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ABSTRACT

Thermal modeling techniques are demonstrated as they apply to a generic integrated microsensor with a heat source located on an anisotropically etched silicon diaphragm. One- and two-dimensional conduction formulas, boundary layer convective heat transfer, a method of images approach for chip-level conduction, and the finite-element method are discussed, with examples, as they apply to the generic microsensor.

I. INTRODUCTION

Thermal management of digital and analog integrated circuits has received considerable attention [1-4]. The additional thermal management challenge presented by integrated microsensors has received considerably less attention [5]. This paper demonstrates the development of a thermal model for a heated chemically sensitive microsensor and addresses issues relevant to microsensor integration. The geometry of the generic heated microsensor treated here is given in Sec. II. Simple conduction and convection models are presented in Secs. III and IV. A chip-level model of thermal conduction which leads to an important conclusion about integrability is introduced in Sec. V, and the resulting lumped electric-circuit analogue is discussed in Sec. VI. Section VII demonstrates the use of the finite-element method as a means of achieving increased accuracy in calculations of conduction heat flow.

II. GEOMETRY OF A GENERIC HEATED MICROSENSOR

The generic microsensor considered here is shown in Fig. 1. It consists of an $80\text{ }\mu\text{m} \times 120\text{ }\mu\text{m}$ poly-Si heater located in a buried layer on a $250\text{ }\mu\text{m}$ square Si membrane. The heater is connected by two $1\text{ }\mu\text{m}$ -thick Al leads and is used to heat a thin, chemically sensitive element, which also has two leads. The membrane is taken as a $2\text{ }\mu\text{m}$ -thick B^+ -implanted Si diaphragm fabricated on a $300\text{ }\mu\text{m}$ thick $\times 3\text{ mm}$ square $\langle 100 \rangle$ -Si chip. Two $1\text{ }\mu\text{m}$ -thick layers of silicon dioxide are used as dielectric insulators. Typical values for the temperature-dependent thermal conductivities of the various materials are taken from the literature [6,7].

Fig. 1 Schematic representation of heat flow paths away from the heater.

III. SIMPLE CONDUCTION MODELS

Because of the planar character of many device geometries, simple one- and two-dimensional heat conduction formulas are often appropriate. This is especially true if the thermal conductivities of adjacent materials are significantly different, as in the case of boron-doped Si and SiO_2 , where the ratio of conductivities can be 50:1. For example, in Fig. 1 the downward-flowing heat currents from the poly-Si heater will flow through the SiO_2 with little lateral deviation. The thermal resistance of this region can then be modeled adequately as the resistance of a rectangular parallelepiped. Similarly, the heat currents through the silicon membrane are essentially lateral and simple formulas for the thermal resistance of rectangular annuli can be applied to obtain an estimate for this region. The more complex regions can be treated with the lumped-element approach by partitioning the structure in some more or less reasonable way, often using average, or equivalent, lengths and areas. This is commonly done [7], although caution must be exercised and accuracy cannot be expected.

IV. CONVECTION

For IC's, convection is generally not an important issue at the chip level, but must be considered in selecting an appropriate package [8]. For the sensing element under consideration here, however, we allow that convection could be important. The Rayleigh number must exceed a geometry-dependent critical value for natural convection to occur. If our microsensor is mounted in a typical TO-style can (with holes for gas flow), then the Rayleigh number turns out to be approximately 6000, well above the usual range of critical values, 1100-1700 [9]. Thus, natural convection is likely. The thermal resistance, $R = \Delta T/Q$, of the natural convection path is given by $L/(A \cdot \text{Nu} \cdot \kappa)$ [10], where L is the length of the side and A is the area of the square membrane region, Nu is the Nusselt number for the geometry, κ is the thermal conductivity of air, ΔT is the difference in temperature between the heated region and the can, and Q is the total heat flux due to natural convection. Using $L = 250\text{ }\mu\text{m}$, $A = 6.25 \times 10^{-4}\text{ (}\mu\text{m)}^2$, $\Delta T = 400^\circ\text{C}$, the value of κ at 200°C , and the range 5 to 8 of average values for the Nusselt number [11], R is estimated to be in the range 13 000 to 21 000 K/W. This range of thermal resistances is large compared to other thermal paths for this sensor. In addition, natural convection is unlikely underneath the diaphragm region because the Rayleigh number there is well below 1000. Thus, natural convection is not an important thermal conduction mechanism for this microsensor.

Forced convection, however, may also be present, especially if the sensor is not protected from flows being directed across the heated sensing element. A complete analysis of the heat transport under forced flow conditions is complex. An estimate can be obtained for specific cases, however, and this is done here assuming a flow speed of 100 mm/s over the chip.

Following the standard integral method used in boundary layer theory [12], the heat flux per unit area from an infinitely long heated strip of arbitrary, continuous wall temperature in the presence of a steady parallel flow of gas (see Fig. 2) is given by

$$q_w(x) = 0.323 \kappa \left(\frac{U_\infty}{\eta} \right)^{1/2} \text{Pr}^{1/3} \int_{x^*=x_0}^x \frac{(dT_w(x^*)/dx^*) dx^*}{x^{1/2} [1 - (x_0/x)^{3/4}]^{1/3}},$$

where $T_w(x^*)$ is the wall temperature a distance x^* downstream from the leading edge of the chip; x_0 is the unheated length of the chip upstream from the heated strip; ρ and η are the density and dynamic viscosity, respectively; U is the free-stream flow speed of the gas; $\text{Pr} = \eta C_p / \kappa$ is the Prandtl number for the system, which includes the specific heat of the flowing gas; and it is assumed that the temperature of the leading edge of the chip is at the ambient temperature. For the case at hand $\text{Pr} \approx 0.73$. Applying this formulation to a diaphragm assumed to be at 400°C , except at its leading and trailing edges, where surface temperature gradients of $\pm 20^\circ\text{C}/\mu\text{m}$ are assumed to connect the membrane to the rest of the chip at 0°C , gives

$$q_w(x) = 0.323 \kappa \left(\frac{U_\infty}{\eta} \right)^{1/2} \left(\frac{4}{3} \text{Pr} \right)^{1/3} (20 \text{ K}/\mu\text{m}) r_w(x)$$

Fig. 2 Geometry considered in the estimate of heat lost to forced convection. The origin of the coordinate system is at the leading edge of the chip, as shown.

The function $r_w(x)$ is shown plotted in Fig. 3 and demonstrates the relative rates of surface heat flux from the membrane as a function of downwind position. The negative values of $r_w(x)$ on the extreme downwind side indicate a net influx of heat in that region of the diaphragm. This procedure gives an estimate of the upper bound on the forced-convection heat flux likely for this heated membrane region, 1.1 mW. This is a minor heat loss path for this sensor. Extension of this procedure to the entire chip surface, assumed for the purposes of the estimate to be at 100°C and cooled by a flow at 0°C , gives 22 mW, which is significant. Turbulent convection, if present, would be expected to increase the heat flux appreciably.

Fig. 3 The convection integral $r_w(x)$ plotted versus position along the $250 \mu\text{m}$ long membrane region, which is centered on the 3 mm square chip. The integral has units $(\mu\text{m})^{1/2}$.

V. CHIP-LEVEL CONDUCTION

The surface temperature field of a chip is of importance for integration of a microsensor with circuitry. Bilotti's work discusses some of the physical concepts of importance [13], and a number of other authors have made analyses [14]. A simple physical picture that has proven useful conceptually and computationally is presented. Essentially, an alternating series of image sources, Fig. 4, leads to the Green's function for surface heat sources in an infinite slab model with an isothermal lower surface [15]. The temperature anywhere in the slab is found by summing up the contributions from the source and image sources.

$$T = T_0 + \sum_i \frac{2Q_i}{\kappa R_i}$$

Fig. 4 Placement of real and image sources in the method of images approach. In (a) the filled circles represent the real source and image sources with the same sign and magnitude as the real source, and unfilled circles represent image sources with the opposite sign. In (b) all the sources have magnitude twice that of the real source.

The sum is over the infinite string of charges, R_i is the distance from charge i to the field point in the chip, and T_0 is the temperature of the lower surface. The resulting top-surface temperature is shown in Fig. 5, which demonstrates the exponential falloff in surface temperature [13] and the near-field dependence, which is R^{-1} for $R < t$, where t is the thickness of the chip. Vertical temperature profiles are given in Fig. 6.

There is only one length scale in the steady-state problem, t , and therefore finite-chip size effects are only of importance within about one thickness unit of the chip edge. Thus, Figs. 5 and 6 give an accurate picture of the temperature everywhere of interest. Extended sources of arbitrary shape can be treated by superposition. Sources not on the surface can be treated in a similar fashion. The essential point is that there is a fast, 13.6 dB/thickness unit, falloff of temperature in the far field, and that any desired surface temperature ($T > T_{\text{header}}$) can be obtained by moving away from sources in steps of size t . Also, because of the single-length-scale character of the problem, defects in the isothermal boundary assumption will be annealed out in a distance $\sim t$ away from the defect, and the essential result is unchanged in the presence of bonding defects that cover a small fraction of the area of the lower surface [16].

Fig. 5 Surface chip temperature as a function of distance from a point source of surface heat flux. The exponential falloff of 13.6 dB per thickness unit is evident for distances greater than one thickness unit.

Fig. 6 Temperature profiles, from thermally anchored heat sink to chip surface, parameterized by the lateral distance from a point source of surface heat flux. Taken together, Figs. 5 and 6 give the temperature everywhere in a semi-infinite chip model.

The sensor being considered contains a membrane region which can be considered as a heat source connected by a small area A to a chip with a hole in it, as in Fig. 7a. For $A \ll 2\pi t^2$ the heat flow is in the $T-R^{-1}$ regime. What matters, then, is the surface area through which the heat in the membrane pours out into the chip proper. The picture in Fig. 7a can thus be replaced by the one in Fig. 7b, for which the thermal resistance is simply $R = (2\pi\kappa r)^{-1}$. For the generic microsensor, this gives $R_{\text{chip}} \sim 60 \text{ K/W}$.

Fig. 7 The thermal resistance of the chip is defined as $\Delta T/Q$, where ΔT is the temperature difference between the isothermal lower boundary and the edge of the diaphragm region, as shown schematically in (a). To estimate the thermal resistance a hemisphere of radius r is introduced that has the same contact area between source region and chip region.

VI. LUMPED MODEL

The above calculations can be used to construct a lumped model, as in Fig. 8. For this sensor, the thermal resistance of greatest importance in reducing power consumption is that of the membrane, and the thermal resistance of the package is of greatest importance to keeping the chip circuitry near ambient temperature. Use of such lumped-element models is common, but cannot be expected to give accurate results, due to obvious limitations.

Fig. 8 Lumped thermal model of the generic microsensor. Approximate thermal resistances of heat flow paths are given in kelvins per watt. The most important resistances are those in the left-most resistance channel. Forced convection can be important when flow is present.

VII. FINITE-ELEMENT METHOD

Off-the-shelf finite-element codes are capable of readily providing $<5\%$ error in a solution of the heat conduction equation with complex boundary and forcing conditions, even with temperature-dependent thermal conductivity. We have demonstrated this with MSC-NASTRAN [17] as the code using simple right parallelepipeds to model the relatively complex membrane region. The pre- and post-processing software is available that can reduce the model building and analysis time considerably. An example of the output is shown in the left-hand side of Fig. 9, which shows isothermal lines on the chip surface for one quadrant

of our model. Design changes can be tested quickly. For example, the effect of introducing a micromachined hole in order to increase the thermal isolation of the sensing element can be explored by simply removing appropriate elements and rerunning the code. The right-hand side of Fig. 9 shows isothermal lines for a quadrant of the membrane with such a hole, and the effect of the hole is apparent. Other design changes, or changes in assumptions about boundary conditions, forcing functions, or material properties (including temperature-dependence) can be explored in a similar manner. Also, transient analysis is available in some codes, as are boundary conditions that take into account convective and radiative heat flow.

Fig. 9 Finite-element-method generated isotherms for a three-dimensional model of a generic micro-sensor fabricated on a micromachined silicon membrane. In the left figure is shown one quarter of the model, which includes a buried heater located in the lower right-hand corner. In the figure on the right are plotted isotherms for a mirror image of the model after a micromachined hole has been introduced. The finite-element method allows for efficient investigation of sensor design modification.

In the membrane model without the hole the thermal resistance is 1020 K/W ($\pm 5\%$) which is $\sim 30\%$ higher than the value obtained from the lumped model of Sec. VI. About one-half of this difference is due to explicit inclusion of temperature-dependent material properties in the finite-element calculation. The other half is due to the ability of the finite-element method to deal with thermal conduction in a quasi-continuous, as opposed to lumped-element, manner. Continued improvement in pre- and post-processing capabilities will allow increased accessibility to the finite-element method to sensor designers and modelers, and increased computational speed and storage capabilities will allow increased accuracy of computations.

VIII. SUMMARY

The heat currents in a generic heated microsensor have been analyzed using one-dimensional conduction models, an integral technique for forced convective heat transport, a method-of-images approach for determining the temperature field of a chip, and the finite-element method for increased accuracy of conduction calculations. It is expected that the finite-element method will be used increasingly in the areas of micro-sensor design and modeling.

IX. REFERENCES

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