NONEXISTANCE OF BIANCHI TYPE – I UNIVERSE WITH DOMAIN WALL COUPLE WITH ELECTROMAGNETIC FIELD IN BIMETRIC RELATIVITY

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Abstract: In this paper we have studied Bianchi type – I cosmological model in bimetric theory of relativity and we have considered the matter domain wall coupled with electromagnetic field in x-direction and we observed that, the resulting space – time is free from singularity at cosmic time $t = 0$.

Keywords: Bianchi Type-I model, Domain wall, Electromagnetic field, Bimetric theory of relativity

Introduction:

When we study the universe, study of Bianchi type model plays an important role, therefore these models are very suitable model for our universe. LR.S. Bianchi type – I model has been studied widely, it is important to study the problem relating to inhomogeneous and anisotropic space time because of these Bianchi type – I model. These models have been studied by number of researchers namely Reddy (1987)(14), Reddy and Venkatwarlu (1987)(15), Lorenz Petzold (1989)(10), Venkatwarlu and Reddy (1990)(19), Singh and Agrawal (1991)(18), Gujman (1997)(8), Chimento et al (1997)(4), Bali and Gokhroo (2001)(2) Bali and Keshav (2003)(3), Bali and Anjali (2004)(1), Kiline (2004)(9), Pradhan and Pandey (2006)(13) in general theory of relativity.

In addition to this Bianchi type – I model are also studied by number of researcher in the context of alternative theory of gravitation viz. Bimetric theory of relativity. Axially symmetric Bianchi – I space time is studied by S.D. Deo (5) and he obtained the vacuum solution in bimetric relativity. Higher five dimensional axially symmetric Bianchi I cosmic strings coupled with maxwells field are studied by Mahur pawar and S.D. Deo (11) in Bimetric relativity. Further, in continuation of this study Mahur pawar and S.D. Deo (12) have obtained the null contribution of cosmic strings to the higher six dimensional axially symmetric Bianchi – I space time in bimetric relativity and observed that the resulting space time is free from singularity, S.D. Deo and R.K. Rajurkar (6) also studied the

Bainchi Type – I Cosmological model in biometric relativity and further they (7) also studied anisotropic Bianchi Type – I model with perfect fluid in biometric relativity.

In continuations of this in 13 paper, we have considered Bianchi type cosmological model in which source of matter distribution is domain wall coupled with electromagnetic field.

Model and field equations :

The Bianchi type - I space time is described by the line element

$$ds^2 = -dt^2 + A^2 dx^2 - B^2 [dy^2 + dz^2] \quad (1)$$

Where a and b are the functions of t only, The background flat metric corresponding to equation (1) is

$$d\sigma^2 = -dt^2 + dx^2 + dy^2 + dz^2 \quad (2)$$

The field equation in bimetric theory of gravitation proposed by Rosen N. (16,17) are

$$K_{ij} = N_{ij} - \frac{1}{2} N g_{ij} = -8\pi k T_{ij} \quad (3)$$

Where, $N_{ij} = \frac{1}{2} \gamma^{\alpha\beta} [g^{hj} g_{hil} \alpha_{\alpha}^i]_{\beta}$

and $N = N^{\alpha} = N_1 \frac{1}{\alpha} + N_2^2 + N_3^3 + N_4^4$

$$K = \sqrt{\frac{g}{\gamma}} \quad (4)$$

Here the vertical bar (|) denotes the covariant differentiation with respect to γ_{ij} and T_{ij} is the energy momentum tensor of the matter field.

Using equations (1) – (4), Rosen's field equation in bimetric theory for the metric (1) can be written in the form

$$K_1^1 = -\frac{1}{2} \left(\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2} \right) + \left(\frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2} \right) = -8\pi k T_1^1 = +8\pi k \left(\rho - \frac{1}{2} \frac{F^2}{B^4} \right) \quad (5)$$

$$K_2^2 = -\frac{1}{2} \left(\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2} \right) = -8\pi k T_2^2 = 8\pi k \left(-\rho + \frac{1}{2} \frac{F^2}{B^4} \right) \quad (6)$$

$$K_3^3 = -\frac{1}{2} \left(\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2} \right) = -8\pi k T_3^3 = 8\pi k \left(-\rho + \frac{1}{2} \frac{F^2}{B^4} \right) \quad (7)$$

$$K_4^4 = -\frac{1}{2} \left(\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2} \right) + \left(\frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2} \right) = -8\pi k T_4^4 = 8\pi k \left(-\rho - \frac{1}{2} \frac{F^2}{B^4} \right) \quad (8)$$

We define $V = (AB^2)^{1/3}$ as the average scale factor so that the Hubble's parameter in anisotropic model may be defined as

$$H = \frac{\dot{V}}{V} = \frac{1}{3} \left(\frac{\dot{A}}{A} + 2 \frac{\dot{B}}{B} \right) \quad (9)$$

Where dot denotes derivative with respect to the cosmic time t

Also we have

$$H = \frac{1}{3} (H_1 + H_2 + H_3) \quad (10)$$

$$H_1 = \frac{\dot{A}}{A}, \quad H_2 = H_3 = \frac{\dot{B}}{B}$$

Where

are directional Hubble's factors in the direction of x , y and z respectively.

Law of variation for hubble' sparameter :

Here we propose that a law to be examine for the variation of Hubble's parameter which yield constant value of deceleration parameter in anisotropic Bianchi type – I model is

$$H = R (AB^2)^{-n/3} \quad (11)$$

Where $R > 0$

and $n > 0$ are constant

The deceleration parameter q is defined by

$$q = \frac{-\dot{V} \ddot{V}}{\dot{V}^2} \quad (12)$$

From equations (9) and (12) we get

$$\frac{1}{3} \left[\frac{\ddot{A}}{A} + 2 \frac{\ddot{B}}{B} \right] = R (AB^2)^{-\frac{n}{3}} \quad (13)$$

Which after integration

$$AB^2 = (nRt + c_1)^{3/n} \text{ for } n \neq 0 \quad (14)$$

$$\text{and } AB^2 = c_2 e^{Rt} \text{ for } n = 0 \quad (15)$$

Here c_1 and c_2 are positive constants of integration

Substituting value of AB^2 from (14) in (12)

We get

$$q = n - 1 \quad (16)$$

This shows that the deceleration parameter is constant for this model. It may be pointed that the above law refers to anisotropic Bianchi – I space time in any context.

Solution of perfect fluid equation:

The energy momentum tensor for domain wall couple with electromagnetic field is defined as

$$T_{ij} = T_{ij(dw)} + T_{ij(emf)} \quad (17)$$

Where

$$T_{ij(dw)} = \rho (g_{ij} + w_i w_j) + p w_i w_j \quad (18)$$

Where ρ is energy density of wall, p is the pressure of the direction normal to the plane of wall and w_i is the unit space like vector in the same direction with

$$w_i w_j = -1 \quad (19)$$

In particular $w_1 w^1 = -1$

Now,

$$T_{ij(emf)} = E_{ij} = -F_{ir} F_{jr} + \frac{1}{4} F_{ab} F^{ab} g_{ij} \quad (20)$$

Where E_{ij} is electromagnetic energy tensor

F_{ij} is electromagnetic field tensor

The magnetic field is taken along the x – direction, so that the only nonvanishing components of F_{ij} are

$$F_{23} = -F_{32}$$

from, Maxwell's equation we have

$$F_{ij,k} + F_{jk,i} + F_{ki,j} = 0 \quad (21)$$

$$\text{give rise to } F_{23} = -F_{32} = F \text{ (Constant)} \quad (22)$$

Non vanishing energy momentum tensors for domain wall are

$$T_1^1 = -p, \quad T_2^2 = T_3^3 = T_4^4 = \rho \quad (23)$$

and non vanishing energy momentum tensor forelectromagnetic field are

$$\begin{aligned} E_1^1 &= \frac{1}{2} \frac{F^2}{B^4} = E_4^4 \\ E_2^2 &= E_3^3 = -\frac{1}{2} \frac{F^2}{B^4} \end{aligned} \quad (24)$$

Using equation (1 – 4) & (17 – 24) we get

$$\frac{1}{2} \left(\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2} \right) - 2 \left(\frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2} \right) = -8\pi k \left(p - \frac{1}{2} \frac{F^2}{B^4} \right) \quad (25)$$

$$\frac{1}{2} \left(\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2} \right) = -8\pi k \left(\rho - \frac{1}{2} \frac{F^2}{B^4} \right) \quad (26)$$

$$\frac{1}{2} \left(\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2} \right) + \left(\frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2} \right) = -8\pi k \left(\rho + \frac{1}{2} \frac{F^2}{B^4} \right) \quad (27)$$

Solving these equations, we get

$$p + 3\rho + \frac{F^2}{B^4} = 0 \quad (28)$$

Equation (28) gives each term LHS of (5) vanishes separately

$$\dot{p} + 3\dot{\rho} = 0 \text{ and } F_2 = 0 \Rightarrow F_{23} = 0, \quad P = 0, \quad \rho = 0$$

which gives us $P = \rho = 0$ & $F_{23} = 0$

i.e. Domain wall and electromagnetic field does not exist.

Hence putting values of $p = \rho = 0$, $F_{23} = F = 0$ in (25) – (27) then we get

$$\frac{1}{2} \left(\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2} \right) - \left(\frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2} \right) = 0 \quad (29)$$

$$\frac{1}{2} \left(\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2} \right) = 0 \quad (30)$$

$$\frac{1}{2} \left(\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2} \right) + \left(\frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2} \right) = 0 \quad (31)$$

Which gives us

$$\frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2} = 0 \text{ and } \frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2} = 0 \quad (32)$$

After integration, we get

$$\frac{d}{dt} \left(\frac{\dot{A}}{A} \right) = 0, \text{ and } \frac{d}{dt} \left(\frac{\dot{B}}{B} \right) = 0$$

$$\frac{\dot{A}}{A} = C_1 \text{ and } \frac{\dot{B}}{B} = C_2 \quad (33)$$

$$\log A = C_1 t + \log C_3, \log B = C_2 t + \log C_4$$

$$A = C_3 e^{C_1 t} \quad B = C_4 e^{C_2 t} \quad (34)$$

With the help of equation (34) metric (1) becomes.

$$ds^2 = -dt^2 + C_3^2 e^{2C_1 t} dx^2 + C_4^2 e^{2C_2 t} (dy^2 + dz^2)$$

without loss of generality we assure that $C_1 = C_2 = C$ & absoritity the const C_3 & C_4 in differential then we get

$$ds^2 = -dt^2 + e^{2Ct} (dx^2 + dy^2 + dz^2) \quad (35)$$

and It is interesting to note that this model (35) is free from singularity at $t = 0$

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