On fuzzy Supra Semi-Baire Spaces

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ABSTRACT: In this paper the concepts of fuzzy supra semi-nowhere dense set, fuzzy supra semi-first category, fuzzy supra semi second category, fuzzy supra semi α -baire space are introduced and the characterizations of fuzzy supra semi-Baire spaces are studied. Several examples are given to illustrate the concepts introduced in this paper.

KEYWORDS: Fuzzy supra semi-open set, Fuzzy supra semi-nowhere dense set, Fuzzy supra semi- first category, Fuzzy supra semi-second category and Fuzzy supra, semi -Baire spaces.

<https://doi.org/10.29294/IJASE.5.3.2019.1052-1055>

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1. INTRODUCTION

Abd El-Monsef et al. [1] introduced Fuzzy supra Topological space. The natural generalizations of classical supra topological spaces introduced by Mashhour. Due to the importance and need for the fuzzification of weaker forms of classical topological notions, several works has been found in this field among the various existing fuzzy topological spaces in recent years, Fuzzy semi-continuous functions are investigated by Won Keun Min [2]. Moreover, Ahmed et al. [3] worked on fuzzy semi-open and fuzzy semi-closed mapping and established number of characterizations.

The fuzzy concept has invaded almost all branches of Mathematics ever since the introduction of fuzzy set by Zadeh [4]. Chang [5] introduced the theory of fuzzy topological spaces. Since then much attention have been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy Topology has been found to be very useful in solving many practical problems. It has been shown that the fuzzy kahler manifolds, which are based on a topology, play the important role in ε^∞ theory [6].

The concept of Baire space method studied extensively in classical topology [7-10]. The concept of Baire spaces in fuzzy setting was introduced and studied by Thangaraj and Anjalmose [11]. The aim of this paper is to introduce the concepts of fuzzy supra semi-Baireness in fuzzy topological spaces.

The aim of this paper is to further contribute to the study of fuzzy supra semi-open and fuzzy supra semi-closed sets defined by Sahidul Ahmed and Biman Chandra chetia [12] by establishing several important fundamental identities and inequalities supported by counter examples. We define fuzzy supra semi-nowhere dense set and fuzzy supra semi-baire space and establish some interesting of these spaces.

2. PRELIMINARIES

Now we introduce some Basic notions to be used in the sequel. In this work we shall mean a non- empty set X together with a fuzzy supra topology T^* and denote it by (X, T^*) .

Definition 2.1[4]: A Fuzzy set in X is a map $f: X \rightarrow [0,1] = I$. The family of fuzzy sets of X is denoted by I^X .

Following are some basic operations in fuzzy sets in X . For the fuzzy sets f and g in X ,

- 1) $f = g$ if and only if $f(x) = g(x)$ for all $x \in X$.
- 2) $f \leq g$ if and only if $f(x) \leq g(x)$ for all $x \in X$.
- 3) $(f \vee g)(x) = \max \{f(x), g(x)\}$ for all $x \in X$.
- 4) $(f \wedge g)(x) = \min \{f(x), g(x)\}$ for all $x \in X$.
- 5) $f^c(x) = 1 - f(x)$ for all $x \in X$ here f^c denotes the complement of f .
- 6) For a family $\{f_\lambda / \lambda \in \Lambda\}$ of fuzzy sets defined on a set X ,
 $(\bigvee_{\lambda \in \Lambda} f_\lambda)(x) = \bigvee_{\lambda \in \Lambda} (f_\lambda(x))$
 $(\bigwedge_{\lambda \in \Lambda} f_\lambda)(x) = \bigwedge_{\lambda \in \Lambda} (f_\lambda(x))$.
- 7) For any $\alpha \in I$, the constant fuzzy set α in X is a fuzzy set in X defined by $\alpha(x) = \alpha$ for all $x \in X$.
- 8) 0 denotes null fuzzy set in X and 1 denotes universal fuzzy set in X .

Definition 2.2[5]: A fuzzy topological space is a pair (X, δ) where X is a nonempty set and δ is a family of 0 fuzzy set on X satisfying the following properties:

- 1) The constant functions 0 and 1 belong to δ .
- 2) $f, g \in \delta$ implies $f \wedge g \in \delta$.
- 3) $f_\lambda \in \delta$ for each $\lambda \in \Lambda$ implies $(\bigvee_{\lambda \in \Lambda} f_\lambda) \in \delta$.

Then δ is called a fuzzy topology on X . Every member of δ is called fuzzy open set. The complement of a fuzzy open set called fuzzy closed set.

Definition 2.3[5]: The closure and interior of a fuzzy set $f \in I^X$ are defined respectively as,

$$cl(f) = \bigwedge \{g / g \text{ is a fuzzy closed set in } X \text{ and } f \leq g\}$$

$$int(f) = \bigvee \{g / g \text{ is a fuzzy open set in } X \text{ and } g \leq f\}$$

Clearly $cl(f)$ is the smallest fuzzy closed set containing f and $int(f)$ is the largest fuzzy open set contained in f .

Definition 2.4[11]: A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy nowhere dense if there exists no non - zero fuzzy open set μ in (X, T) such that $\mu < cl(\lambda)$. That is, $int(cl(\lambda)) = 0$.

Definition 2.5[11]: Let λ be a fuzzy first category set in a fuzzy topological space (X, T) . Then $1 - \lambda$ is called a fuzzy residual set in (X, T) .

Definition 2.6[11]: Let (X, T) be a fuzzy topological space. Then (X, T) is called a fuzzy Baire space if int

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Received: 02.01.2019

Accepted: 18.02.2019

Published on 27.02.2019

Poongothai and Priya

$(\bigvee_{i=1}^{\infty}(\lambda_i)) = 0$, where λ_i 's are fuzzy nowhere dense sets in (X, T) .

Definition 2.7[12]: Let (X, T) be any fuzzy topological space and λ be any fuzzy set in (X, T) .

We define $cl(\lambda) = \bigwedge \{ \mu / \lambda \leq \mu, 1 - \mu \in T \}$ and $int(\lambda) = \bigoplus \{ \mu / \mu \leq \lambda, \mu \in T \}$.

For any fuzzy set in a fuzzy topological space (X, T) it is easy to see that $1 - cl(\lambda) = int(1 - \lambda)$ and $1 - int(\lambda) = cl(1 - \lambda)$.

Definition 2.8[15]: A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy first category if $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$ where λ_i 's are fuzzy nowhere dense sets in (X, T) . Any other fuzzy set in (X, T) is said to be of fuzzy second category.

Definition 2.9[15]: A fuzzy topological space (X, T) is called fuzzy first category space if $1 = \bigvee_{i=1}^{\infty}(\lambda_i)$ where λ_i 's are fuzzy nowhere dense sets in (X, T) . A topological space, which is not of fuzzy first category, is said to be of fuzzy second category space.

Definition 2.10[16]: A fuzzy set λ in a fuzzy topological space (X, T) is called fuzz α - dense if there exists no fuzzy α - closed set μ in (X, T) such that $\lambda < \mu < 1$.

Definition 2.11[17]: A collection δ^* of fuzzy sets in a set X is called fuzzy supra topology on X if the following conditions are satisfied :

- 1) 0 and 1 belongs to δ^* .
- 2) $f_\lambda \in \delta^*$ for each $\lambda \in \wedge$ implies $(\bigvee_{\lambda \in \wedge} f_\lambda) \in \delta^*$.

The pair (X, δ^*) is called a fuzzy supra topological space. The elements of δ^* called fuzzy supra open sets and the complement of a fuzzy supra open set is called fuzzy supra closed set.

Definition 2.12[17]: let (X, δ^*) be a fuzzy supra topological space and f be a fuzzy set in X , then the fuzzy supra closure and fuzzy supra interior of f defined respectively as

$cl^*(f) = \bigwedge \{ g / g \text{ is a fuzzy supra closed set in } X \text{ and } f \leq g \}$
 $int^*(f) = \bigvee \{ g / g \text{ is a fuzzy supra open set in } X \text{ and } g \leq f \}$.

Definition 2.13[17]: let (X, δ) be a fuzzy topological space and δ^* be a fuzzy supra topology on X . We call δ^* is a fuzzy supra topology associated with δ if $\delta \leq \delta^*$.

Remark 2.14[17]:

- 1) The fuzzy supra closure of a fuzzy set f in a fuzzy supra topological space is the smallest fuzzy supra closed set containing f .
- 2) The fuzzy supra interior of a fuzzy set f in a fuzzy supra topological space is the largest fuzzy supra open set contained in f .
- 3) If (X, δ^*) is an associated fuzzy supra topological space with the fuzzy topological space (X, δ) and f is any fuzzy set in X , then $int(f) \leq int^*(f) \leq f \leq cl^*(f) \leq cl(f)$.

Theorem 2.15[17]: For any fuzzy sets f and g in a fuzzy supra topological space (X, δ^*) ,

- 1) f is a fuzzy supra closed set if and only if $cl^*(f) = f$.
- 2) f is a fuzzy supra open set if and only if $int^*(f) = f$.
- 3) $f \leq g$ implies $int^*(f) \leq int^*(g)$ and $cl^*(f) \leq cl^*(g)$
- 4) $cl^*(cl^*(f)) = cl^*(f)$ and

$int^*(int^*(f)) = int^*(f)$.

- 5) $cl^*(f \vee g) \geq cl^*(f) \vee cl^*(g)$
- 6) $cl^*(f \wedge g) \leq cl^*(f) \wedge cl^*(g)$
- 7) $int^*(f \vee g) \geq int^*(f) \vee int^*(g)$
- 8) $int^*(f \wedge g) \leq int^*(f) \wedge int^*(g)$
- 9) $cl^*(f^c) = [int^*(f)]^c$
- 10) $int^*(f^c) = [cl^*(f)]^c$.

Definition 2.16[17]: Let (X, δ^*) be a fuzzy supra topological space. A fuzzy set f is called fuzzy supra semi open set if, $f \leq cl^*(int^*(f))$. The complement of a fuzzy supra semi open set is called a fuzzy supra semi closed set.

Definition 2.17[19]: Let (X, T^*) be a fuzzy supra topological space. A fuzzy set f is called fuzzy supra α -open set, if $f \leq int^*[cl^*(int^*(f))]$ and a fuzzy set f is called fuzzy supra α - closed set if $cl^*[int^*(cl^*(f))] \leq f$.

Definition 2.18: Let (X, T_1^*) and (Y, T_2^*) be two FSTS's. A Mapping $f: X \rightarrow Y$ is called

- 1) fuzzy supra semi-open if $f(A)$ is fuzzy supra open in Y for each $A \in FSSCS(X)$,
- 2) fuzzy supra semi-closed if $f(A)$ is fuzzy supra closed in Y for each $A \in FSSOS(X)$.

Theorem 2.19: Let $f: X \rightarrow Y$ be a mapping between two FSTS's. Then the following are equivalent:

- 1) f is fuzzy supra semi-open,
- 2) $f(int^*(A)) \leq int^*(f(A))$ for each fuzzy set A of X ,
- 3) $sint^*(f^{-1}(B)) \leq f^{-1}(int^*(B))$ for each fuzzy set B of Y ,
- 4) $f^{-1}(cl^*(B)) \leq scl^*(f^{-1}(B))$ for each fuzzy set B of Y .

3. FUZZY SUPRA SEMI-NOWHERE DENSE SETS

Definition 3.1: Let (X, T^*) be a fuzzy supra topological space. A fuzzy set λ in (X, T^*) is called a fuzzy supra semi-nowhere dense set if there exists no non-zero fuzzy supra semi -open set μ in (X, T^*) Such that $\mu < scl^*(\lambda)$. That is, $sint^*scl^*(\lambda) = 0$.

Example 3.1: Let $X = \{a, b, c\}$. The fuzzy sets $\lambda, \mu, \gamma, \alpha$ and β are defined on X as follows:

$\lambda: X \rightarrow [0, 1]$ define as $\lambda(a) = 0.7; \lambda(b) = 0.3; \lambda(c) = 0.5$,

$\mu: X \rightarrow [0, 1]$ define as $\mu(a) = 0.2; \mu(b) = 0.8; \mu(c) = 0.6$,

$\gamma: X \rightarrow [0, 1]$ define as $\gamma(a) = 0.5; \gamma(b) = 0.7; \gamma(c) = 0.8$,

$\alpha: X \rightarrow [0, 1]$ define as $\alpha(a) = 0.9; \alpha(b) = 0.8; \alpha(c) = 0.7$,

$\beta: X \rightarrow [0, 1]$ define as $\beta(a) = 0.9; \beta(b) = 0.9; \beta(c) = 0.6$.

Then $T = \{0, \lambda, \mu, \gamma, (\lambda \vee \mu), (\lambda \wedge \mu), (\lambda \vee \gamma), (\lambda \wedge \gamma), (\mu \vee \gamma), (\mu \wedge \gamma), \lambda \vee \mu \vee \lambda, \mu \vee (\lambda \wedge \gamma), \lambda \vee (\mu \vee \gamma), \gamma \wedge (\lambda \vee \mu), 1\}$ is a fuzzy topology on X . The non-zero fuzzy supra semi-open sets

in (X, T^*) are $\lambda, \mu, \gamma, (\lambda \vee \mu), (\lambda \vee \gamma), (\mu \vee \gamma), (\lambda \wedge \mu), (\lambda \wedge \gamma), (\mu \wedge \gamma), \lambda \vee \mu \vee \lambda, \mu \vee (\lambda \wedge \gamma), \lambda \vee (\mu \vee \gamma), \gamma \wedge (\lambda \vee \mu)$. Now the fuzzy sets $1 - \lambda, 1 - \mu, 1 - \gamma, 1 - (\lambda \vee \mu), 1 - (\lambda \vee \gamma), 1 - \lambda \wedge \gamma,$

$1 - (\mu \vee \gamma), 1 - \lambda \vee (\mu \wedge \gamma), 1 - (\mu \vee (\lambda \wedge \gamma)), 1 - \lambda \vee \mu \vee \lambda, 1 - \gamma \wedge \lambda \vee \mu,$

$1 - \alpha, 1 - \beta$ are fuzzy supra semi-nowhere dense sets in (X, T^*) .

Proposition 3.1: The complement of a fuzzy suprasemi-nowhere dense set in a fuzzy topological space (X, T^*) need not be fuzzy supra semi-nowhere dense set.

Proof: For example 3.1, $(1 - \alpha)$ is a fuzzy supra semi-nowhere dense set in (X, T^*) where

as $\alpha = 1 - (1 - \alpha)$ is not a fuzzy supra semi-nowhere dense set in (X, T^*) .

Poongothai and Priya

Proposition 3.2: If λ and μ are fuzzy supra semi-nowhere dense set in a fuzzy topological space (X, T^*) , then $\lambda \vee \mu$ need not be fuzzy supra semi-nowhere dense set in (X, T^*) .

Proof: For, in Example 3.1, $1-\alpha$, $1-\lambda$ are fuzzy supra semi-nowhere dense set in (X, T^*) . But $(1-\alpha) \vee (1-\lambda) = \delta$ implies that $\text{int}^* \text{scl}^*(\delta) \neq 0$. Therefore union of fuzzy supra semi-nowhere dense sets need not be fuzzy supra semi-nowhere dense set in (X, T^*) .

Proposition 3.3: If the fuzzy sets λ and μ are fuzzy supra semi-nowhere dense sets in a Fuzzy topological space (X, T^*) then $\lambda \wedge \mu$ is a fuzzy suprasemi-nowhere dense set in (X, T^*) .

Proof: Let the fuzzy sets λ and μ be fuzzy supra semi-nowhere dense set in (X, T^*) . Now $\text{int}^* \text{scl}^*(\lambda \wedge \mu) \leq \text{int}^* \text{scl}^*(\lambda) \wedge \text{int}^* \text{scl}^*(\mu) \leq 0 \wedge 0$ (since $\text{int}^* \text{scl}^*(\lambda) = 0$ and $\text{int}^* \text{scl}^*(\mu) = 0$). That is, $\text{int}^* \text{scl}^*(\lambda \wedge \mu) = 0$. Hence $(\lambda \wedge \mu)$ is a fuzzy supra semi-nowhere dense set in (X, T^*) .

Proposition 3.4: If λ is a fuzzy supra semi-nowhere dense sets in a fuzzy topological space (X, T^*) then $\text{Sint}^*(\lambda) = 0$.

Proof: Let λ be a fuzzy supra semi-nowhere dense set in (X, T^*) . Then we have $\text{int}^* \text{scl}^*(\lambda) = 0$. Now $\lambda \leq \text{scl}^*(\lambda)$ we have $\text{int}^*(\lambda) \leq \text{int}^* \text{scl}^*(\lambda) = 0$. Hence $\text{int}^*(\lambda) = 0$.

Proposition 3.5: If λ is a fuzzy nowhere dense set in fuzzy topological space (X, T^*) then $\text{int}^* \text{scl}^*(\lambda) = 0$.

Proof: Let λ be a fuzzy nowhere dense set in (X, T^*) . Then, we have $\text{int}^* \text{cl}^*(\lambda) = 0$ and $\text{int}^*(\lambda) = 0$. Now $\text{Scl}^*(\lambda) = \lambda$. Since λ is fuzzy supra semi-closed set in (X, T^*) [15] implies that $\text{int}^* \text{scl}^*(\lambda) = \text{int}^*(\lambda) = 0$. Hence $\text{int}^* \text{scl}^*(\lambda) = 0$.

Proposition 3.6: If λ is a fuzzy supra semi-nowhere dense set and μ is any fuzzy set in a fuzzy topological space (X, T^*) , then $(\lambda \wedge \mu)$ is a fuzzy supra semi-nowhere dense set in (X, T^*) .

Proof: Let λ be a fuzzy supra semi-nowhere dense set in (X, T^*) .

Then $\text{int}^* \text{scl}^*(\lambda) = 0$. Now $\text{int}^* \text{scl}^*(\lambda \wedge \mu) \leq \text{int}^* \text{scl}^*(\lambda) \wedge \text{int}^* \text{scl}^*(\mu) \leq 0 \wedge \text{int}^* \text{scl}^*(\mu) = 0$.

That is, $\text{int}^* \text{scl}^*(\lambda \wedge \mu) = 0$. Hence $(\lambda \wedge \mu)$ is a fuzzy supra semi-nowhere dense set in (X, T^*) .

Definition 3.2[11]: A fuzzy set λ in a fuzzy topological space (X, T^*) is called fuzzy supra semi-dense if there exists no fuzzy semi-closed set μ in (X, T^*) such that $\lambda < \mu < 1$. That is $\text{scl}^*(\lambda) = 1$.

Proposition 3.7: If λ is a fuzzy supra semi-dense and fuzzy supra semi-open set in a fuzzy topological space (X, T^*) and if $\mu \leq 1-\lambda$, then μ is a fuzzy supra semi-nowhere dense set in (X, T^*) .

Proof: Let λ be a fuzzy supra semi-dense set in (X, T^*) . Then we have $\text{Scl}^*(\lambda) = 1$ and $\text{int}^*(\lambda) = \lambda$. Now $\mu \leq 1-\lambda$, implies that $\text{Scl}^*(\mu) \leq \text{Scl}^*(1-\lambda)$. Then $\text{Scl}^*(\mu) \leq 1-\text{int}^*(\lambda) = 1-\lambda$.

Hence $\text{Scl}^*(\mu) \leq \text{scl}^*(1-\lambda)$, which implies that $\text{int}^* \text{scl}^*(\mu) \leq \text{int}^*(1-\lambda) = 1-\text{scl}^*(\lambda) = 1-1=0$.

That is, $\text{int}^* \text{scl}^*(\mu) = 0$. Hence μ is a fuzzy supra semi-nowhere dense set in (X, T^*) .

Proposition 3.8: If λ is a fuzzy supra semi-nowhere dense set in a fuzzy topological space (X, T^*) . Then $1-\lambda$, is a fuzzy supra semi-dense set in (X, T^*) .

Proof: Let λ be a fuzzy supra semi-nowhere dense set in (X, T^*) . Then, $\text{int}^* \text{scl}^*(\lambda) = 0$. Now $\lambda \leq \text{scl}^*(\lambda)$ implies that $\text{int}^*(\lambda) \leq \text{int}^* \text{scl}^*(\lambda) = 0$. Then $\text{int}^*(\lambda) = 0$ and $\text{scl}^*(1-\lambda) = 1-\text{int}^*(\lambda) = 1-0=1$ and hence $1-\lambda$ is a fuzzy supra semi-dense set in (X, T^*) .

4. FUZZY SUPRA SEMI-BAIRE SPACES

Definition 4.1: Let (X, T^*) be a fuzzy topological space. Then (X, T^*) is called a fuzzy supra semi-Baire space if $\text{int}^*[\bigvee_{i=1}^{\infty} (\lambda_i)] = 0$, where λ_i 's are fuzzy supra semi-nowhere dense sets in (X, T^*) .

Proposition 4.1: If $\text{int}^* (\bigvee_{i=1}^{\infty} (\lambda_i)) = 0$ where $\text{int}^*(\lambda_i) = 0$ and λ_i 's are fuzzy supra semi-closed sets in a fuzzy topological space (X, T^*) , then (X, T^*) is a fuzzy supra semi-Baire space.

Proof: Let λ_i 's be fuzzy supra semi-closed sets in (X, T^*) . since $\text{int}^*(\lambda_i) = 0$, By theorem 3.13, the λ_i 's are fuzzy supra semi-nowhere dense sets in (X, T^*) , therefore we have $\text{int}^* (\bigvee_{i=1}^{\infty} (\lambda_i)) = 0$, where λ_i 's are fuzzy supra semi-nowhere dense sets in (X, T^*) . Hence (X, T^*) is a fuzzy supra semi-Baire space.

Proposition 4.2: If $\text{scl}^*(\bigwedge_{i=1}^{\infty} (\lambda_i)) = 1$, where λ_i 's are fuzzy supra semi-nowhere dense and fuzzy supra semi-open sets in a fuzzy topological space (X, T^*) , then (X, T^*) is a fuzzy supra semi-Baire space.

Proof: Now $\text{scl}^*(\bigwedge_{i=1}^{\infty} (\lambda_i)) = 1$ implies that $1-\text{scl}^*(\bigwedge_{i=1}^{\infty} (\lambda_i)) = 0$. Then we have $\text{int}^*(1-\bigwedge_{i=1}^{\infty} (\lambda_i)) = 0$, which implies that $\text{int}^*(\bigvee_{i=1}^{\infty} (1-\lambda_i)) = 0$. Since λ_i 's are fuzzy supra semi-dense sets in (X, T^*) , $\text{scl}^*(\lambda_i) = 1$ and $\text{int}^*(1-\lambda_i) = 1-\text{scl}^*(\lambda_i) = 1-1=0$. Hence we have $\text{int}^*(\bigvee_{i=1}^{\infty} (1-\lambda_i)) = 0$, where $\text{int}^*(1-\lambda_i) = 0$ and $(1-\lambda_i)$'s are fuzzy supra semi-closed sets in (X, T^*) . Then, by proposition 4.1, (X, T^*) is a fuzzy supra semi-Baire space.

Proposition 4.3: Let (X, T^*) be a fuzzy topological space. Then the following are equivalent:

- 1) (X, T^*) is a fuzzy supra semi-Baire space.
- 2) $\text{int}^*(\lambda) = 0$ for every fuzzy supra semi-first category set λ in (X, T^*) .
- 3) $\text{scl}^*(\mu) = 1$ for every fuzzy supra semi-residual set in (X, T^*) .

Proof: (1) $\rightarrow$ (2). Let λ be a fuzzy supra semi-first category set in (X, T^*) . Then $\lambda = \bigvee_{i=1}^{\infty} (\lambda_i)$ where λ_i 's are fuzzy supra semi-nowhere dense set in (X, T^*) . Now $\text{int}^*(\lambda) = \text{int}^*(\bigvee_{i=1}^{\infty} (\lambda_i)) = 0$ (since (X, T^*) is a fuzzy supra semi-Baire space).

Therefore $\text{int}^*(\lambda) = 0$.

(2) $\rightarrow$ (3). Let μ be a fuzzy supra semi-residual set in (X, T^*) . Then $1-\mu$ is a fuzzy supra semi-first category set in (X, T^*) . By hypothesis, $\text{int}^*(1-\mu) = 0$. Which implies that $1-\text{scl}^*(\mu) = 0$.

Hence we have $\text{scl}^*(\mu) = 1$.

(3) $\rightarrow$ (1). Let λ be a fuzzy supra semi-first category set in (X, T^*) . Then $\lambda = \bigvee_{i=1}^{\infty} (\lambda_i)$ where λ_i are fuzzy supra semi-

nowhere dense set in (X, T^*) . $1-\lambda$ is a fuzzy supra semi-residual set in (X, T^*) . Since λ is a fuzzy supra semi-first category set in (X, T^*) . By hypothesis, we have $\text{scl}^*(1-\lambda)=1$. Then $1-\text{sint}^*(\lambda)=1$, which implies that $\text{sint}^*(\lambda)=0$. Hence $\text{sint}^*(\bigvee_{i=1}^{\infty}(\lambda_i))=0$ where λ_i 's are fuzzy supra semi-nowhere dense sets in (X, T^*) . Hence (X, T^*) is a fuzzy supra semi-Baire space.

Proposition 4.4: If a fuzzy topological space (X, T^*) is a fuzzy supra semi-Baire space, then (X, T^*) is a fuzzy supra semi-second category space.

Proof: Let (X, T^*) be a fuzzy supra semi-Baire space. Then $\text{sint}^*(\bigvee_{i=1}^{\infty}(\lambda_i))=0$ where λ_i 's are fuzzy supra semi-nowhere dense sets in (X, T^*) . Then $(\bigvee_{i=1}^{\infty}(\lambda_i)) \neq 1_X$. (Suppose, $(\bigvee_{i=1}^{\infty}(\lambda_i)) = 1_X$ implies that $\text{sint}^*(\bigvee_{i=1}^{\infty}(\lambda_i)) = \text{sint}^*(1_X)$ which implies that $0 = 1$, a contradiction). Hence (X, T^*) is a fuzzy supra semi-second category space.

Proposition 4.5: If a fuzzy topological space (X, T^*) is a fuzzy supra semi-Baire space, Then no non-zero fuzzy supra semi-open sets in (X, T^*) is a fuzzy supra semi-first category set in (X, T^*) .

Proof: suppose that λ is a non-zero fuzzy supra semi-open set in (X, T^*) such that $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where λ_i 's are fuzzy supra semi-nowhere dense set in (X, T^*) . Then we have $\text{sint}^*(\lambda) = \text{sint}^*(\bigvee_{i=1}^{\infty}(\lambda_i))$. since λ is a non-zero fuzzy supra semi-open set in (X, T^*) . $\text{sint}^*(\lambda) = \lambda$. Then $\text{sint}^*(\bigvee_{i=1}^{\infty}(\lambda_i)) = \lambda \neq 0$. But this is a contradiction to (X, T^*) being a fuzzy supra semi-Baire space in which $\text{sint}^*(\bigvee_{i=1}^{\infty}(\lambda_i)) = 0$ where λ_i 's are fuzzy suprasemi-nowhere dense set in (X, T^*) . Hence, we must have $\lambda \neq (\bigvee_{i=1}^{\infty}(\lambda_i))$. Therefore no non-zero fuzzy supra semi-open set in (X, T^*) is a fuzzy supra semi-first category set in (X, T^*) .

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