WEAK SIGNAL DETECTION BASED ON MULTIPLE AUTO-CORRELATION TECHNIQUES

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Abstract: This study presents the performance of the auto-correlation methods for detecting weak signals, where the signal level is much less than the noise level. Double and triple auto-correlation techniques are used to improve the detection performance compared with the single autocorrelation. Simulation results obtained by MATLAB programs show that the multiple correlation techniques outperform the single correlation in terms of probability of detection and probability of false alarm versus signal to noise ratio SNR.

Keywords: *weak signal detection, signal to noise ratio (SNR), auto-correlation method, triple auto-correlation.*

1. Introduction

Recently, there has been an increase in the studies of acquiring, analyzing and processing all kinds of information in a deeper way, and in particular there is a great interest in discovering useful weak signals embedded in strong noise. A weak signal is a signal that has a signal-to-noise ratio less than zero dB, meaning the noise level is bigger than the signal level. The importance of these researches is evident in its wide applications such as very long-distance communication when the signal power becomes very low like satellite communication, seismic testing, pipeline leakage, fault diagnosis of mechanical system etc. Hence, the detection of the weak signal under low signal-to-noise ratio becomes very important. Accordingly, several methods of processing the weak signal, including neural network, wavelet analysis method and fuzzy set theory [1,2], have been developed.

Le et al. [3] suggested the inner product vector for detection where cross correlation functions between response measurements is used. Where the response quantity is due to white noise excitation. Avramchuk et al. [4] made the analysis of the correlation of two signals, to determine valid signal frequency limits and to set on this basis frequency filters parameters to enhance the signal-to-noise ratio. Liu et al. [5] suggested a new method to detect unknown weak signals in the satellite applications. These methods depends on cross-correlation algorithm, which does not need previous knowledge of the signal, only resorts to the features of signal to filter noise and detect if the unknown signal exists or not. On the other hand, some methods of weak signal detection need a certain previous knowledge of the signal, i.e., some information of the signal must be known right from the beginning, such as frequency, phase, etc. Blind detection techniques based on auto-correlation algorithms are given in [6,7] where these techniques do not require prior knowledge of the signal parameters.

Yuandong et al. [8] suggested a detector based on Ensemble Average of Autocorrelated Envelopes to identify the early occurrence faults in rolling element bearings, of which the fault caused vibration signals are contaminated or masked by additive background noise and random phase

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noise (or slippage between bearing components). To improve the cyclostationary characteristics, it uses the phase synchronization property of autocorrelation for aligning the cyclostationary signals in the lag domain to obtain an effective ensemble average which permits both types of random effects to be eliminated significantly. Duona et al. [9] suggested the learning modulation filter networks to enhance the detection performance. The approach depends on a two-stage optimization scheme for filter learning, attention mechanism and classification in a unified framework.

The main challenging problem in detection of weak signals is to get high detect probability with low probability of false alarm at low signal to noise ratio.

In this paper, multiple autocorrelation techniques are used to improve the detection performance over the single auto-correlation techniques. MATLAB simulation results assure that the double and triple auto-correlation techniques outperform the single autocorrelation in terms of probability of detection P_d and probability of false alarm P_{fa} versus the signal to noise ratio SNR.

The rest of this paper proceeds as follows: Section 2 discusses the theory of auto-correlation; Section 3 discusses the simulation results of the techniques; Section 4 gives some conclusions.

2. Weak Signal Detection Based on Auto-Correlation Technique

2.1. Auto-Correlation Method

The techniques based on correlation method to detect weak signals depends on the statistical characteristics of the SNR, and the correlation function is a measure of the similarity between two signals [10].

Often the values of deterministic signal have a stronger correlation at different times. On the contrary, the noise values are random and therefore has less correlation. By using these properties (the difference between the signal and the noise), the deterministic signal and random noise can be distinguished. So, we can detect the weak signals from strong noise based on this theory. An autocorrelation function is expressed as [11]:

$$R_{xx}(\tau) = \int_{-\infty}^{\infty} f(t) * f(\tau + t) dt \quad (1)$$

Where R_{xx} is autocorrelation function, τ is lag index, $f(t)$ is received signal, $*$ is complex conjugate of the signal. Autocorrelation method can be achieved in time or frequency domain [12]. Autocorrelation function has another property, which is the symmetry with respect to the origin, maximum at ($\tau=0$), periodicity in the case of pure signals, and attenuation in the case of AWGN noise as shown in Fig. 1, where:

$$R_{xx} \neq 0 \dots \tau = 0 \text{ (max)}, \quad (2)$$

$$R_{xx} \approx 0 \dots \tau \neq 0 \quad (3)$$

Fig. 2 shows autocorrelation of the periodic signal where it has spike form of correlation function at ($\tau=0$), and other lags values for ($\tau \neq 0$). These lag values roll away from lag ($\tau=0$). The magnitudes of these lags depend on the feature of the data which consists of the interested signal.

$$R_{xx} \neq 0 \dots \tau = 0 \text{ (max)}, \quad (4)$$

$$R_{xx} \neq 0 \dots (\tau \text{ closet to } 0) \quad (5)$$

Hence, and as shown in Fig.1, one can use other than the zero lag to detect the presence of the signal.

Figure 1. Auto-correlation function of (a) AWGN; (b) signal $f(t)$.

2.2. Multiple Auto-correlation

However, and under very low signal to noise ratio, this non-zero lag value may be deeply embedded in noise and may be lost.

The detection performance of the signal $s(t)$ embedded in the noise $n(t)$ can be further improved if double or even triple autocorrelation computation are carried out i.e., the process is

shows block diagram for the three levels of autocorrelation.

Figure 2. Auto-correlation function of signal (a) single auto-correlation; (b) double auto-correlation; (c) triple auto-correlation.

Figure 3. Block diagram for multiple Auto-correlation.

Repeated to obtain an improvement in the signal-to-noise ratio and to determine the spectral content of the signal [11]. By using the previous properties of auto-correlation function, it is possible to detect weak signal better than a single and double auto-correlation. Where the number of harmonics doubles and the amplitude of the signal increases dramatically when the correlation is repeated as shown in Fig.2. Fig.3

3. Simulation Results

According to what was mentioned in the previous paragraph of the characteristics of auto-correlation, where all the harmonics of the noise are almost zero except for lag ($\tau=0$), and also the auto-correlation function of the periodic signal gives a large harmonic value, but it is smaller as it moves away from lag ($\tau=0$). Using this feature, the detection process is done by taking a specific

lag peak (say $\tau=1$) and operating the system with noise only, setting a threshold value to obtain the required probability of false alarm P_{fa} , and then the signal to be detected is added to the noise.

The system uses the same threshold value set to obtain a certain P_{fa} when the signal is embedded in the noise and the system has to discover the presence of the information signal with a certain P_d . The de-noising performance of the double and triple auto-correlation is evaluated and compared with single auto-correlation. The tested signal is a sinusoid with amplitude $A=1$ volt and normalized frequency $f_0=1\text{Hz}$ with the data length $N=10000$ as shown in Fig. 4(a). An AWGN with $\text{SNR}=-4\text{dB}$ is added to the pure signal as illustrated in Fig. 4(b). Fig. 4(c)-(d) display the single and double auto-correlation method respectively, for which the periodical harmonic signal is almost recovered from the noisy signal. By employing the triple auto-correlation method, the periodical harmonic signal is better than the result obtained in single and double auto-correlation method as demonstrated in Fig. 4(e) where the detection performance is largely improved. Fig. 5 is similar to Fig. 4 with $\text{SNR}=-20\text{dB}$.

The performance of P_d versus SNR under different P_{fa} are given in Fig. 6(a)-(b)-(c). The input SNR varies from -20 to 0dB, from -23 to 0dB and -28 to 0dB for single, double and triple auto-correlation respectively. As an example, for $\text{SNR}=-20\text{dB}$ with $P_{fa}=0.05$ at single auto-correlation $P_d=20\%$ and at double auto-correlation $P_d=100\%$, and at $\text{SNR}=-25\text{dB}$ with the same P_{fa} at double auto-correlation $P_d=29\%$ and at triple auto-correlation $P_d=79\%$.

To further evaluating the detection performance, the receiver operating characteristic (ROC) curves at $\text{SNR}=-13, -15$ and -17dB are presented in Fig. 7(a) for single auto-correlation. Fig. 7(b) presented the ROC curves for double auto-correlation at $\text{SNR}=-21, -22$ and -23dB . Fig. 7(c) shows the ROC curves for triple auto-correlation under lower $\text{SNR}=-24, -25$ and -27dB .

Figure 4. Auto-correlation function of signals with $\text{SNR}=-4\text{dB}$ (a) pure signal; (b) noisy signal; (c) Auto-correlation to noisy signal after first stage of correlation (d) Auto-correlation to noisy signal after second stage of correlation. (e) Auto-correlation to noisy signal after third stage of correlation.

Figure 5. Auto-correlation function of signals with SNR=-20dB (a) pure signal; (b) noisy signal; (c) Auto-correlation to noisy signal after first stage of correlation (d) Auto-correlation to noisy signal after second stage of correlation. (e) Auto-correlation to noisy signal after third stage of correlation.

Fig. 8 demonstrates the differences between single, double and triple correlation techniques at $P_{fa}=0.05$. One can easily notice that for $P_d=0.9$, the double auto-correlation is approximately 6dB better than single and triple auto-correlation is even better than double by 3dB. Table (1) shows a comparison of the SNR requirement for $P_d=0.9$ for $P_{fa} = 0.01, 0.05$ and 0.1 .

Figure 6. Probability of detection versus SNR for single tone detection at using autocorrelation, (a) single auto-correlation; (b) double auto-correlation; (c) triple auto-correlation.

Table 1. A comparison of SNR requirement at $P_d=0.9$ to keep P_{fa} a specific value.

	single correlation	double correlation	triple correlation
SNR (dB) required for $P_{fa} = 0.01$	-14.2	-21.1	-23.8
SNR (dB) required for $P_{fa} = 0.05$	-15.1	-21.7	-24.2
SNR (dB) required for $P_{fa} = 0.1$	-15.8	-22.4	-24.5

Figure 7. ROC curves of detection using Autocorrelation, (a) single auto-correlation; (b) double auto-correlation; (c) triple auto-correlation.

Figure 8. Comparison between multiple auto-correlation function based on probability detection versus SNR for single tone detection.

4. Conclusions

This paper investigates the performance of multiple autocorrelation techniques in detecting weak signals. Triple auto-correlation technique showed the best performance over single and double correlation techniques. For $P_d=0.9$ and $P_{fa}=0.05$, the double auto-correlation is approximately 6dB better than single correlation and triple auto-correlation is even better than double correlation by about 3dB. This gives about 9dB improvement of the triple correlation technique compared with the single correlation. Multiple correlation technique, as the single correlation, does not require any pre-information about the signal to be detected. Finally, it was found during simulation that more than triple correlation does not give a significant improvement in the performance of the detector.

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Conflict of interest

The authors declare that this research does not conflict with any other work.

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