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Abstract: We present two-dimensional numerical solutions of carrier transport (comprising the electrostatic potential and current density) in semiconductor magnetic sensors in the presence of nonuniform, locally inverted induction $B_z(x,y)$. The variations in B and device geometries considered restrict the application of previous analytical methods. We illustrate our results by considering a split-drain device structure and studying its response for various induction configurations.

Introduction

A magnetic field sensor (MFS) is an input transducer that converts the magnetic field $\vec{H}$ into a useful electronic signal. Semiconductor MFS exploit the galvanomagnetic effects (such as Hall voltage, carrier deflection, magnetoresistance, and magnetoconcentration), which arise from the action of the Lorentz force $\vec{F} = q(\vec{v} \times \vec{B})$ on moving charge carriers (electrons and holes). Here q denotes the elementary charge, $\vec{v}$ the velocity of charge carrier and $\vec{B} = \mu\mu_0\vec{H}$ is the magnetic induction. The magnetic permeability $\mu\mu_0$ for semiconductors (Si, GaAs, InSb) is approximately 1. The magnitude of the sensor response is controlled by the product of the carrier Hall mobility and the magnetic induction B . Hence, high carrier mobility is crucial for achieving high sensitivity [1].

MFS detecting uniform fields are used in a variety of applications [1]. In the presence of nonuniform, locally inverted magnetic induction $B_z(x,y)$, an extended range of applications is also possible. Such nonuniform magnetic field configurations are crucial if semiconductor MFS are to be used for the detection of submicron displacements, vibrations, bubble domain velocities, magnetic ink patterns in banknotes and credit cards [2]. The variations in the device structures and $B_z(x,y)$ imply that standard analytical or related modeling methods are no longer applicable except in the limiting cases (see [2] and references therein).

In this paper, we present two-dimensional numerical solutions of the carrier transport (comprising the electrostatic potential and current density distributions) in semiconductor MFS, in the presence of nonuniform, locally inverted magnetic fields. This enables us to deal with general device geometries and induction configurations, not subject to the restrictions found in earlier work.

Model Equations

The action of the Lorentz force manifests itself in the carrier transport equations. We consider N-type Hall devices (bulk or linear MOSFET's) with the magnetic field $\vec{B} = (0, 0, B_z)$ perpendicular to the direction of current density $\vec{J}$ and $B_z = B_z(x,y)$.

We assume that diffusion currents and temperature gradients are negligible. For not too large magnetic induction B_z , the two-dimensional electron current density obeys the equation

$$\vec{J}_n(x,y) = -q\mu_n n \vec{\nabla}\Psi(x,y) + \mu_n^* [\vec{B} \times \vec{J}_n(x,y)] \quad (1)$$

where μ_n^* and μ_n denote the Hall and drift mobility of electrons, respectively, n the carrier concentration and $\Psi(x,y)$ the two-dimensional electrostatic potential. Equation (1) comprises the *isothermal* galvanomagnetic effects for electrons, accounting for the direct effects of temperature on carrier concentration and mobility. But it does not include thermomagnetic or thermoelectric effects [1]. Equation (1) corresponds to the leading terms of a weak field expansion and involves relative errors of $(\mu_n^* B_z)^{-1}$ [3]. They provide a good approximation ($\mu_n^* B_z \leq 0.3$) for N-type bulk material; hence it can be used for $|\vec{B}| \leq 2T$ for Si, $|\vec{B}| \leq 600$ mT for GaAs and $|\vec{B}| \leq 40$ mT for InSb [1].

Assuming uniform mobilities, μ_n^* and μ_n and zero net recombination, the divergence of $\vec{J}_n(x,y)$ in (1) yields

$$\begin{aligned} \frac{\partial}{\partial x} [(1 + \beta^2)^{-1} (\frac{\partial \Psi}{\partial x} + \beta \frac{\partial \Psi}{\partial y})] \\ + \frac{\partial}{\partial y} [(1 + \beta^2)^{-1} (-\beta \frac{\partial \Psi}{\partial x} + \frac{\partial \Psi}{\partial y})] = 0 \end{aligned} \quad (2)$$

where $\beta = \mu_n^* B_z$. Note that β is x,y dependent because $B_z = B_z(x,y)$. The boundary conditions for Ψ in (2) consist of a Dirichlet condition prescribed by the applied voltage, V_a at the electrodes (see Fig. 1). Everywhere else the standard condition, $\vec{J}_n \cdot \vec{n} = 0$ is adopted, where $\vec{n}$ is the outward normal vector. Subject to the above conditions, equation (2) is then discretized on a triangularized domain (see Fig. 1) using a Galerkin finite element approximation, and the resulting matrix is solved by an adaptive Successive Over Relaxation (SOR) procedure [4].

Results and Discussion

Our results are illustrated by considering three material types of bulk Hall devices: Si, GaAs and InSb with their mobilities assumed to be $0.08 \text{ m}^2/\text{Vs}$, $0.5 \text{ m}^2/\text{Vs}$, and $7.1 \text{ m}^2/\text{Vs}$ respectively. We have also assumed that the free carriers are electrons with a concentration of 10^{17} cm^{-3} . Split-drain device structures, $W = L = 100 \text{ }\mu\text{m}$, $d = 20 \text{ }\mu\text{m}$, are considered (see Fig. 1) with a locally inverted magnetic induction perpendicular to the device plane. As opposed to the conventional Hall plate structure illustrated in [4] where the Hall voltage is the computed signal, the

the split-drain device is biased with constant voltage sources and the relative current imbalance, $\delta = (I_1 - I_2) / (I_1 + I_2)$ is computed.

Equipotential and current lines are shown in Figs. 2 to 4 for different orientations of induction "inversion". For the device with a longitudinal strip domain (Fig. 2), we see that Hall effect prevails (current flow is parallel to the "inversion" boundaries and potential lines are skewed). In the case of the transverse strip domain (Fig. 3), the potential lines are parallel to the inversion boundaries and carrier deflection prevails (current lines are skewed by the local Hall angle). In the general case of the bubble domain (Fig. 4), the operating mechanisms of the device can be visualized neither in terms of Hall effect nor carrier deflection; these mechanisms are involved in a complex way on both sides of the inversion boundary. Similar to the results discussed in [4], at regions far away from the electrodes and inversion boundaries (e.g., in the device centre), equation (2) reduces to Laplace's equation and smooth divergence free electric fields are observed as predicted by analytical models [2]. The behavior of the fields become complicated at the vicinity of the electrodes and at the inversion boundaries.

Figure 5 shows the relative current imbalance as the bubble domain moves along the x-direction at two different positions in the device; $y = 75 \mu\text{m}$ (curve A) and $y = 50 \mu\text{m}$ (curve B). The values on the abscissa denote the distance between the centre of the domain and the centre of the device. The relative current imbalance behaves more or less linearly between $+10 \mu\text{m}$ and $+50 \mu\text{m}$ (Fig. 5). We note that for bubble positions remote from the drain vicinity, there is a reduction in δ as well as its gradient. With respect to spatial resolution, the optimum bubble position is at the vicinity of the drains. Previous theoretical calculations of the Hall output were performed only for the bubble located at the device centre. These calculations were based on an *ad hoc* weighted combination of the Hall outputs for the longitudinal and transverse cases [2].

Conclusion

In this paper, we have reported the two-dimensional numerical solutions of carrier transport in semiconductor magnetic field sensors under nonuniform, locally inverted magnetic fields. The effects of the device boundaries and "inversion" boundaries, on the electric field and current density, restricts simpler models solely based on Hall effect or carrier deflection, to very specific cases of device geometry and magnetic field.

References

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Fig. 1. Two-dimensional geometry of split-drain device with an adaptively generated finite element grid of 621 nodes and 1168 elements. This grid was employed in the simulation of device with bubble domain of radius $20 \mu\text{m}$, located in the centre of the device. The device is biased with voltage sources, $V_a = 1.0 \text{ V}$.

Fig. 2. Equipotential and current lines for device with longitudinal strip domain; $(\delta/B)_{\text{Si}} = -2.2\%/T$, $(\delta/B)_{\text{GaAs}} = -13.9\%/T$, $(\delta/B)_{\text{InSb}} = 0.20\%/mT$.

Fig. 3. Equipotential and current lines for device with transverse strip domain; $(\delta/B)_{Si} = -0.88\%/T$, $(\delta/B)_{GaAs} = -5.5\%/T$, $(\delta/B)_{InSb} = -0.08\%/T$.

Fig. 4. Equipotential and current lines for device with bubble domain (Fig. 1); $(\delta/B)_{Si} = -3.0\%/T$, $(\delta/B)_{GaAs} = -18.9\%/T$, $(\delta/B)_{InSb} = -0.27\%/T$.

Fig. 5. The relative current imbalance, δ as a function of bubble domain location in device for; A: bubble centred at $Y = 75 \mu m$. B: bubble centred at $Y = 50 \mu m$.