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THE INFLUENCE OF THE ELASTIC MEDIUM ON THE BOUNDARY EQUILIBRIUM OF ELASTIC PLASTIC CYLINDRICAL SHELL WITH INTERNAL CRACK

Kostenko Iryna, Nykolyshyn Myron, Nykolyshyn Taras, Hoshko Zinoviia, Pelekh Yaroslav

ВПЛИВ ПРУЖНОГО СЕРЕДОВИЩА НА ГРАНИЧНУ РІВНОВАГУ ПРУЖНО-ПЛАСТИЧНОЇ ЦИЛІНДРИЧНОЇ ОБОЛОНКИ З ВНУТРІШНЬОЮ ТРІЩИНОЮ

Костенко І.С., Николишин М.М., Николишин Т.М., Гошко З.О., Пелех Я.М.

The algorithm of numerical solving of a system of nonlinear singular integral equations of the problem about the boundary equilibrium of a closed elastic-plastic cylindrical shell located in an elastic medium and weakened by an internal crack is proposed.

The influence of the elastic medium and mechanical parameters on the crack opening and the size of the plastic zones are calculated.

Key words: *elastic-plastic cylindrical shell, internal crack, integral equations, opening of the front crack*

Задачу про граничну рівновагу ослабленої внутрішньою тріщиною замкнутої пружно-пластичної циліндричної оболонки, що знаходиться в пружному середовищі, зведено до розв'язання системи нелінійних сингулярних інтегральних рівнянь. Побудовано алгоритм числового розв'язання отриманої системи сумісно з умовами пластичності тонких оболонок, умовами обмеженості напружень та умовами однозначності переміщень. Досліджено вплив пружного середовища, геометричних та механічних параметрів на розкриття тріщини і розміри пластичних зон.

Ключові слова: *пружно-пластична циліндрична оболонка, внутрішня тріщина, інтегральні рівняння, розкриття тріщини*

The thin-walled elements of structures often interact with the environment in which they are located. They are modeled by shells, which are affected by the additional load as a reaction of the medium.

If this load is directed along the normal to the middle surface of the shell and its value is proportional to the normal component of displacements, then this medium is modeled by Winkler's hypothesis.

Such computational models are used, for example, in the study of various tanks and pipelines placed in soil or liquid, during the connection of shells with elastic bodies, and so on.

Therefore, the development of methods for calculating shells that are in an elastic medium and weakened by defects such as cracks and structural cuts, is of great practical importance. Quite a complete overview of the works on thin-walled elements of structures that are in an elastic medium has been made before [1-6].

But they relate, as a rule, to shells without pointed concentrators. Here the influence of the reaction of the elastic medium on the opening of the shores of the internal flat crack of arbitrary configuration, which is found in an elastically-plastic closed cylindrical shell, is investigated here

Let in such an environment there is a closed elastic-plastic isotropic cylindrical shell of thickness $2h$. We assign this shell to a three-orthogonal coordinate system α, β, γ . Let the shell be weakened by a longitudinal internal crack of length $2l_0$, which is located in the section $\beta = 0$ and is limited by continuous curves $d_1(\alpha)$ and $d_2(\alpha)$, which set the distance from the crack to the outer and inner surfaces of the shell.

We will assume that the shell and the cracks are loaded only symmetrically with respect to the plane of the crack by forces and moments. During the deformation of the shore, the cracks do not touch. We restrict ourselves to considering deep enough cracks ($d_3 = d_1 + d_2 \leq 0,6h$). The dimensions of the crack, the level of total loading and the properties of the material are assumed to be such that in the vicinity of the crack along the entire depth of the shell a narrow strip develops plastic deformations.

Now, according to the analogue δ_c -model [5, 6], the zones of plastic deformations will be replaced by the surfaces of the elastic displacement and turning angles, and the reaction of the material of the plastic zone with the elastic - by the corresponding forces and moments.

We assume that the continuation of the crack in depth to the outer and inner surfaces of the shell, that is, in the area $x \in]-x_0; x_0[$, $\gamma \in [-h; -h + 2d_1] \cup [h - 2d_2; h]$, there are constant stresses $\sigma_0 = (\sigma_B + \sigma_T)/2$, where σ_B and σ_T is the tensile strength and yield strength of the material. In plastic zones on the continuation of the crack along the length, that is, in areas $\gamma \in [-h; h]$, $\alpha \in]-\alpha_p; \alpha_0[\cup]\alpha_0; \alpha_p[$, $\alpha_p = l_p/R$; (l_p - the length of the plastic zone on the continuation of the crack), there are unknown normal efforts N and bending moment M , which, for an ideally elastic-plastic material, satisfy one of the conditions of plasticity of the Treska:

- in the form of the condition of plasticity of the surface layer

$$\frac{N}{2h\sigma_T} + \frac{3|M|}{2h^2\sigma_T} = 1, \quad (1)$$

- or the condition of the plastic hinge [9, 10]

$$\left[\frac{N}{2h\sigma_T} \right]^2 + \frac{|M|}{h^2\sigma_T} = 1. \quad (2)$$

And for the material with the strengthening, conditions of plasticity are

$$\begin{aligned} N(\alpha) &= P[(1-m^*)(|\alpha| - \alpha_0)/(\alpha_1 - \alpha_0) + m^*], \\ M(\alpha) &= H[(1-m^*)(|\alpha| - \alpha_0)/(\alpha_1 - \alpha_0) + m^*], \\ \alpha_0 &\leq |\alpha| \leq \alpha_1, \quad m^* = \sigma_B/\sigma_T, \quad \alpha_1 = \alpha_0 + \alpha_p, \end{aligned} \quad (3)$$

where $2\alpha_0$ - the length of a real crack, and $2\alpha_1$ - fictitious; P, H - unknown steels, which must satisfy the given condition of plasticity, for example, the condition of plastic hinge

$$[P/(2h\sigma_T)]^2 + |H|/(h^2\sigma_T^*) = 1.$$

Then

$$\begin{aligned} \left(\frac{N(\alpha)}{2h\sigma^*(\alpha)} \right)^2 + \frac{|M(\alpha)|}{h^2\sigma^*(\alpha)} &= 1, \quad \alpha_0 \leq |\alpha| \leq \alpha_1, \\ \sigma^*(\alpha) &= (\sigma_T - \sigma_B)(|\alpha| - \alpha_0)/(\alpha_1 - \alpha_0) + \sigma_B. \end{aligned}$$

Thus, within the framework of the adopted model, a non-through crack of length $2\alpha_0$ is replaced by a through crack of unknown length $2\alpha_1$, on the banks of which the conditions are fulfilled

$$\begin{aligned} N_s(\alpha) &= \begin{cases} N_s^{(i)} + N^l - N_s^0, & |\alpha| < \alpha_0, \\ N - N_s^0, & \alpha_0 \leq |\alpha| \leq \alpha_1, \end{cases} \\ M_s(\alpha) &= \begin{cases} M_s^{(i)} + M^l - M_s^0, & |\alpha| < \alpha_0, \\ M - M_s^0, & \alpha_0 \leq |\alpha| \leq \alpha_1, \end{cases} \end{aligned} \quad (4)$$

where N^l , M^l - the normal efforts and bending moment, which is the reaction of the material to break the internal bonds over and under the crack and which according to accepted assumptions about the stresses in these zones are determined by the formulas

$$N^l = \sigma^0 \int_{-h}^{d_1(\alpha)} d\gamma + \sigma_0 \int_{d_2(\alpha)}^h d\gamma, \quad M^l = \sigma^0 \int_{-h}^{d_1(\alpha)} \gamma d\gamma + \sigma_0 \int_{d_2(\alpha)}^h \gamma d\gamma,$$

N_2^0 , M_2^0 - efforts and moment of the main stressed state (shell without a crack); $N_s^{(1)}$, $M_s^{(1)}$ - the effort and moment applied to the shores of the crack.

The main relations, weakened by the crack of the cylindrical shell in an elastic medium.

Consider the shell loaded symmetrically with respect to the plane of the crack. The tense state of a shell without a crack (main) is considered to be well-known. Determination of perturbed stressed state caused by a crack is reduced to the solution of the system of differential equations in displacements:

$$L_{ij}u_i = q_i(\varepsilon_{22}^0, \kappa_{22}^0) - \frac{R^2}{B}ku_i, \quad (i, j = \overline{1,3}), \quad (5)$$

Here $u_1 = u$, $u_2 = v$, $u_3 = w$, k - is the coefficient of resistance of the elastic medium to the normal displacement of the median surface of the shell; operators L_{ij} , q_i , and functionals ε_{22}^0 , κ_{22}^0 described in [7].

Having solved the system (5) by the operator method [7, 8], we define the components of the displacement of the median surface of the shell:

$$\begin{aligned} u_i &= R(L_{2u_i}^{(1)}\varphi_2 + P_{2u_i}^{(1)}\psi_2), \quad L_{2u_i}^{(1)} = L_{2u_i} + L_{2u_i}^*, \quad P_{2u_i}^{(1)} = P_{2u_i} + P_{2u_i}^*, \\ L_{2u}^* &= 2\eta/(1-\nu)\nu\partial_1\nabla^2\nabla^2, \quad L_{2v}^* = 2\eta/(1-\nu)\partial_2\nabla^2\nabla^2, \quad L_{2w}^* = \eta\partial_1^2/c_1^2, \\ P_{2u}^* &= 2\eta\nu/(1-\nu)(\nu\partial_1^3 + \partial_1\partial_2^2), \quad P_{2v}^* = 2\eta/(1-\nu)(\nu\partial_1^2\partial_2 + \partial_2^3), \\ P_{2w}^* &= -\eta\nu\partial_1^4 - (1+\nu)/(1-\nu)\eta\partial_1^2\partial_2^2 - 2\eta_1/(1-\nu)\partial_2^4, \\ \eta &= R^2k/B, \quad c_1^2 = h^2/(3R^2), \end{aligned} \quad (6)$$

Operators L_{u_i} , P_{u_i} are given in [8], and functions φ_2 , ψ_2 are determined from equations

$$D\varphi_2 = \varepsilon_{22}^0, \quad D\psi_2 = R\kappa_{22}^0, \quad (7)$$

The operator D has the form:

$$D = \nabla^2\nabla^2\nabla^2\nabla + c^{-2}\partial_1^4 + b_1\partial_1^6 + b_2\partial_1^4\partial_2^2 + b_3\partial_1^2\partial_2^4 + b_4\partial_2^6 + b_8\partial_1^2, \quad (8)$$

where

$$\begin{aligned} b_1 &= \eta, \quad b_2 = 2\eta(2-\nu), \quad b_3 = 2\eta/(1-\nu), \quad b_4 = 2\eta(5-\nu)/2/(1-\nu), \\ b_8 &= \eta/c_1^2, \quad c^2 = c_1^2/(1-\nu^2). \\ b_1 &= \eta, \quad b_2 = 2\eta(2-\nu), \quad b_3 = 2\eta/(1-\nu), \quad b_4 = 2\eta(5-\nu)/2/(1-\nu), \\ b_8 &= \eta/c_1^2, \quad c^2 = c_1^2/(1-\nu^2). \end{aligned}$$

The field of incompatible deformations ε_{22}^0 , κ_{22}^0 , in this case, will take the form

$$\begin{aligned} \varepsilon_{22}^0(\alpha, \beta) &= R^{-1}[\nu(\alpha)]\delta(\beta), \quad \kappa_{22}^0(\alpha, \beta) = -R^{-1}[\theta(\alpha)]\delta(\beta), \quad |\alpha| < \alpha_0, \\ \varepsilon_{22}^0(\alpha, \beta) &= \kappa_{22}^0(\alpha, \beta) = 0, \quad |\alpha| \geq \alpha_0. \end{aligned} \quad (9)$$

Having constructed a 2π -periodic solution of equations (7), taking into account (9), we find the efforts and moments in the shell caused by the unknown density of jumps of displacements $[\nu]$, and angles of rotation $[\theta_2]$ on the line of the crack. If, before the efforts and moments of the perturbed tense state, the effort and moments in the shell without a cut (the main stress state) are added and require that the total values satisfy the conditions of the free shores of the crack

$$N_2(\alpha, 0) + N_2^0(\alpha, 0) = 0, \quad M_2(\alpha, 0) + M_2^0(\alpha, 0) = 0, \quad |\alpha| \leq \alpha_0, \quad (10)$$

then to determine the derivatives of functions $[\nu]$, $[\theta_2]$, we obtain a system of singular integral equations:

$$\int_{-\alpha_1}^{\alpha_1} \left\{ F_1(u) \left[\frac{1}{u-s} + K_{11}(\alpha_0(s-u)) \right] + F_2(u) K_{12}(\alpha_0(s-u)) \right\} du = 2\pi f_1^0(s), \quad (11)$$

$$\int_{-\alpha_1}^{\alpha_1} \left\{ F_1(u) K_{21}(\alpha_0(s-u)) + F_2(u) \left[K_{22}(\alpha_0(s-u)) + \frac{a_{22}}{u-s} \right] \right\} du = 2\pi f_2^0(s).$$

Here

$$F_1(\alpha) = R^{-1} \frac{\partial}{\partial \alpha} [\nu(\alpha)]; \quad F_2(\alpha) = c \frac{\partial}{\partial \alpha} [\theta_2(\alpha)]; \quad a_{22} = 3 - 2\nu - \nu^2;$$

$$f_1^0(\alpha) = -N_2^0(\alpha)/(Eh); \quad f_2^0(\alpha) = -M_2^0(\alpha)/(RcEh);$$

$$K_{11}(z) = \alpha_0 [1/z - 1/2 \operatorname{cth}(z/2) - K_{11}^0(z)]; \quad K_{12}(\alpha) = -\alpha_0 K_{12}^0(z);$$

$$K_{21}(z) = -\alpha_0 K_{21}^0(z); \quad K_{22}(z) = \alpha_0 [a_{22}(1/z - 1/2 \operatorname{cth}(z/2)) - K_{33}^0(z)].$$

Regular parts of nuclei $K_{11}^0(z)$, $K_{12}^0(z)$, $K_{21}^0(z)$, $K_{22}^0(z)$ were obtained when solving an elastic problem and expressed through a_{jn} , b_{jn} an imaginary and valid part of the complex roots of the characteristic equation

$$S_n^8 + (4n^2 - b_1)S_n^6 + (6n^4 + c^{-2} - b_2n^2)S_n^4 + (4n^6 - b_3n^4 - b_8)S_n^2 + (n^8 - b_4n^6) = 0 \quad (12)$$

Note that at $n=0$ the root of equation (12) have the form

$$S_{1,2} = \pm ia_{10}, \quad S_{3,4} = \pm iC_{10}, \quad S_{5,6,7,8} = \pm b_{20} \pm ia_{20}.$$

Note that in the system (11) the limits of integration are unknown (unknown length of the plastic zone l_p). In addition, in the right parts of the unknown effort N and the moment M . Therefore, the system (11) is solved together with the conditions of plasticity (1) or (2) for an ideally elastic-plastic material, or with conditions (3) for a material with the strengthening.

We require that the condition of stress limitations at the end of the plastic zones be fulfilled. To do this, it is sufficient that the coefficients of the intensity of the efforts and the moment at the end of the fictitious crack are equal to zero, i.e. $k_N(\alpha_1) = k_M(\alpha_1) = 0$. The algorithm for numerical solution of such systems with additional conditions is given in [7].

After integrating the solution of system (11), the formula for finding the opening of the crack at its arbitrary point will be given in the form

$$\delta(\alpha, \gamma) = [\nu(\alpha)] + \gamma[\theta_2(\alpha)]. \quad (13)$$

If in formula (13) we replace $\delta(\alpha, \gamma)$ by δ_c , then we obtain the criterion relation that relates the critical load, the crack length, the geometrical parameters of the shell and the resistance coefficients of the linear medium.

The algorithm for numerical solution of system (6) in conjunction with the conditions (7), (8) is implemented for a shell that is under the influence of internal pressure of intensity p ($N_2^0 = Rp$, $M_2^0 = 0$). Shell material is ideally elastic-plastic ($\sigma_B = \sigma_T$). The shell is weakened by an internal parabolic crack (Fig. 1)

Fig.1. Scheme of the location of the crack and plastic zones in a cylindrical shell.

$$d_1(\alpha) = \frac{1}{\tau_0^2} (h - d'_1 - d'_2) \alpha^2 - h + d'_1, \quad d_2(\alpha) = \frac{1}{\tau_0^2} (h - d'_1 - d'_2) \alpha^2 - h + d'_2,$$

where d'_1 and d'_2 is the distance from the top of the corresponding parabola to the inner and outer surface of the shell, $\tau_0 = l_0/l_1$. In Fig. 2 shows graphs of dependence of the relative crack opening $\delta^* = \delta(l_0/R, 0) E / (l_0 \sigma_T)$ at point A from the relative load $n^0 = N_2^0 / (2h\sigma_T) = pR / (2h\sigma_T)$. Numerical analysis was carried out at $\nu = 0,3$; $d'_1/h = 0,15$; $d'_2/h = 0,25$. Curves 1-4 correspond to the values of the relative length of the crack. The solid curves correspond to the value $\eta = 0,01$, and the dashed curves correspond to $\eta = 0,1$.

Fig. 2. Dependence of the relative crack opening from the load.

Fig. 3. Dependence of the length of the zone on the relative length of the real crack.

In Fig. 3 shows the dependence of the length of the plastic zone τ_0 on the relative length of the real crack, at $n^0 = 0,2$. Other parameters are the same as in Fig. 2.

In the case when the shell is weakened by the internal rectangular crack ($d_1(\alpha) = 0,15h$; $d_2(\alpha) = 0,25h$) of its opening at point A 2-3 times larger than in a parabolic crack. It depends on the load, geometric and mechanical parameters. At point B for a rectangular crack, the opening is 15-20 times more than for a parabolic. If the rectangular and parabolic cracks have the same area, then this difference is much smaller.

The choice of the plasticity condition for a perfectly elastic-plastic material has little effect on the result. If the material of the shell is characterized by strengthening, then the opening of the crack and the length of the plastic zones decreases by 10-15%.

Consideration of the reaction of the elastic medium on the cylindrical shell with an internal fracture leads to a decrease in the opening of the cracks and the size of the plastic zones, that is, to increase the bearing capacity of this shell. The choice of the plasticity condition for a perfectly elastic-plastic material has little effect on the result. If the material of the shell is characterized by hardening, then the opening of the crack and the length of the plastic zones decreases by 10-15%.

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