Numerical Study of Heat Transfer of Water Flow through Pipe with Property Variations

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Microchannels, with their small size and high heat dissipation capacity, play an important role in electronic devices. The present study deals with steady, laminar, incompressible, single phase liquid water flow, with constant wall heat flux, through a micro-pipe. Numerical simulations are performed for two-dimensional axisymmetric pipe for developing flow at the entrance. The effect of constant and variable thermo-physical properties, at the inlet boundary, on heat transfer and hydrodynamic characteristics is examined with variations of radius, inlet velocities, and constant wall heat flux. It is observed that the Nusselt number increases with an increase in radii and inlet velocities and depicts higher values with variable property flows as compared to constant property flows.

Keywords: Heat flux, Mean temperature, Micropipe, Nusselt number, Thermal conductivity, Viscosity.

Introduction

Liquid flow through micro ducts is used to dissipate heat in miniature engineering systems, which is of paramount importance for their performance. Fluid flow, through conventional macro size ducts, has been extensively studied to develop the well-established analytical relations of heat transfer (Weilin et al., 2000; Sobhan et al., 2001; Morini, 2004; Hetsroni et al., 2005a; Mahmoud and Karayiannis, 2013). Unfortunately, there exist many discrepancies between the mechanism of fluid flow and heat transfer in macro ducts and micro ducts. This opens a broad door to unveil the flow physics in the area of micro-convection heat transfer.

The first micro channel experimental studies performed by Tuckerman and Pease (1981), demonstrated that greater heat transfer rate could be achieved with smaller diameters. Subsequently, several investigations have been conducted over the past years to practically analyze the flow through micro channels (Kohl et al., 2005; Liu and Garimella, 2007; Dirker et al., 2014; Asadi et al., 2014). However, experimental measurements for micro channel pipes face a major challenge till date. As an alternative, numerical simulations have been employed by researchers, to overcome this limitation (Molho et al., 2005;

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Hetsroni et al., 2005b; Gulhane and Mahulikar, 2010; Gulhane and Mahulikar, 2011; Dixit and Ghosh, 2015).

Generally, constant properties (CP) are assumed in fluid flow and heat transfer analytical calculations for duct flows. The effect of fluid property variations with temperature such as density $\rho(T)$, thermal conductivity $k(T)$, specific heat at constant pressure $C_p(T)$ and viscosity $\mu(T)$, are generally neglected, that account for large temperature differences when compared with constant properties and around 30% deviation in Nusselt number (Nu) (Herwig and Mahulikar, 2006; Ozalp, 2010). Experimental evidence with a comprehensive review of the literature were provided by researchers showing the discrepancies present in literature for frictional factor and pressure drop for flows through micro channels (Muzychka and Yovanovich, 1998; Steinke and Kandlikar, 2005).

Our main objective is to investigate the effects of temperature variation of thermo physical properties like $\rho(T)$, $k(T)$, $C_p(T)$ and $\mu(T)$, on heat transfer and hydrodynamic characteristics i.e. Nusselt number (Nu), pressure drop Δp , Darcy friction factor f_D and skin friction coefficient C_f for developing water flow through the micro pipe. Also, the effect of variation in diameter, inlet velocities, and constant wall heat flux is studied on these characteristics.

The paper is organized as follows. First, the simulation methodology is discussed. In this section, governing equations, geometry details, grid convergence study, numerical method, and formulas are discussed. Next, the computed velocity and temperature profiles are discussed for constant and variable properties (CP and VP) respectively and compared to analytical conventional profiles of pipe flow. Next, the isolation and combined effects of variable properties (VP) on heat transfer and fluid flow characteristics are studied and compared to constant property (CP) numerical results. Next, the effect of different radii, inlet velocities and wall heat wall fluxes on these characteristics is studied for constant and variable properties (CP and VP). Finally, the conclusions are discussed.

Simulation Methodology

Governing Equations

The conservation equations of mass, momentum, and energy used in numerical simulations are described below. The single-phase liquid flow is assumed to be laminar, steady, and incompressible (Rohsenow et al., 1998).

Continuity equation:

$$\frac{1}{r} \frac{\partial(r u_r)}{\partial r} + \frac{\partial u_z}{\partial z} = 0 \quad (1)$$

r-component momentum equation:

$$\rho(u_r \frac{\partial u_r}{\partial r} + u_z \frac{\partial u_r}{\partial z}) = -\frac{\partial p}{\partial r} + \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_r}{\partial r} \right) - \frac{u_r}{r^2} + \frac{\partial^2 u_r}{\partial z^2} \right] \quad (2)$$

z-component momentum equation:

$$\rho(u_r \frac{\partial u_z}{\partial r} + u_z \frac{\partial u_z}{\partial z}) = -\frac{\partial p}{\partial z} + \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_z}{\partial r} \right) + \frac{\partial^2 u_z}{\partial z^2} \right] \quad (3)$$

Energy equation:

$$\rho C_p (u_r \frac{\partial T}{\partial r} + u_z \frac{\partial T}{\partial z}) = k \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right] + 2\mu \left(\frac{\partial u_r}{\partial r} \right)^2 + \left(\frac{\partial u_z}{\partial z} \right)^2 + \mu \left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z} \right)^2 \quad (4)$$

Here, r and z are radial and axial coordinates, u_r is the velocity component in the radial direction and u_z is the velocity component in the axial direction as shown in Figure 1, and p is the pressure. In incompressible flow, the density ρ , viscosity μ , thermal conductivity k , specific heat capacity at constant pressure C_p are assumed as constants, which is a poor assumption for micro channels. A more accurate approach to solve these equations would be to assume the dependence of these properties on temperature. In Eq. 1, the left-hand side is the total mass flux passing in axial and radial directions through the control surface per unit volume. The left-hand terms in Eq. 2 and 3 represent the rate of momentum transfer by convection per unit volume in axial and radial directions. The first and second terms on right-hand side represent the pressure and viscous forces acting on control surface per unit volume in axial and radial directions. The gravitational force is neglected. The left-hand side of Eq. 4, is the total energy lost by convection while the first term on the right-hand side is the amount of heat transfer by conduction per unit volume. It is given by Fourier's law, $q = -k\nabla T$. Where k is the thermal conductivity and T is the temperature. The second term represents the work done by shear forces.

Numerical Details

Figure 1. The Two-dimensional Axisymmetric Geometry of the Micro Pipe showing Temperature and Velocity Profiles with Constant Heat Flux Boundary Condition on the Wall

Two-dimensional axisymmetric geometry is generated using Pointwise V17.2 R2 package. The domain extends to a length of $L = 0.018$ m in the axial

direction and to a diameter of $D = 10 \times 10^{-5}$ m in the radial direction as shown in Figure 1. Based on grid convergence study (see Figure 2), a grid size of 400×100 in axial and radial directions is used in the numerical simulations. The grid is clustered at an inlet and near the wall. A first cell distance of 5×10^{-6} m is taken to capture the velocity and temperature gradients.

Figure 2. Variation of Nusselt Number Nu , with Different Grids for Water Flow, through the Micro Pipe of Radius $R = 5 \times 10^{-5}$ m, Inlet Velocity $u_{in} = 3$ m/s, and Constant Wall Heat flux, $q_w = 100$ W/cm²

An in-house code is used to carry out the numerical simulations. The post-processing is done using Tecplot 360 package. The governing equations are discretized using second-order finite volume technique. The SIMPLE scheme is used to couple the velocity and pressure variables (Patankar, 1980). The implicit method is used to reach the steady-state solutions. The no-slip velocity and uniformly distributed constant heat flux boundary condition are used on the wall. The inlet boundary conditions of velocity $u_{in} = 3$ m/s, temperature $T_{in} = 273.65$ K are taken. At the outlet pressure $p_{out} = 1.01325 \times 10^5$ Pa, is assumed. At center line, an axisymmetric boundary condition i.e. gradients in the axial direction, $\partial(p)/\partial z$, $\partial(T)/\partial z$ and $\partial(\rho)/\partial z$ are assumed to be zero.

Thermophysical Properties

The thermophysical properties of water, i.e. $\rho(T)$, $k(T)$, $C_p(T)$ and $\mu(T)$ are assumed to depend only on temperature. The polynomial functions of second and third order are used to curve fit the data (see Figure 3) in the temperature range of 278.15 to 372 K at 1 bar pressure (Bergman and Incropera, 2011).

$$\rho(T) = 765.33 + 1.8142(T) - 0.0035(T)^2 \quad (5)$$

$$C_p(T) = 1.095e^4 - 59.27(T) + 0.171(T)^2 - 0.0001623(T)^3 \quad (6)$$

$$k(T) = -0.5752 + 6.3967 \times 10^{-3}(T) - 8.151 \times 10^{-6}(T)^2 \quad (7)$$

$$\mu(T) = 9.67 \times 10^{-2} - 8.207 \times 10^{-4}(T) + 2.344 \times 10^{-6}(T)^2 - 2.244 \times 10^{-9}(T)^3 \quad (8)$$

Figure 3 shows that as temperature increases, $k(T)$ increases and both $\rho(T)$ and $\mu(T)$ decreases. $C_p(T)$ first decreases with increase in temperature and then increases. These thermo physical variable properties (VP) used in our numerical simulations differ from formulations used in Ref. (Gulhane and Mahulikar, 2011). Also, the constant properties (CP) in our simulations are calculated at the fluid mean temperature in contrary to the simulations by Gulhane and Mahulikar (2011) where CP calculations are performed at inlet temperature.

Figure 3. Comparison of Polynomial Function Curves to the Available Data (Bergman and Incropera, 2011) for Variation of Thermophysical Properties with Temperature

Formulas

A numerical sample calculation is explained as follows. The mass flow rate of water flow through a micro pipe of radius $R = 5 \times 10^{-5} \text{ m}$ with an inlet velocity of $u_{in} = 3 \text{ m/s}$ is given as $m = \rho A u_{in}$. Here, $A = \pi R^2/2$ is cross sectional

area of pipe. The local mean bulk temperature at any location along the z-axis, $T_{m(z)}$ and exit temperature T_e is calculated by

$$T_{m(z)} = \frac{2}{u_m R^2} \int_0^R T(r) u(r) r dr ; \quad T_e = T_{in} + \frac{q_w A_w}{m C_p} \quad (9)$$

Here, r is the radius of pipe, T_{in} is inlet temperature, q_w is constant wall heat flux and T_e is exit temperature. The constant properties (CP), density ρ , viscosity μ , thermal conductivity k and specific heat C_p are calculated at bulk fluid temperature i.e. $T_b = (T_{in} + T_e)/2$ is 289.64 K, where T_{in} is taken as 273.65 K. The fluid properties at T_b are $\rho = 998.66$ (kg/m³), $C_p = 4184$ (J/kg-K), $k = 0.5926$ (W/m-K), and $\mu = 0.00108353$ (kg/m-s). The Prandtl number, $Pr = 7.65$, therefore the momentum transport by diffusion in velocity boundary layer is higher than energy transport by diffusion in the thermal boundary layer. The Reynolds number, Re based on the hydraulic diameter $D = 2R$, is calculated as

$$Re = \frac{\rho u_{in} D}{\mu} = 276.501 \quad (10)$$

Since, $Re \leq 2300$, the flow is assumed to be laminar. The hydro dynamically developed flow length, L_h and thermally developed flow length, L_t is calculated by

$$L_h = 0.05 Re D = 0.00138 \quad (11)$$

$$L_t = L_h Pr = 0.0105 \quad (12)$$

The L_h is calculated, at the location when the velocity profile does not change in the axial direction i.e. $\partial u / \partial z = 0$. The L_t is calculated where the temperature profile is only a function of radial direction. The local surface temperature of the pipe at along z is calculated as

$$T_{w(z)} = (q_w / h) T_{m(z)} \quad (13)$$

The Nusselt number Nu is calculated as the ratio of convective to conductive heat transfer normal to the boundary. The Nusselt number for constant properties (Nu_{CP}) and variable properties (Nu_{VP}) is calculated numerically for fully developed flow as follows:

$$Nu_{CP} = \frac{hD}{k} = \frac{q_w D}{k(T_w - T_m)_{CP}} = 4.53 \quad (14)$$

$$Nu_{VP} = \frac{hD}{k(T)} = \frac{q_w D}{k_{VP}(T_w - T_m)_{VP}} = 5.25 \quad (15)$$

Here, $k(T)$ has direct effect on Nu_{VP} , whereas $\rho(T)$ and $\mu(T)$ and $C_p(T)$ effects the wall and mean temperature (T_w and T_m respectively), and consequently has an indirect effect on Nusselt number, Nu .

The skin friction coefficient for constant property flow $C_{f_{CP}}$ and variable property flow $C_{f_{VP}}$ are calculated as follows.

$$C_{f_{CP}} = \frac{\mu \left(\frac{\partial u}{\partial r} \right)_{CP}}{0.5 \rho_{in} u_{in}^2} \quad (16)$$

$$C_{f_{VP}} = \frac{\mu(T) \left(\frac{\partial u}{\partial r} \right)_{VP}}{0.5 \rho_{in} u_{in}^2} \quad (17)$$

The Darcy friction factor f_D is defined in terms of non-dimensional mean pressure gradient $(dp/dz)_m$ along the length of the channel by,

$$f_D = \frac{D \left(\frac{\partial p}{\partial z} \right)_m}{0.5 \rho_m u_m^2} \quad (18)$$

Where, ρ_m is the mean density. The mean pressure drop along the length of the channel L is defined in terms of inlet pressure P_{in} and outlet pressure P_{out} .

$$\frac{\Delta p}{L} = \frac{P_{in} - P_{out}}{L} \quad (19)$$

Test Cases

In this work three case studies are simulated as given in Tables 1-3. These cases are simulated for both CP and VP to study their effect on flow and heat transfer rates for developing flow in micro pipe. In case-1 study, radius of pipe is varied keeping all the other properties constant. Similarly, in Cases 2 and 3, the inlet velocities and wall heat flux are varied.

Table 1. Variation of Radius (m)

Case-1	
Radius (m)	5×10^{-5} , 1.5×10^{-4} , 2×10^{-4}
Inlet temperature	273.65 K
Inlet velocity	3 m/s
Wall heat flux	100 W/cm ²
Outlet pressure	1.01325×10^5 Pa

Table 2. Variation of Inlet Velocity (m/s)

Case-2	
Radius	5×10^{-5} m
Inlet temperature	273.65 K
Inlet velocity (m/s)	3, 7.5, 15
Wall heat flux	100 W/cm ²
Outlet pressure	1.01325×10^5 Pa

Table 3. Variation of Wall Heat Flux (W/cm²)

Case-3	
Radius	5×10^{-5} m
Inlet temperature	273.65 K
Inlet velocity	3 m/s
Wall heat flux, W/cm ²	50, 100, 140
Outlet pressure	1.01325×10^5 Pa

Results and Discussion

Velocity and Temperature Profiles

In this section, we numerically study the effect of variable properties (VP) $C_p(T)$, $\rho(T)$, $k(T)$, $\mu(T)$ in isolation and combination to investigate their effect on heat transfer rates. A pipe radius of 5×10^{-5} m at inlet velocity and temperature 3 m/s and 273.65 K at uniform wall heat wall flux of 100 W/cm² is taken in the simulations. An outlet pressure of 1.01325×10^5 Pa is assigned at pipe exit. The results are compared to constant properties (CP) and theoretical data of conventional pipe flows. At the inlet, the uniform velocity profile is assumed. As the fluid flows along the length of the pipe, velocity reaches its maximum value at centre and attains a parabolic profile. At this point, the flow is said to be hydro dynamically fully developed. Beyond this point, the velocity does not change along z-axis i.e. $\partial u / \partial z = 0$, remains constant throughout.

In the present calculations, it is observed that the flow is hydro dynamically developed at $z/D = 20$ with a maximum centre velocity of 5.9934 m/s for CP as shown in Figure 4. The flow develops later for VP near the pipe exit with a maximum velocity at centre line = 5.99447 m/s. Theoretically, for conventional pipes, the flow becomes fully developed at $u_r = 2u_m[1-(r/R)^2]$ and its maximum value at centre of the pipe is given by 6 m/s (Bergman and Incropera, 2011). This value is higher as compared to the flow through micro pipes for CP and VP.

Figure 4. Velocity Profile for Developing Flow with $u_{in} = 3 \text{ m/s}$, $R = 5 \times 10^{-5} \text{ m}$ and $q_w = 100 \text{ W/cm}^2$ for CP and VP compared to Theory (Bergman and Incropera, 2011)

Figure 5. Variation of Radial Velocity in m/s, with $u_{in} = 3 \text{ m/s}$, $R = 5 \times 10^{-5} \text{ m}$ and $q_w = 100 \text{ W/cm}^2$ with Constant Property (CP), Properties $C_p(T)$, $\rho(T)$, $k(T)$ and $\mu(T)$ in Isolation and their Combination (VP)

Figure 5 shows that the radial velocity is negligible for CP hence convection is zero in radial direction with CP. There is a small magnitude of the radial velocity with VP indicating radial convection. The major effect is due to $\mu(T)$ and minor is due to $\rho(T)$. The $k(T)$ and $C_p(T)$ variations have a negligible effect on the radial velocity profile.

Figure 6. Temperature Profile for Developing Flow with $u_{in} = 3\text{m/s}$, $R = 5 \times 10^{-5}\text{m}$ and $q_w = 100\text{ W/cm}^2$ for Constant Property (CP) and Variable Property (VP) compared to Conventional Theory

Uniform temperature profile is given at inlet with $T_{in} = 273.65\text{ K}$. As the liquid flows along the length of the pipe, the temperature rises due to constant heat flux of $q_w = 100\text{ W/cm}^2$ at the wall. When the temperature profile does not change after certain distance from entrance then it is said to be thermally fully developed flow. In our calculations, the flow is still developing for CP and VP as shown in Figure 6. The theoretical thermally fully developed flow for conventional pipes is given by

$$T(r, x) = T_w(x) - \frac{2u_m R^2}{\alpha} \frac{dT_m}{dx} \left[\left(\frac{3}{16} \right) + \frac{1}{16} \left(\frac{r}{R} \right)^4 - \frac{1}{4} \left(\frac{r}{R} \right)^2 \right] \quad (16)$$

$$T_m(x) = T_w(x) - \frac{11u_m R^2}{48\alpha} \frac{dT_m}{dx} \quad (17)$$

Here $\alpha = k/(\rho C_p)$. Since the Prandtl number is more than one for water, the velocity boundary layer develops faster as compared to thermal developing

boundary layer for CP as shown in Figures 4 and 6. Note that both the CP and VP fluid flows show deviation from the calculated theoretical temperature at pipe exit in Figure 6.

Effects of Constant Properties and Temperature Variable Thermophysical Properties on Nusselt Number in Isolation and Combination

The thermo physical fluid properties such as $\rho(T)$, $C_p(T)$, $k(T)$ and $\mu(T)$ are dependent on temperature and their effect on heat transfer cannot be neglected. The individual and combined effects of these properties are analysed on mean T_m and wall temperature T_w , and their temperature difference $(T_w - T_m)$, and are compared to the solution with constant properties (CP) in Figure 7. It is observed that the variation of $\mu(T)$ and $k(T)$ has major influence on wall temperature and negligible effects are observed with variation of $C_p(T)$ and $\rho(T)$ as compared to CP. The combined effects of these properties (VP) predict lower values of T_w as compared to CP. The CP and VP solution for T_m matches.

Figure 7. Water Flow through Pipe with $u_{in} = 3 \text{ m/s}$, $R = 5 \times 10^{-5} \text{ m}$ and $q_w = 100 \text{ W/cm}^2$ for Constant Properties CP, Variable Properties in Isolation and their Combination (VP) Depicting Variation of (a) Wall Temperature T_w and Mean Temperature T_m and (b) Temperature Difference $\Delta T = (T_w - T_m)$

The overall effect of wall and mean temperatures on the Nusselt number, Nu is shown in Figure 8 for both CP and VP. The Nu depends on two parameters; k and temperature difference, $\Delta T = (T_w - T_m)$ as given by Eqs 14. and 15. Firstly, the constant heat addition from wall pipe increases temperature in axial direction of pipe. This increase in temperature, increases $k(T)$ as shown in Figure 1. The effect of this increase in $k(T)$, decreases the Nu_{VP} as compared to Nu_{CP} . Secondly, the overall effect of $C_p(T)$, $\rho(T)$, $k(T)$ and $\mu(T)$ as compared to constant properties results in $\Delta T_{CP} > \Delta T_{VP}$, thereby resulting in decrease in Nu_{CP} as compared to Nu_{VP} . The net effect of these both parameters results in $Nu_{CP} < Nu_{VP}$, with the dominant effect of ΔT rather than k . It is observed that Nu values are predicted higher with VP as compared to CP by 13.55%. Table 4

shows that at $z/D = 100$, the Nu is predicted higher for VP and CP, as compared to conventional theoretical value of 4.36.

Figure 8. Variation of Nusselt Number along the Flow with $u_{in} = 3 \text{ m/s}$, $R = 5 \times 10^{-5} \text{ m}$ and $q_w = 100 \text{ W/cm}^2$ with CP, Properties $C_p(T)$, $\rho(T)$, $k(T)$ and $\mu(T)$ in Isolation and Combination [$C_p(T) + \rho(T) + k(T) + \mu(T) = VP$]

Table 4. Wall Temperature T_w , Mean Temperature T_m and Nusselt Number Values at $z/D = 180$ with Isolation Properties and their Combination (VP) compared to CP with $u_{in} = 3 \text{ m/s}$, $R = 5 \times 10^{-5} \text{ m}$ and $q_w = 100 \text{ W/cm}^2$

Property	T_w	T_m	Nu number
CP	369.01	331.15	4.34
$\rho(T)$	368.56	331.09	4.36
$C_p(T)$	368.45	331.08	4.35
$k(T)$	366.14	331.17	4.42
$\mu(T)$	366.03	331.08	4.67
VP	363.88	330.96	4.70

Figure 9. Variation of (a) Skin Friction Coefficient C_f (b) Darcy Friction Factor f_D and (c) Normalized Pressure for Constant Properties (CP) and, Variable Properties $C_p(T)$, $\rho(T)$, $k(T)$ and $\mu(T)$ in Isolation, and in Combination [$C_p(T) + \rho(T) + k(T) + \mu(T) = VP$]

The C_f in Figure 9a with CP decreases along the pipe to $z/D = 24$ and remains constant downstream but C_f with VP decreases to the end of the pipe and shows lower values at exit as compared to CP. The Darcy friction factor in Figure 9b increases upto $z/D = 24$ for both CP and VP. Downstream of it remains constant for CP and decreases for VP. The pressure in Figure 9c shows linear drop with CP and non-linear drop with VP. The variation of $\mu(T)$ along the pipe changes in temperature hence changes the value of pressure with VP as compared to CP. The other properties do not affect the pressure. The velocity profiles in Figure 4 shows that the velocity gradients are higher with VP $z/D = 50$ as compared to CP, therefore predicts higher values of C_f as shown in Figure 9a. Downstream of $z/D = 50$, although the velocity gradients remain constant with CP and have lower values as compared to VP, the decrease in $\mu(T)$ with an increase in temperature along the pipe gives lower values of C_f with VP as compared to CP as per Eq. 17.

Case 1: Variation of Radius

Numerical simulations are performed with varying radii of $R = 5 \times 10^{-5}$ m, 10×10^{-5} m, 15×10^{-5} m and 20×10^{-5} m at a constant velocity and heat flux (see Table 1) In general, from Eqs 14 and 15, we infer that an increase in radius and decrease in ΔT results in an increase in Nu. Figure 10 shows that an increase in radii results in an increase of wall temperature T_w and decrease of mean temperature T_m for both CP and VP. This results in increase of $\Delta T = (T_w - T_m)$ as shown in Figure 11, thereby reducing Nu. But, the Figure 12 shows that Nu increases with increase in radii at CP and VP. This shows that increase in the dimension of pipe i.e. radius has influential effect as compared to ΔT effect on Nu. The difference in Nu between CP and VP increases with increase in radii. A difference of 14.50%, 17.33% and 19.20% is observed between $R = 5 \times 10^{-5}$ m, 1.5×10^{-4} m and 2×10^{-4} m respectively. Table 5 shows that Nu at $z/L = 1$ has higher values for higher radii in developing flow regime. It is observed that Nu with $R = 5 \times 10^{-5}$ m is close to the conventional fully developed flow value of 4.36 for CP at exit and asymptotes to the correlation (Grigull and Tratz, 1965) at $z/L = 0.5$, whereas for VP the Nu is higher at $Z/L = 0.5$ and is still developing.

Figure 10. Variation of Wall Temperature T_w and Mean Temperature T_m compared for Constant Properties (CP) and Variable Properties (VP) along the Flow with Variable Radii at $u_{in} = 3$ m/s and $q_w = 100$ W/cm²

Figure 11. Variation of $\Delta T = (T_w - T_m)$ compared for Constant Properties (CP) and Variable Properties (VP) along the Flow with Variable Radii at $u_{in} = 3 \text{ m/s}$ and $q_w = 100 \text{ W/cm}^2$

Figure 12. Comparison of Nusselt Number along the Flow with Variable Radii at $u_{in} = 3 \text{ m/s}$, and $q_w = 100 \text{ W/cm}^2$ with Constant Properties (CP) and Variable Properties (VP)

Table 5. Comparison of Wall Temperature T_w , Mean Temperature T_m and Nusselt Number for CP and VP at $z/L = 1$ with Variable Radii at $u_{in} = 3 \text{ m/s}$, and $q_w = 100 \text{ W/cm}^2$

Radius (m)	T_w	T_m	Nu number
CP 5×10^{-5}	342.74	305.55	4.54
VP 5×10^{-5}	337.94	305.58	5.25
CP 15×10^{-5}	351.61	283.63	7.48
VP 15×10^{-5}	340.47	283.63	8.90
CP 20×10^{-5}	355.19	281.62	9.13
VP 20×10^{-5}	342.65	281.66	11.07

Figure 13. Variation of Computed (a) Friction Factor (b) Skin Friction (c) and Pressure Drop along the Pipe for Different Micro-pipe Radii compared with CP and VP at $u_{in} = 3 \text{ m/s}$ and $q_w = 100 \text{ W/cm}^2$

Figure 13 shows variation of skin friction C_f , friction factor f_D and pressure drop Δp over pipe length L for CP and VP at inlet velocity of $u_{in} = 3 \text{ m/s}$ and $q_w = 100 \text{ W/cm}^2$. The z -axis is non-dimensioned with $D = 10^{-4} \text{ m}$. Figure 13a shows that as the diameter of pipe is increased the C_f decreases with VP and shows lower values. The flow exhibits lower velocity gradients for higher diameters, therefore results in lower values of C_f . Figure 13b and 13c shows a decrease in Δp and a decrease in f_D . This decrease is due to the fact that the contact surface area reduces for the same inlet velocity and mass flow rate.

Moreover for smaller radii, pressure drop and friction factor being more and this reduces with increasing radii. The friction factor shows a constant trend for all the radii but for VP it reduces continuously.

Case 2: Variation of Inlet Velocities

In this section we study the effect of different inlet velocities with $u_{in} = 3 \text{ m/s}$, 7.5 m/s and 15 m/s as shown in Table 1. This corresponds to different Reynolds number flows of 276, 690, 1383. Figure 14 shows that with increase in inlet velocities, T_w and T_m for both VP and CP decrease, resulting in decrease in ΔT as shown in Figure 15. This results in higher values of Nu for higher inlet velocities for both CP and VP and is shown in Figure 16. It is to be noted that Nu with 3 m/s inlet velocity is almost close to the conventional fully developed flow value of 4.36 for CP at exit and asymptotes to the correlation (Grigull and Tratz, 1965) for developing flow at $Z/D = 100$. The differences between Nu for CP and VP are 13.93%, 11.11% and 8.82% for $u_{in} = 3 \text{ m/s}$, 7.5 m/s and 15 m/s respectively. Table 6 shows that higher inlet velocities lead to delay in fully thermally developed flow.

Figure 14. Variation of Wall Temperature T_w and Mean Temperature T_m compared for Constant Properties (CP) and Variable Properties (VP) along the Flow with Variable Inlet Velocities at $R = 5 \times 10^{-5} \text{ m}$ and $q_w = 100 \text{ W/cm}^2$

Figure 15. Variation of $\Delta T = (T_w - T_m)$ compared for Constant Properties (CP) and Variable Properties (VP) along the Flow with Variable Inlet Velocities at $R = 5 \times 10^{-5} \text{ m}$ and $q_w = 100 \text{ W/cm}^2$

Figure 16. Variation of Nusselt Number along the Flow with Variable Inlet Velocities at $R = 5 \times 10^{-5} \text{ m}$ and $q_w = 100 \text{ W/cm}^2$ compared with Constant Properties (CP) and Variable Properties (VP)

Figure 17. Comparison of Computed (a) Friction Factor (b) Skin Friction (c) Pressure Drop for Different Inlet Velocities with CP and VP for $R = 5 \times 10^{-5}$ and $q_w = 100 \text{ W/cm}^2$

Table 6. Wall Temperature T_w , Mean Temperature T_m and Nusselt Number Values at $z/D = 100$ compared to CP and VP, with Variable Inlet Velocities at $R = 5 \times 10^{-5} \text{ m}$, and $q_w = 100 \text{ W/cm}^2$

Velocity m/s	T_w	T_m	Nu number
CP 3	342.73	305.55	4.54
VP 3	337.93	305.56	5.22
CP 7.5	318.45	286.42	5.27
VP 7.5	315.08	286.42	5.89
CP 15	306.95	280.04	6.28
VP 15	304.66	280.04	6.86

Figure 17a shows that as the inlet velocity is increased the C_f decreases for VP for $R = 5 \times 10^{-5} \text{ m}$ and $q_w = 100 \text{ W/cm}^2$. Figure 17b shows a friction factor f_D plot with different inlet velocities. With smaller velocities i.e. 3 m/s the friction factor is very high and there is a huge deviation between CP and VP compared to other velocities. As the velocities are increasing the friction decreases and becomes constant for CP while for VP it drops continuously. Figure 17c

shows that pressure drop increases drastically with an increase in inlet velocity and the deviation of pressure drop from CP to VP increases as the inlet velocity increases. The values of pressure drop are lower for VP as compared to CP.

Case 3: Variation of Wall Heat Flux

Figure 18. Variation of Wall Temperature T_w and Mean Temperature T_m compared for Constant Properties (CP) and Variable (VP) along the Flow with Variable Wall Heat Flux at $u_{in} = 3 \text{ m/s}$, and $R = 5 \times 10^{-5} \text{ m}$

Figure 19. Variation of $\Delta T = (T_w - T_m)$ compared for Constant Properties (CP) and Variable Properties (VP) along the Flow with Variable Wall Heat Flux at $u_{in} = 3 \text{ m/s}$, and $R = 5 \times 10^{-5} \text{ m}$

Figure 20. Variation of Nusselt Number along the Flow with Variable Wall Heat Flux at $u_{in} = 3$ m/s, and $R = 5 \times 10^{-5}$ m compared with Constant Properties (CP) and Variable Properties (VP)

Table 7. Wall Temperature T_w , Mean Temperature T_m and Nusselt Number Values at $z/D = 100$ compared to CP and VP, with Variable Wall Heat Fluxes, at $u_{in} = 3$ m/s, and $R = 5 \times 10^{-5}$ m

Heat flux q_w (W/cm ²)	T_w	T_m	Nu
CP 50	308.19	289.56	4.53
VP 50	306.89	289.60	4.87
CP 100	342.73	305.55	4.55
VP 100	337.93	305.51	5.23
CP 140	370.25	318.32	4.88
VP 140	362.36	318.19	5.76

In this case study (see Table 1) different wall heat flux values q_w of 50 W/cm², 100 W/cm² and 140 W/cm² are assumed. In this case, the Nu majorly depends on two parameters; q_w and temperature difference, $\Delta T = (T_w - T_m)$ as given by Eqs 14 and 15. Figure 18 shows that as q_w is increased, both T_w and T_m increases for CP and VP. This results in increase of ΔT as shown in Figure 19 and causes decrease in values of Nu. But the amount of increase in q_w dominates and causes an increase in Nu as shown in Figure 20. A difference of 7.23%, 13.98%, 16.54% in Nu is observed at $q_w = 50$ W/cm², 100 W/cm² and 140 W/cm² for CP and VP respectively. Table 7 shows that higher values of q_w

results in higher lengths of thermally developed flow based on Nusselt number values.

Figure 21. Variation of (a) Skin Friction Coefficient (b) Darcy Friction Factor and (c) Pressure Drop for Different Heat Fluxes compared with CP and VP with Variable Wall Heat Fluxes, at $u_{in} = 3 \text{ m/s}$, and $R = 5 \times 10^{-5} \text{ m}$

Figure 21a shows that for a $5 \times 10^{-5} \text{ m}$ pipe radius and inlet velocity of $u_{in} = 3 \text{ m/s}$, increasing the heat flux, decreases the skin friction C_f . The higher values of heat flux results in higher wall temperature thereby reduce the values of $\mu(T)$, hence the C_f . In Figure 21b, the friction factor decreases with increasing heat fluxes but shows a constant trend for all the given heat fluxes for CP. But for VP it drops continuously below the CP values. Figure 21c shows that for a $5 \times 10^{-5} \text{ m}$ pipe radius and inlet velocity of $u_{in} = 3 \text{ m/s}$ increasing heat flux, pressure drop continuously decreases. It is seen that pressure drop for both the CP and VP is almost linearly reducing, but with a huge difference between them. This is because of large temperature gradient q_w at the wall, which reduces the viscosity, reducing the frictional pressure drop.

Correlation of Nusselt Number with Reynolds Numbers and Heat Flux

Figure 22 shows inlet Reynolds number, Re_∞ variation with the Nusselt number, Nu . The Re_∞ is calculated for different velocities in case-1, and at different diameters for Case 2. It is observed that as the velocity increases, the Nu increases. Moreover, Nu being greater for VP than CP. But as the radius increases, there is a huge increase in Nu number compared to an increase in velocities. In Figure 23 the Nu is compared with increasing heat flux values. It is observed that Nu is greater for VP as compared to CP and both deviate with an increase in heat flux values.

Figure 22. Correlation of Nusselt Number at Pipe Exit with Variation of Radius and Inlet Velocities for Cases 1 and 2

Figure 23. Correlation of Nusselt Number at Pipe Exit with Variation of Heat Flux Rates for Case 3

Conclusions

Numerical simulations are performed for two-dimensional axisymmetric micro pipe with entrance effects, for steady, laminar, incompressible and single phase liquid water with constant properties CP and variable properties VP flows.

First, the isolation effect of $C_p(T)$, $\rho(T)$, $k(T)$ and $\mu(T)$ and their combined effect on mean T_m and wall temperature T_w is studied. It is observed that the wall temperature obtained with $k(T)$ and $\mu(T)$ variations shows lower values as compared to CP flows and the negligible effect is observed with variation of $C_p(T)$ and $\rho(T)$. Therefore the combined effect of variations in properties $C_p(T)$, $\rho(T)$, $k(T)$ and $\mu(T)$ give lower values of T_w as compared to CP flows. The variation of T_m is not affected and is same for both CP and VP. Therefore, the Nusselt number at variable property flows Nu_{VP} is predicted higher as compared to Nusselt number at constant property Nu_{CP} flows. It is observed that $\mu(T)$ has a major influence on skin friction C_f , Darcy friction factor f_D and the pressure with VP flows. The C_f and f_D are not affected by $C_p(T)$, $\rho(T)$ and $k(T)$ flows.

In the Case 1 study, an increase in pipe radius results in; a decrease of Nu with $Nu_{VP} > Nu_{CP}$ and, lower values of C_f and f_D at pipe exit with VP as compared to CP flow.

In the Case 2 simulations, an increase in inlet velocities gives; an increase in Nu with $Nu_{VP} > Nu_{CP}$ and, C_f and f_D show lower values at pipe exit with VP as compared to CP flows. The pressure drop Δp increases with an increase in inlet velocities showing lower values with VP as compared to CP.

In the last Case study 3, an increase in constant wall flux q_w results in; an increase of Nu with $Nu_{VP} > Nu_{CP}$ and, C_f and f_D show lower values at pipe exit with VP as compared to CP flows. An increase in heat flux results in a decrease of pressure drop Δp along pipe length with lower values of Δp with VP as compared to CP.

The correlation between Nusselt number Nu and Reynolds number Re_∞ shows that Nu increases with an increase in Re_∞ for an increase in radii and inlet velocities, and Nu with VP shows higher values as compared to CP. The Nu has negligible effect with an increase in heat flux and Nu shows higher values with VP as compared to CP.

Nomenclature

C_f	: Skin friction coefficient
$C_p(T)$	: Temperature dependent specific heat capacity, $J\ kg^{-1}K^{-1}$
D	: Pipe diameter, m
f_D	: Darcy friction factor
$k(T)$	: Temperature dependent thermal conductivity, $Wm^{-1}K^{-1}$
L	: Length of micro-tube, m
Δp	: pressure drop
q_w	: Constant wall heat flux, Wcm^{-2}
r, z	: Radial and axial cylindrical coordinate, m
R	: Pipe radius, m
Re	: Reynolds number
T_m	: Bulk mean temperature, K
T_w	: Wall temperature, K
$\Delta T = (T_w - T_m)$	: Temperature difference, K
T_{in}	: Inlet temperature, K
T_{out}	: Outlet temperature, K
u_{in}	: Inlet velocity, m/s
u_m	: Bulk mean velocity, m/s
$\rho(T)$	: Temperature dependent density, Kg/m^3
$\mu(T)$	: Temperature dependent viscosity, Nsm^{-2}

Subscripts

CP	: Constant properties
VP	: Variable properties
m	: Mean value
w	: Wall
in	: Inlet
e	: Exit

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