 are the thickness and unperturbed density of the polymer and d and ρ_1 are the thickness and density of the bridge. For the SAW device we assume that the velocity components of the Rayleigh wave are constant through the thin polymer film. Consequently, the integral of the kinetic energy density reduces to $\rho h v^2$ with h being the film thickness:

$$S = -\frac{1}{2} \left[\frac{\frac{1}{2} \frac{d\rho}{dc} h (|v_x|^2 + |v_y|^2 + |v_z|^2)}{KE_0} \right]$$

The kinetic energy KE_0 is the unperturbed kinetic energy per unit surface area. The sensitivities of both vapor sensors are proportional to the kinetic energy in the sensitive film. Thus, it is desirable in the case of the microbridge to increase the ratio of film to bridge thickness, or in the case of the SAW to increase the film thickness to Rayleigh wave-length ratio. In pursuing this approach, the effects of the elastic properties of the film on the original resonant frequency and quality factor must be considered. The changes in elastic properties of the film as it interacts with the vapor may also affect the sensitivity.

C. Force/Pressure Sensors

The majority of resonant force and pressure sensors use perturbations of potential energy for sensing. We consider three general types; resonant-diaphragm pressure sensors, resonant-ribbon/quartz-microbeam force and pressure sensors, and SAW cantilever/diaphragm force and pressure sensors.

The resonant-diaphragm pressure sensor uses perturbations of the stretching energy term discussed in the previous section to measure pressure[25-27]. A static pressure induces a deflection of the diaphragm which introduces stretching. The vibrational potential energy of the diaphragm for a static deflection y (much greater than the vibration amplitude) is given by:

$$PE = k_1 + k_2 y^2$$

where k_1 and k_2 are constants[4]. Calculating the sensitivity of this structure to static deflection we find:

$$S = \frac{1}{2} \left[\frac{1}{PE_0} \right] 2k_2 y$$

The sensitivity is zero about the zero deflection point, because at very low deflections the stretching term does not affect resonant frequency. Thus, the sensitivity is a non-linear function of the displacement and requires a bias pressure.

An attractive approach to force or pressure measurement is to mechanically couple into the axial tension of a resonating element[15-18,29]. The sensitivity to variations in axial tension in a ribbon or microbeam is:

$$S = \frac{1}{2} \left[\frac{\frac{1}{2} \int_0^L Y^2(x) dx}{PE_0} \right]$$

The sensitivity is constant in the applied load. Examples of this type of sensor are the quartz microbeam/tuning fork devices.

A final example of potential energy change force

sensors are the SAW cantilever/diaphragm sensors[20-24]. The sensitivity is due to the third-order elastic coefficients which are functions of stress and strain in addition to strain induced geometry changes, and is typically analyzed using perturbation methods.

D. Other Resonant Sensors

Temperature sensors have been made using resonant structures[1,2]. These typically rely on the temperature sensitivity of a material property to alter the potential energy. It is also possible to make sensors which monitor the elastic property changes of materials. An example is the polymer glass-transition-temperature SAW sensor reported by Wohltjen[30].

V. Important Issues

In addition to ultimate sensitivity, there are many other materials, reliability and packaging issues that will dictate the applications of resonant microsensors.

The quality factor, both intrinsic and extrinsic, is important in determining performance. Quality factors of beams made of a variety of common microelectronic materials have been reported, and in general crystalline materials have substantially higher intrinsic Q than polycrystalline or amorphous films[33]. The extrinsic quality of resonance will depend strongly on geometry and packaging due to viscous damping. An example of this is a torsionally resonant silicon plate which has been reported to resonate with a Q of 10,000 in vacuums of 10^{-3} Torr but ceases to resonate above 1 Torr[28].

Many of the materials used in microfabrication have residual stresses which cause buckling and warpage[31,32]. Residual stresses can dominate the potential energy of vibration and the control of these stresses is critical to sensor performance. Both the quartz tuning fork and SAW devices have encountered substantial aging problems due to creep and fatigue of initially stressed structures. These problems motivated the development of the quartz microbeam resonator as a replacement for the tensioned string[15]. A microbeam does not require static tension in order to support vibration, thereby eliminating the potential for creep.

The method of driving and transducing resonance can substantially impact the reliability of a resonant sensor. As an example, stresses in deposited electrodes are critical in determining the aging of SAW delay lines and quartz resonators[35]. In general, excitation and detection mechanisms that do the least "damage" to the resonating structure are desirable.

As in all sensor applications, packaging poses special challenges. The way in which a resonating element is mounted can severely affect the quality factor because of poor rigidity. Dual tuning fork designs have been developed which eliminate this problem by canceling out any moments that would be imparted to the mount[18]. It is often desirable to keep the resonating element away from the environment being sensed. An example of this is the work by Greenwood which separates the resonating plate from the sensor diaphragm[28]. Finally, an example of a creative packaging scheme is the mounting of an axial-force-sensitive quartz beam in a metal package[34]. Axial forces induced by mismatch in thermal expansion coefficients cause the frequency to vary with temperature.

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