Noise in Piezoresistive and Capacitive Sensors

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Abstract

An analysis has been performed of the minimum resolvable input (*MRI*) for piezoresistive and capacitive pressure sensors as determined by fundamental noise mechanisms. Even though capacitive sensors have raw sensitivities that are an order of magnitude or more higher than those of piezoresistive sensors, the *MRI* of piezoresistive sensors (ignoring temperature coefficients and offsets) can be much lower than that of comparable capacitive sensors. Piezoresistive sensors may thus be more promising than capacitive sensors for applications requiring a large dynamic range (*DR*) and/or small *MRI*.

Glossary of Symbols

B_m	Measurement bandwidth (Hertz)
C_s	Capacitance of a capacitive pressure sensor (Farads)
d	Distance between plates of capacitor with $P = 0$ (μm)
DR	Dynamic range (P_{max}/MRI)
E	Young's modulus (mmHg)
h	Diaphragm thickness (μm)
k	Boltzmann's constant (Joules/K)
kT	Thermal energy of one electron (Joules)
L	Length of one side of a square diaphragm (μm)
MRI	Minimum Resolvable Input (mmHg)
P	Pressure on diaphragm (mmHg)
P_{max}	Maximum allowable input pressure (mmHg)
q	charge on an electron (Coulombs)
R	Resistance (Ω)
r	Radius of a circular diaphragm (μm)
S_{cap}	Capacitive sensitivity, defined as change in capacitance divided by reference-pressure capacitance (ppm/mmHg)
S_{piezo}	Piezoresistive sensitivity, defined as change in resistance divided by reference-pressure resistance (ppm/mmHg)
t	time
T	Absolute temperature (K)
V_b	Bias voltage applied to sensor (V)
ϵ	Permittivity of dielectric used for C_s and C_r ($F/\mu\text{m}$)
ν	Poisson's ratio
σ_{fract}	Fracture stress of silicon (mmHg)
σ_{max}	Maximum stress in diaphragm (mmHg)

This work was supported by Ford Corporation and the National Institutes of Health. Bruce Fleischer is supported by an AT&T Bell Laboratories PhD scholarship.

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Introduction

The purpose of this investigation is to determine the minimum detectable signal for a pressure sensor assuming that all systematic errors (*e.g.*, temperature coefficient and offset) have been eliminated. Capacitive and piezoresistive sensors are classified based on the method used to measure ΔC or ΔR . Noise in the detection and bias circuitry has been separated from noise in the sensor element proper where possible. In some configurations, however, circuit noise is an inherent part of the sensing method and so is included in the analysis.

Several comparisons of piezoresistive and capacitive sensors have been published (*e.g.*, [1,2,3]). Only the work by Chau and Wise [3] deals specifically with noise mechanisms, and that work did not consider the effects of different measurement techniques. Chau and Wise did show that Brownian noise is insignificant in all practical cases and it will not be considered here.

Capacitive Sensors

Chau and Wise imply that kT/C noise is dominant in all capacitive sensors. However, for many capacitive sensors kT/C noise is confined to a narrow bandwidth and does not affect the achievable resolution. Depending on the measurement technique used, the limit is determined by either shot noise integrated by the capacitor, kT/C noise, or noise in the switching operation (if a ratiometric scheme is used).

Detection circuitry for capacitive sensors often uses a current-controlled oscillator [4,1]. Since the currents used to ramp the oscillator must be switched, the dominant noise mechanism will be determined by the type of switches used. If the switches are MOS devices then the analysis is the same as for the RC oscillator circuit described below. If the switches are bipolar devices then their shot noise determines the fundamental limit to the performance.

With bipolar switches the only kT/C noise present is due to the resistance of the dielectric and has a bandwidth on the order of 10^{-3} Hz. The lower limit of the measurement bandwidth is much greater than the upper limit of the noise bandwidth and kT/C noise is not detectable. The minimum resolvable input for this shot noise limited case is

$$MRI = \frac{2}{S_{cap}} \sqrt{\frac{q}{C_s V_b}}. \quad (1)$$

The plate-touching pressure is

$$P_{max} = \frac{5.33dEh^3}{r^4(1-\nu^2)} \quad (2)$$

and the dynamic range is

$$DR = 0.295r \sqrt{\frac{\epsilon V_b}{dq}}. \quad (3)$$

Only two of these three quantities are independent (the dynamic range is defined as the maximum input pressure divided by the *MRI*). P_{max} is derived from the circular-diaphragm sensitivity equation in [3] and the equation for the deflection at the center of the diaphragm in [5].

Equations have also been derived for capacitive sensors where kT/C noise is the dominant mechanism (*e.g.*, RC relaxation oscillators). The maximum input pressure is the same as above and the other equations are

$$MRI = \frac{5.44}{S_{cap} V_b} \sqrt{\frac{kT}{C_s}} \quad (4)$$

and

$$DR = .167 V_b \sqrt{\frac{C_s}{kT}}. \quad (5)$$

These equations were derived by assuming that $t = RC_s$ is the parameter measured. The jitter in t was found using a method similar to Abidi's [6].

If a ratiometric measurement scheme is used then the MRI is determined by the circuit implementation. In a simple MOS-switch charge-sharing approach the kT/C noise resulting from the switch resistance is dominant. If more complicated switched capacitor schemes are used then the MRI is determined by op-amp noise, random errors in charge cancellation, and other sources determined by the particular design. Although promising, this technique is more complicated than the others discussed and no sensors using it have been described in the literature.

Piezoresistive Sensors

For piezoresistive sensors two general measurement techniques are possible: a ratiometric scheme (including the classic bridge configuration), or an RC oscillator. In both cases the dominant noise mechanism is thermal noise in the piezoresistor. P_{maz} , defined to be the rupture pressure, is found using equations for S_{piezo} , σ_{peak} , and σ_{fract} [1].

$$P_{maz} \approx 8.96 \times 10^6 \left(\frac{h}{L} \right)^2 \quad (6)$$

For piezoresistive RC sensors the MRI is the same as noted above for capacitive RC sensors except that the appropriate sensitivity must be used:

$$MRI = \frac{5.44}{S_{piezo} V_b} \sqrt{\frac{kT}{C}}, \quad (7)$$

and

$$DR \approx .03 V_b \sqrt{\frac{C}{kT}}. \quad (8)$$

For the ratiometric piezoresistive sensor (assuming a standard bridge configuration)

$$MRI = \frac{2\sqrt{4kTRB_m}}{S_{piezo} V_b}, \quad (9)$$

and

$$DR \approx \frac{.022 V_b}{\sqrt{4kTRB_m}}. \quad (10)$$

Summary

Using these equations it is possible to compare the performance limits of capacitive and piezoresistive sensors. These equations also show how the MRI , DR , and P_{maz} scale as long as the assumptions used in deriving them remain valid. The table below compares the calculated MRI for four representative sensors (adapted from [3]).

Minimum Resolvable Input ¹ (mmHg)				
Sensing Method	Ultraminiature		Ultrasensitive	
	Piezo (C1)	Cap (A1)	Piezo (D1)	Cap (B1)
Ratiometric	1.6×10^{-4}	see note 2	7.1×10^{-6}	see note 2
RC Oscillator	5.9×10^{-4}	1.7×10^{-2}	2.8×10^{-4}	1.0×10^{-2}
Current Controlled Oscillator ³	N/A	6.7×10^{-2}	N/A	4.1×10^{-2}

Notes:

1. Unless stated otherwise, a 3 V supply and a 100 Hz B_m have been used.
2. Depends on exact circuit configuration, see text.
3. These numbers do not use a 100 Hz B_m and can be improved slightly. How much improvement is possible depends entirely on circuit design and device performance, but the MRI will always be larger than for the piezoresistive case.

Acknowledgements

We thank Lyn Bowman for his participation in the early discussions of this work.

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