ANISOTROPIC BIANCHI TYPE – I MODEL WITH PERFECT FLUID IN BIMETRIC RELATIVITY

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Abstract:

In this paper we have studied anisotropic Bianchi type – I cosmological model with perfect fluid in bimetric theory of relativity. Further law of variation for Hubble parameter is studied in homogeneous and anisotropic Bianchi type – I cosmological model which yields a constant value of deceleration parameter.

Keywords: Bianchi Type – I, Bimetric relativity, perfect fluid, Hubble parameter, deceleration parameter.

Introduction :

We observed that, the Bianchi type model contain isotropic special cases and they permit arbitrarily small anisotropic level at some instant of cosmic times. Therefore these models are very suitable models of our universe, because of this study of Bianchi type – I model creates much more interest and for simplification and description of large scale behavior of the universe. LRS Bianchi – I models has been studied widely. In order to study the problem like formation of galaxies of the process of homogenization and isotropization of the universe. It is important to study the problem relating to inhomogeneous and anisotropic space-time because of these Bianchi type models have been considered number of authors like Reddy (1987) (13), Reddy and Venkatswarlu (1987)(14), Lorenz – petzold (1989) (9), venkatswarlu and Reddy (1990) (18), Singh and Agrawal (1991) (17), Gujman (1997) (7), Chimento et.al (1997) (4), Bali and Gokhroo (2001) (2), Bali and Kashav (2003) (3), Bali and Anjali (2004) (1), Kiline (2004) (8), Pradhan and pandey (2006) (12) in general relativity. In addition to this Bianchi type models are also

studied by number of researcher in the context of bimetric theory of relativity. Axially symmetric Bianchi-I space – time is studied by S.D. Deo (5) and he obtained the vacuum solution in bimetric relativity. Higher five dimensional axially symmetric Bianchi I cosmic strings coupled with Maxwell's field are studied by Mahurpawar and Deo (10) in Bimetric relativity. Further, in continuation of this study Mahurpwar and Deo (11) have obtained the null contribution of cosmic strings to the higher six dimensional axially symmetric Bianchi – I space – time in Bimetric relativity and observed that the resulting space – time is free from singularity S.D. Deo & R.K. Rajurkar also studied the Bianchi Type – I cosmological model in Bimetric relativity (6).

In this paper we consider a spatially homogeneous and anisotropic Bianchi type – I cosmological model in which source of matter distribution is perfect fluid. Further an attempt has been made to formulate a law of variation for Hubble parameter in anisotropic Bianchi – I space-time that yields a constant value of deceleration parameter.

Model And Field Equations :

The spatially homogenous and anisotropic Bianchi type – I space time is described by the line element

$$ds^2 = dt^2 - a^2 dx^2 - b^2 dy^2 - c^2 dz^2 \quad (1)$$

Where a, b and c are the functions of t only, The background flat metric corresponding to equation (1) is

$$d\sigma^2 = dt^2 - dx^2 - dy^2 - dz^2 \quad (2)$$

The field equation in bimetric theory of gravitation proposed by Rosen N. (15,16) are

$$K_{ij} = N_{ij} - \frac{1}{2} N g_{ij} = -8\pi k T_{ij} \quad (3)$$

$$k = \left(\frac{g}{\gamma}\right)^{1/2} \text{ Where, } N_{ij} = \frac{1}{2} \gamma^{\alpha\beta} [g^{hj} g_{hi} - g^{hi} g_{hj}]$$

$$\mathbf{N} = \mathbf{N} \text{ and } \propto \quad (4)$$

Together with $g = \det (g_{ij})$ and $\mathbf{y} = \det (\mathbf{y}_{ij})$

Here the vertical bar (|) denotes the covariant differentiation with respect to $\mathbf{y}_{ij}$ and $\mathbf{T}_{ij}$ is the energy momentum tensor of the matter field.

Using equations (1) – (4), Rosen's field equation in bimetric theory for the metric (1) can be written in the form

$$K_1^1 = \left(\frac{\dot{a}}{a}\right) - \left(\frac{\dot{b}}{b}\right) - \left(\frac{\dot{c}}{c}\right) = -16\pi k T_1^1 \quad (5)$$

$$K_2^2 = \left(\frac{\dot{a}}{a}\right) + \left(\frac{\dot{b}}{b}\right) - \left(\frac{\dot{c}}{c}\right) = -16\pi k T_2^2 \quad (6)$$

$$K_3^3 = \left(\frac{\dot{a}}{a}\right) - \left(\frac{\dot{b}}{b}\right) + \left(\frac{\dot{c}}{c}\right) = -16\pi k T_3^3 \quad (7)$$

$$K_4^4 = \left(\frac{\dot{a}}{a}\right) + \left(\frac{\dot{b}}{b}\right) + \left(\frac{\dot{c}}{c}\right) = -16\pi k T_4^4 \quad (8)$$

$$\text{Here } \dot{a} = \frac{da}{dt}, \quad \ddot{a} = \frac{d^2a}{dt^2} \text{ etc.}$$

The spatial volume of this model is given by

$$V^3 = abc$$

We define $V = (abc)^{1/3}$ as the average scale factor so that the Hubble's parameter in anisotropic models may be defined as

$$H = \frac{\dot{V}}{V} = \frac{1}{3} \left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c} \right) \quad (9)$$

Where an over dot denotes derivative with respect to the cosmic time t

Also we have

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$$H = \frac{1}{3} (H_1 + H_2 + H_3) \quad (10)$$

Where

$$H_1 = \frac{\dot{a}}{a}, \quad H_2 = \frac{\dot{b}}{b}, \quad H_3 = \frac{\dot{c}}{c}$$

are directional Hubble's factors in the direction of x , y and z respectively

Law Of Variation For Hubble's Parameter :

Here we propose that a law to be examine for the variation of Hubble's parameter which yield constant value of deceleration parameter in anisotropic Bianchi type I model is

$$H = R (abc)^{-n/3} \quad \text{Where } R > 0 \quad (11)$$

and $n > 0$ are constant

The deceleration parameter q is defined by

$$q = \frac{-V \ddot{V}}{\dot{V}^2} \quad (12)$$

From equations (9) and (12) we get

$$\frac{1}{3} \left[\frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c} \right] = R (abc)^{-\frac{n}{3}} \quad (13)$$

Which after integration

$$abc = (nRt + c_1)^{3/n} \quad \text{for } n \neq 0 \quad (14)$$

$$\text{And } abc = c_2 e^{Rt} \quad \text{for } n = 0 \quad (15)$$

Here c_1 and c_2 are positive constants of integration

Substituting value of abc from (14) in (12)

We get

$$q = n - 1 \quad (16)$$

This shows that the deceleration parameter is constant for this model. It may be pointed that the above law refers to anisotropic Bianchi I space time in any context i.e. in any theory that is based on anisotropic Bianchi – I space – time

4. Solution Of Perfect Fluid Equation :

The energy momentum tensor for perfect fluid is defined as

$$T_{ij} = (p + \rho) V_i V_j + p g_{ij} \quad (17)$$

together with an orthogonality condition

$$V_4 V^4 = -1 \quad (18)$$

Then field equation is

$$K_1^1 = \left(\frac{\dot{a}}{a}\right) - \left(\frac{\dot{b}}{b}\right) - \left(\frac{\dot{c}}{c}\right) = -16\pi k p \quad (19)$$

$$K_2^2 = -\left(\frac{\dot{a}}{a}\right) + \left(\frac{\dot{b}}{b}\right) - \left(\frac{\dot{c}}{c}\right) = -16\pi k p \quad (20)$$

$$K_3^3 = -\left(\frac{\dot{a}}{a}\right) - \left(\frac{\dot{b}}{b}\right) + \left(\frac{\dot{c}}{c}\right) = -16\pi k p \quad (21)$$

$$K_4^4 = \left(\frac{\dot{a}}{a}\right) + \left(\frac{\dot{b}}{b}\right) + \left(\frac{\dot{c}}{c}\right) = 16\pi k \rho \quad (22)$$

Adding (19) – (22)

We get

$$3p - \rho = 0 \quad (23)$$

Which is equation of state for perfect fluid and in view of relativity condition $p \geq 0$, $\rho > 0$

Which give us $p = 0$, $\rho = 0$ (24)

Thus perfect fluid does not exists in Bianchi type I cosmological models

Hence vacuum Solution can be obtained

Hence putting value of $p = \rho = 0$ in (19) –(22) ,

we get

$$\left(\frac{\dot{a}}{a}\right) - \left(\frac{\dot{b}}{b}\right) - \left(\frac{\dot{c}}{c}\right) = 0 \quad (25)$$

$$\left(\frac{\dot{a}}{a}\right) - \left(\frac{\dot{b}}{b}\right) + \left(\frac{\dot{c}}{c}\right) = 0 \quad (26)$$

$$\left(\frac{\dot{a}}{a}\right) + \left(\frac{\dot{b}}{b}\right) + \left(\frac{\dot{c}}{c}\right) = 0 \quad (27)$$

$$\left(\frac{\dot{a}}{a}\right) + \left(\frac{\dot{b}}{b}\right) - \left(\frac{\dot{c}}{c}\right) = 0 \quad (28)$$

Solving (25) – (28)**We get**

$$\frac{\dot{a}}{a} = \frac{\dot{b}}{b} = \frac{\dot{c}}{c} = m \text{ Integrating } \left(\frac{\dot{a}}{a}\right)^{\cdot} = \left(\frac{\dot{b}}{b}\right)^{\cdot} = \left(\frac{\dot{c}}{c}\right)^{\cdot} = 0$$

Where m is constant of integration

After integration,

We get

a = b = c = c₁e^{mt} where **c₁** is constant of integration.Substituting these value in metric (1) and absorbing the constant c₁ in differential we get

$$ds^2 = dt^2 - e^{2mt} [dx^2 + dy^2 + dz^2]$$

and it is observed that the resulting space time is free from singularity at t = 0

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