Comparative Study of Fuzzy *g – Closed Set with other Fuzzy Closed Sets in Fuzzy Topological Spaces

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ABSTRACT: The aim of this paper is to introduce new class of Fuzzy set, namely, fuzzy *g -closed set in Fuzzy topological spaces (fts). This new class is properly lies between the classes of g -closed and g^* -closed fuzzy sets. Compared the fuzzy *g - closed set with already existing various fuzzy closed sets (introduced and studied by many researchers) in fuzzy topological paces. We studied the properties of obtained closed fuzzy set with necessary counter examples. Further work leads to study and analyses of the concept like, fuzzy *g -continuous, fuzzy *g -irresolute mapping, fuzzy *g -closed maps, fuzzy *g -open maps and fuzzy *g -homeomorphism mapping in Fuzzy topological spaces.

KEYWORDS: *g - closed fuzzy sets, f^*g -open set, fuzzy closure and fuzzy interior.

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1. INTRODUCTION

Zadeh [1] introduced of the concept of 'fuzzy subset' in 1965. As an application of fuzzy subsets Chang [2] introduced the structure of fuzzy topology to general topology in 1968. Subsequently many researchers namely Wong [3], Warren [4], Lowen [5], Mashhour [6], Azad [7], Mukherjee [8], Balasubramanian & Sundaram [9] and many others have contributed to the development of fuzzy topological spaces.

The image and the inverse image of fuzzy subsets of Zadeh [1] functions and their properties was proved by Chang [2] and Warren [4]. They also presented some basic concepts and results on fuzzy topological spaces. Levine [10] introduced generalized closed sets (g -closed sets) in general topology as a generalization of closed sets. Many researchers have worked on this and related problems both in general and fuzzy topology.

Veer Kumar [11] and Sadanand Patil [12-14] and Devi and Muthamil Selvan [15] are introduced and studied $g^{\#}$ -closed fuzzy sets and continuous maps. The class of g^* -closed fuzzy sets is properly placed between the class of closed fuzzy sets and the class of g -closed fuzzy sets. Generalization of fuzzy closed and open sets in fts had influenced the notion like continuity of the function. So the continuity of the function are characterized in terms of open or closed sets in fts.

2. PRELIMINARIES

Throughout this paper (X, T) , (Y, σ) & (Z, η) or (simply X, Y , & Z) represents non-empty fuzzy topological spaces on which no separation axiom is assumed unless explicitly stated. For a subset A of a space (X, T) , $cl(A)$, $int(A)$ & $C(A)$ denotes the closure, interior and the compliment of A respectively.

Definition 2.1: A fuzzy set A of a fts (X, T) is called:

- 1) a semi-open fuzzy set, if $A \leq cl(int(A))$ and a semi-

closed fuzzy set, if $int(cl(A)) \leq A$ [13]

- 2) a pre-open fuzzy set, if $A \leq int(cl(A))$ and a pre-closed fuzzy set, if $cl(int(A)) \leq A$ [13]

The semi closure (respectively pre-closure, α -closure) of a fuzzy set A in a fts (X, T) is the intersection of all semi closed (respectively pre closed fuzzy set, α -closed fuzzy set) fuzzy sets containing A and is denoted by $scl(A)$ (respectively $pcl(A)$).

Definition 2.2: A fuzzy set A of a fts (X, T) is called:

- 1) a generalized closed (g -closed) fuzzy set, if $cl(A) \leq U$, whenever $A \leq U$ and U is open fuzzy set in (X, T) . [2]
- 2) a generalized pre-closed (gp -closed) fuzzy set, if $pcl(A) \leq U$, whenever $A \leq U$ and U is open fuzzy set in (X, T) . [13]
- 3) a generalized semi pre closed (gsp -closed) fuzzy set, if $spcl(A) \leq U$, whenever $A \leq U$ and U is open fuzzy set in (X, T) . [13,14&15]
- 4) a g^* -closed fuzzy set, if $cl(A) \leq U$, whenever $A \leq U$ and U is g -open fuzzy set in (X, T) . [7]
- 5) a $g^{\#}$ -closed fuzzy set, if $cl(A) \leq U$, whenever $A \leq U$ and U is αg -open fuzzy set in (X, T) . [13&14]

Complement of g -closed fuzzy (respectively gp -closed fuzzy set, gsp -closed fuzzy set, g^* -closed fuzzy set and $g^{\#}$ -closed fuzzy set) sets are called g -open (respectively gp -open fuzzy set, gsp -open fuzzy set, g^* -open fuzzy set and $g^{\#}$ -open fuzzy set) sets.

3. FUZZY *g - CLOSED SETS IN FUZZY TOPOLOGICAL SPACES.

Definition 3.01: A fuzzy set A of FTS (X, T) is called *g -closed fuzzy set, if $cl(A) \leq U$ whenever $A \leq U$ & U is open fuzzy set in (X, T) .

Theorem 13.02: Every closed fuzzy set is *g -closed set.

Proof: Let A be a closed fuzzy set in a fts (X, T) , Let $A \leq U$, where U is open fuzzy set in X . Since A is Closed fuzzy

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set that is $cl(A) \leq U$. Hence A is *g - Closed Fuzzy sets.
The converse of the above theorem need not be true as seen from the following example.

Example 3.03: Let $X = \{a, b, c\}$ and the fuzzy sets A & B be defined as follows, Let $A = \{(a, 0.1), (b, 0.2), (c, 0.6)\}$, $B = \{(a, 1), (b, 0.5), (c, 0.8)\}$. Let $T = \{0, 1, A\}$ in a fts , then (X, T) is a fts . Therefore the fuzzy subset B is *g -closed fuzzy set but not a closed fuzzy set.

Theorem 1 3.04: Every semi closed fuzzy set is *g -closed set.

Proof: Let A be a semi closed fuzzy set in a fts X, Let $A \leq U$, where U is open fuzzy set in X and U is open fuzzy set in X. Since A is semi closed fuzzy set. We have, $scl(A) = A = cl(A) \leq U$ ie. $cl(A) \leq U$.

Hence A is *g -closed Fuzzy sets

The converse of the above theorem need not be true as seen from the following example.

Example 3.05: Let $X = \{a, b, c\}$ and the fuzzy sets A & B be defined as follows,

Let $A = \{(a, 0.2), (b, 0.5), (c, 0.6)\}$, $B = \{(a, 1), (b, 0.8), (c, 0.9)\}$. Let $T = \{0, 1, A\}$ in a fts , then (X, T) is a fts . therefore the fuzzy subset B is *g -closed fuzzy set but not a semi-closed fuzzy set.

Theorem 3.06: Every g^* -closed fuzzy set is *g -closed set in fts X.

Proof: Let A be a g^* -closed fuzzy set in X, Let $A \leq U$, where U is g -open fuzzy set in X and U is open fuzzy set in X. Since A is g^* -closed fuzzy set, We have, $cl(A) = A \leq U$ ie. $cl(A) \leq U$.

Hence A is *g - Closed Fuzzy sets.

The converse of the above theorem need not be true as seen from the following example.

Example 3.07: From the example 3.05, the fuzzy subset B is g^* -closed fuzzy set but not a *g -closed set in (X, T) .

Theorem 3.08: Every *g -closed fuzzy set is g^*s -closed set in fts X.

Proof: Let A be *g -closed fuzzy set in X, Let $A \leq U$, where U is open fuzzy set in X and U is g -open fuzzy set in X. Since A is *g -closed fuzzy set, We have, $cl(A) \leq scl(A) = A \leq U$ ie. $cl(A) \leq U$.

Hence A is a g^*s - Closed Fuzzy sets.

The converse of the above theorem need not be true as seen from the following example.

Example 3.09: From the example 3.05, the fuzzy subset B is g^*s -closed fuzzy set but not a *g -closed set in (X, T) .

Theorem 3.10: In any fts X, Every *g -closed fuzzy set is g -closed fuzzy set in fts X.

Proof: Proof arises from definition.

Example 3.11: Let $X = \{a, b, c\}$ and the fuzzy sets A & B be defined as follows,

Let $A = \{(a, 0.2), (b, 0.5), (c, 0.7)\}$, $B = \{(a, 0.4), (b, 0.6), (c, 0.8)\}$. Let $T = \{0, 1, A\}$ in a fts , then (X, T) is a fts . Therefore the fuzzy subset B is g -closed fuzzy set (X, T) .

Theorem 3.12: In any fts X, Every *g -closed fuzzy set is gs -closed fuzzy set in fts X.

Proof: Let A be a *g -closed fuzzy set in X. Let $A \leq U$, where U is open fuzzy set in X. Since A is *g -closed fuzzy set, therefore $cl(A) \leq scl(A) = A \leq U$ ie. $scl(A) \leq U$. Hence A is gs - Closed Fuzzy sets.

The converse of the above theorem need not be true as seen from the following example.

Example 3.13: Let $X = \{a, b, c\}$ and the fuzzy sets A & B be defined as follows,

Let $A = \{(a, 0.1), (b, 0.3), (c, 0.5)\}$, $B = \{(a, 0.2), (b, 0.5), (c, 0.7)\}$. Let $T = \{0, 1, A\}$ in a fts , then (X, T) is a fts . Therefore the fuzzy subset B is gs -closed fuzzy set but not a *g -closed set in (X, T) .

Theorem 3.14: In any fts X, every *g -closed fuzzy set is sg -closed fuzzy set in fts X. But Converse need not be true.

Proof: Omitted

The converse of the above theorem need not be true as seen from the following example.

Example 3.15: From the example 3.13, the fuzzy subset B is sg -closed fuzzy set but not a *g -closed set in (X, T) .

Theorem 3.16: In any fts X, every *g -closed fuzzy set is $gspr$ -closed fuzzy set in fts X. But Converse need not be true.

Proof: Omitted

The converse of the above theorem need not be true as seen from the following example.

Example 3.17: From the example 3.13, the fuzzy subset B is $gspr$ -closed fuzzy set but not a *g -closed set in (X, T) .

Theorem 3.18: In any fts X, every *g -closed fuzzy set is gp -closed fuzzy set in fts X. But Converse need not be true.

Proof: Let A be a *g -closed fuzzy set in X, Let $A \leq U$, where U is open fuzzy set in X. Since A is gp -closed fuzzy set, Therefore $pcl(A) \leq cl(A) = A \leq U$ ie. $cl(A) \leq U$. Hence A is gp - Closed Fuzzy sets.

The converse of the above theorem need not be true as seen from the following example.

Theorem 3.19: In any fts X, every *g -closed fuzzy set is gsp -closed fuzzy set in fts X. But Converse need not be true.

Proof: Omitted

The converse of the above theorem need not be true as seen from the following example.

Example 3.20: From the example 3.13, the fuzzy subset B is gsp -closed fuzzy set but not a *g -closed set in (X, T) .

Theorem 3.21: In any fts X, every *g -closed fuzzy set is ψ -closed fuzzy set in fts X. But Converse need not be true.

Proof: Let A be a *g -closed fuzzy set in X, Let $A \leq U$, where U is open fuzzy set in X and sg -open fuzzy set in X. Since A is ψ -closed fuzzy set, Therefore $cl(A) \leq scl(A) = A \leq U$ ie. $cl(A) \leq U$. Hence A is ψ - Closed Fuzzy sets.

The converse of the above theorem need not be true as seen from the following example.

Example 3.22: Let $X = \{a, b, c\}$ and the fuzzy sets A & B be defined as follows, Let

$A = \{ (a, 0.4), (b, 0.7), (c, 0.9) \}$, $B = \{ (a, 0.6), (b, 0.8), (c, 1) \}$. Let $T = \{ 0, 1, A \}$ in a fts , then (X, T) is a fts . Therefore the fuzzy subset B is ψ -closed fuzzy set but not a *g -closed set in (X, T) .

Theorem 3.23: In any fts X , every *g -closed fuzzy set is ψg -closed fuzzy set in fts X . But Converse need not be true.

Proof: Let A be a *g -closed fuzzy set in X , Let $A \leq U$, where U is open fuzzy set in X . Since A is ψg -closed fuzzy set, Therefore $cl(A) \leq \psi cl(A) = A \leq U$ ie. $cl(A) \leq U$. Hence A is ψg -Closed Fuzzy sets.

The converse of the above theorem need not be true as seen from the following example.

Example 3.24: From the example 3.32, the fuzzy subset B is ψg -closed fuzzy set but not a *g -closed set in (X, T) .

Theorem 3.25: In any fts X , every *g -closed fuzzy set is ψg^* -closed fuzzy set in fts X . But Converse need not be true.

Proof: Follows from previous theorem.

The converse of the above theorem need not be true as seen from the following example.

Example 3.26: From the example 3.32, the fuzzy subset B is ψg^* -closed fuzzy set but not a *g -closed set in (X, T) .

Theorem 3.27: In any fts X , every *g -closed fuzzy set is $(gsp)^*$ -closed fuzzy set in fts X . But Converse need not be true.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 3.28: From the example 3.13, the fuzzy subset B is $(gsp)^*$ -closed fuzzy set but not a *g -closed set in (X, T) .

Theorem 3.29: In any fts X , every *g -closed fuzzy set is gp -closed fuzzy set in fts X . But Converse need not be true.

Proof: Omitted.

Example 3.30: From the example 3.13, the fuzzy subset B is gp -closed fuzzy set but not a *g -closed set in (X, T) .

Theorem 3.31: In any fts X , every *g -closed fuzzy set is g^*p -closed fuzzy set in fts X . But Converse need not be true.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 3.32: From the example 3.13, the fuzzy subset B is g^*p -closed fuzzy set but not a *g -closed set in (X, T) .

Theorem 3.33: In any fts X , every *g -closed fuzzy set is $(g^*p)^*$ -closed fuzzy set in fts X . But Converse need not be true.

Proof: Let A be a *g -closed fuzzy set in X , Let $A \leq U$, where U is open fuzzy set in X and g^*p -open in X . Since A is $(g^*p)^*$ -closed fuzzy set, Therefore $cl(A) \leq U$. Hence A is $(g^*p)^*$ -Closed Fuzzy sets.

The converse of the above theorem need not be true as seen from the following example.

Example 3.34: From the example 3.13, the fuzzy subset B is $(g^*p)^*$ -closed fuzzy set but not a *g -closed set in (X, T) .

Theorem 3.35: In any fts X , every *g -closed fuzzy set is g^{**} -closed fuzzy set in fts X . But Converse need not be true.

Proof: Let A be a *g -closed fuzzy set in X , Let $A \leq U$, where U is open fuzzy set in X and g^* -open in X . Since A is g^{**} -closed fuzzy set, Therefore $cl(A) \leq U$. Hence A is g^{**} -Closed Fuzzy sets.

The converse of the above theorem need not be true as seen from the following example.

Example 3.36: Let $X = \{a, b, c\}$ and the fuzzy sets A & B be defined as follows, Let $A = \{ (a, 0.1), (b, 0.4), (c, 0.6) \}$, $B = \{ (a, 0.3), (b, 0.5), (c, 0.7) \}$. Let $T = \{ 0, 1, A \}$ in a fts , then (X, T) is a fts . Therefore the fuzzy subset B is g^{**} -closed fuzzy set but not a *g -closed set in (X, T) .

Theorem 3.37: In any fts X , every *g -closed fuzzy set is gs^* -closed fuzzy set in fts X . But Converse need not be true.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Theorem 3.38: In any fts X , every *g -closed fuzzy set is $\hat{g}$ -closed fuzzy set in fts X . But Converse need not be true.

Proof: Let A be a *g -closed fuzzy set in X , Let $A \leq U$, where U is open fuzzy set in X and semi open in X . Since A is $\hat{g}$ -closed fuzzy set, Therefore $cl(A) \leq U$. Hence A is $\hat{g}$ -Closed Fuzzy sets.

The converse of the above theorem need not be true as seen from the following example.

Example 3.39: Let $X = \{a, b, c\}$ and the fuzzy sets A & B be defined as follows,

Let $A = \{ (a, 0), (b, 0.3), (c, 0.5) \}$, $B = \{ (a, 0.2), (b, 0.4), (c, 0.6) \}$. Let $T = \{ 0, 1, A \}$ in a fts , then (X, T) is a fts . Therefore the fuzzy subset B is $\hat{g}$ -closed fuzzy set but not a *g -closed set in (X, T) .

Theorem 3.40: In any fts X , every *g -closed fuzzy set is $(gs)^*$ -closed fuzzy set in fts X . But Converse need not be true.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Theorem 3.41: In any fts X , if a fuzzy set A is both open fuzzy set & *g -closed fuzzy set. Then A is closed fuzzy set.

Proof: Let a fuzzy set A of fts X is both open fuzzy set & *g -closed fuzzy set. Now $A \leq A$, where A is open fuzzy set, this implies that $cl(A) \leq A$. Since A is *g -closed fuzzy set in X . And also we have $A \leq cl(A)$ this implies that $cl(A) = A$. Hence A is closed fuzzy set in fts X .

Theorem 3.42: If A is *g -closed fuzzy set in X and $cl(A) \wedge (1 - cl(A)) = 0$. Then there is no non zero closed fuzzy set F , such that $F \leq cl(A) \wedge (1 - A)$

Proof: suppose F is any fuzzy closed set in X , such that $F \leq cl(A) \wedge (1 - A)$, Now $F \leq (1 - A)$, this implies that $A \leq (1 - F)$, where $1 - F$ is fuzzy closed set. Then $cl(A) \leq 1 - F$, As A is *g -closed fuzzy set, this implies that $F \leq 1 - cl(A)$. Then $F \leq cl(A) \wedge F \leq 1 - cl(A)$ therefore

$F \leq cl(A) \wedge (1 - cl(A)) = 0$. Then $F = 0$, hence Theorem proved.

Theorem 3.43: If a fuzzy set A is $*g$ -closed fuzzy set in X , such that $A \leq B \leq cl(A)$. Then B is also $*g$ -closed fuzzy set in X .

Proof: Let U be an open fuzzy set in X , such that $B \leq U$. Then $A \leq U$. Since A is $*g$ -closed fuzzy set in X . Then by definition $cl(A) \leq U$. Now $B \leq cl(A)$, $cl(B) \leq cl(cl(A)) \leq U$ that is $cl(B) \leq U$. Hence B is $*g$ -closed fuzzy set in X .

Theorem 3.44: A finite union of $*g$ -closed fuzzy set is a $*g$ -closed fuzzy set.

Proof: Let A & B be $*g$ -closed fuzzy set in X . To show that $A \vee B$ is $*g$ -closed fuzzy set. Let $A \vee B \leq U$, where U is open fuzzy set. Then $A \leq U, B \leq U$ & $cl(A) \leq U, cl(B) \leq U$. Since A & B are $*g$ -closed fuzzy set this implies $cl(A) \vee cl(B) \leq U$ and so $cl(A \vee B) \leq U$. Hence $A \vee B$ is $*g$ -closed fuzzy set. Then a finite union of $*g$ -closed fuzzy set is a $*g$ -closed fuzzy set in fuzzy topological spaces.

Definition 3.45: A fuzzy set A of $fts(X, T)$ is called $*g$ -open fuzzy sets, if its compliment $1 - A$ is $*g$ -closed fuzzy set.

Theorem 3.46: A fuzzy set A of a $fts X$ is $*g$ -open fuzzy set, iff $F \leq int(A)$ whenever F is closed fuzzy set and $F \leq A$.

Proof: Suppose A is $*g$ -Open Fuzzy set. Then $1 - A$ is $*g$ -closed fuzzy set. Let F be closed fuzzy set in X & $F \leq A$. Then $1 - A \leq 1 - F$ and $1 - F$ is open fuzzy set. Since $1 - A$ is $*g$ -closed fuzzy set, we have $cl(1 - A) \leq 1 - F$, this implies that $F \leq int(A)$, $cl(1 - A) = 1 - int(A)$. Conversely, assume that $F \leq int(A)$, whenever $F \leq A$ and F is closed fuzzy set in $fts X$. Let $1 - A \leq G$, where G is open fuzzy set in X . Then $1 - G \leq A$, where $1 - G$ is closed fuzzy set in X , which implies that $1 - G \leq int(A)$, implies that $1 - int(A) \leq 1 - (1 - G)$. That is $cl(1 - A) \leq G$. Hence $1 - A$ is $*g$ -closed fuzzy set and A is $*g$ -open fuzzy set.

Theorem 3.47: Every open fuzzy set in $fts X$ is a $*g$ -open fuzzy set.

Proof: By definition.

The converse of the above theorem need not be true as seen from the following example.

Example 3.48: Let $X = \{a, b, c\}$ and the fuzzy sets A & B be defined as follows, Let $A = \{(a, 0), (b, 0.6), (c, 0.8)\}$, $B = \{(a, 0.2), (b, 0.7), (c, 0.8)\}$, $1 - B = \{(a, 0.8), (b, 0.3), (c, 0.2)\}$ Let $T = \{0, 1, A\}$ in a fts , then (X, T) is a fts . Thus the fuzzy set B is $*g$ -open Fuzzy set but not a open fuzzy set in $fts X$.

Theorem 3.49: Every semi open fuzzy set in $fts X$ is a $*g$ -open fuzzy set.

Proof: Let A be a semi open fuzzy set in a $fts X$. Then $1 - A$ is semi closed set and $1 - A$ is $*g$ -closed fuzzy set. Hence A is $*g$ -open fuzzy set.

The converse of the above theorem need not be true as seen from the following example.

Example 3.50: From the example 3.48, the fuzzy set B is $*g$ -open Fuzzy set but not a semi-open fuzzy set in $fts X$.

Theorem 3.51: In a X , every $*g$ -open Fuzzy set is $*g$ -open Fuzzy set.

Proof: Let A be $*g$ -open Fuzzy set in a $fts X$. Then $1 - A$ is $*g$ -closed Fuzzy set in X . and $1 - A$ is $*g$ -open - closed Fuzzy set in X . That is A is $*g$ -open Fuzzy set in $fts X$.

The converse of the above theorem need not be true as seen from the following example.

Example 3.52: From the example 3.48, the fuzzy set B is $*g$ -Open Fuzzy set but not a $*g$ -open fuzzy set in $fts (X, T)$.

Theorem 3.53: In a X , every $*g$ -open Fuzzy set is g -open Fuzzy set.

Proof: Let A be $*g$ -open Fuzzy set in a $fts X$. Then $1 - A$ is $*g$ -closed Fuzzy set in X . and $1 - A$ is g -closed Fuzzy set in X . That is A is g -open Fuzzy set in $fts X$.

The converse of the above theorem need not be true as seen from the following example.

Example 3.54: From the example 3.48, the fuzzy set B is g -open Fuzzy set but not a $*g$ -open fuzzy set in $fts (X, T)$.

Theorem 3.55: In a X , every $*g$ -open Fuzzy set is g^*s -open Fuzzy set.

Proof: Let A be $*g$ -open Fuzzy set in a $fts X$. Then $1 - A$ is $*g$ -closed Fuzzy set in X . and $1 - A$ is g^*s -closed Fuzzy set in X . That is A is g^*s -open Fuzzy set in $fts X$.

The converse of the above theorem need not be true as seen from the following example.

Example 3.56: Let $X = \{a, b, c\}$ and the fuzzy sets A & B be defined as follows,

Let $A = \{(a, 0), (b, 0.6), (c, 0.7)\}$,

$B = \{(a, 0.2), (b, 0.4), (c, 1)\}$ $1 - B = \{(a, 0.8), (b, 0.6), (c, 0)\}$.

Let $T = \{0, 1, A\}$ in a fts , then (X, T) is a fts . Thus the fuzzy set B is g^*s -open Fuzzy set but not a $*g$ -open fuzzy set in $fts (X, T)$.

Theorem 3.57: In a X , every $*g$ -open Fuzzy set is gs -open Fuzzy set.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 3.58: From the example 3.56, the fuzzy set B is gs -open Fuzzy set but not a $*g$ -open fuzzy set in $fts (X, T)$.

Theorem 3.59: In a X , every $*g$ -open Fuzzy set is sg -open Fuzzy set.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 3.60: From the example 3.56, the fuzzy set B is sg -open Fuzzy set but not a $*g$ -open fuzzy set in $fts (X, T)$.

Theorem 3.61: In a X , every $*g$ -open Fuzzy set is sgp -open Fuzzy set.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 3.62: From the example 3.48, the fuzzy set B is sgp - open Fuzzy set but not a $*g$ -open fuzzy set in $fts(X, T)$.

Theorem 3.63: In a X , every $*g$ - open Fuzzy set is $(sgp)^*$ - open Fuzzy set.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 3.64: From the example 3.56, the fuzzy set B is $(sgp)^*$ - open Fuzzy set but not a $*g$ -open fuzzy set in $fts(X, T)$.

Theorem 3.65: In a X , every $*g$ - open Fuzzy set is $gspr$ - open Fuzzy set.

Proof: Let A be $*g$ - open Fuzzy set in a $fts X$. Then $1 - A$ is $gspr$ - closed Fuzzy set in X . and $1 - A$ is $*g$ - closed Fuzzy set in X . That is A is $gspr$ - open Fuzzy set in $fts X$.

The converse of the above theorem need not be true as seen from the following example.

Example 3.66: From the example 3.56, the fuzzy set B is $gspr$ - open Fuzzy set but not a $*g$ -open fuzzy set in $fts(X, T)$.

Theorem 3.67: In a X , every $*g$ - open Fuzzy set is ψ - open Fuzzy set.

Proof: Let A be $*g$ - open Fuzzy set in a $fts X$. Then $1 - A$ is ψ - closed Fuzzy set in X . and $1 - A$ is $*g$ - closed Fuzzy set in X . That is A is ψ - open Fuzzy set in $fts X$.

The converse of the above theorem need not be true as seen from the following example.

Example 3.68: Let $X = \{a, b, c\}$ and the fuzzy sets A & B be defined as follows, Let $A = \{(a, 0), (b, 0.6), (c, 0.7)\}$, $B = \{(a, 0.3), (b, 0.4), (c, 0.5)\}$, $1 - B = \{(a, 0.7), (b, 0.6), (c, 0.5)\}$. Let $T = \{0, 1, A\}$ in a fts , then (X, T) is a fts . Thus the fuzzy set B is ψ - open Fuzzy set but not a $*g$ -open fuzzy set in $fts(X, T)$.

Theorem 3.69: In a X , every $*g$ - open Fuzzy set is ψg - open Fuzzy set.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 3.70: From the example 3.68, the fuzzy set B is ψg - open Fuzzy set but not a $*g$ -open fuzzy set in $fts(X, T)$.

Theorem 3.71: In a X , every $*g$ - open Fuzzy set is ψg^* - open Fuzzy set.

Proof: Follows from 3.69..

The converse of the above theorem need not be true as seen from the following example.

Example 3.72: From the example 3.68, the fuzzy set B is ψg^* - open Fuzzy set but not a $*g$ -open fuzzy set in $fts(X, T)$.

Theorem 3.73: In a X , every $*g$ - open Fuzzy set is g^*p - open Fuzzy set.

Proof: Let A be $*g$ - open Fuzzy set in a $fts X$. Then $1 - A$ is g^*p - closed Fuzzy set in X . and $1 - A$ is $*g$ - closed Fuzzy set in X . That is A is g^*p - open Fuzzy set in $fts X$.

The converse of the above theorem need not be true as seen from the following example.

Example 3.74: Let $X = \{a, b, c\}$ and the fuzzy sets A & B be defined as follows, Let $A = \{(a, 0.1), (b, 0.5), (c, 1)\}$, $B = \{(a, 0.4), (b, 0.4), (c, 0.5)\}$, $1 - B = \{(a, 0.6), (b, 0.6), (c, 0.5)\}$. Let $T = \{0, 1, A\}$ in a fts , then (X, T) is a fts . Thus the fuzzy set B is g^*p - open Fuzzy set but not a $*g$ -open fuzzy set in $fts(X, T)$.

Theorem 3.75: In a X , every $*g$ - open Fuzzy set is $(g^*p)^*$ - open Fuzzy set.

Proof: Omitted.

The converse of the above theorem need not be true

Theorem 3.76: In a X , every $*g$ - open Fuzzy set is $(gs)^*$ - open Fuzzy set.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Theorem 3.77: In a X , every $*g$ - open Fuzzy set is gs^* - open Fuzzy set.

Proof: Omitted.

The converse of the above theorem need not be true.

Theorem 3.78: In a X , every $*g$ - open Fuzzy set is $\hat{g}$ - open Fuzzy set.

Proof: Let A be $*g$ - open Fuzzy set in a $fts X$. Then $1 - A$ is $\hat{g}$ - closed Fuzzy set in X . and $1 - A$ is $*g$ - closed Fuzzy set in X . That is A is $\hat{g}$ - open Fuzzy set in $fts X$.

The converse of the above theorem need not be true.

Theorem 3.79: In a X , every $*g$ - open Fuzzy set is g^{**} - open Fuzzy set.

Proof: Let A be $*g$ - open Fuzzy set in a $fts X$. Then $1 - A$ is g^{**} - closed Fuzzy set in X . and $1 - A$ is $*g$ - closed Fuzzy set in X . That is A is g^{**} -open Fuzzy set in $fts X$.

The converse of the above theorem need not be true as seen from the following example.

Example 3.80: Let $X = \{a, b, c\}$ and the fuzzy sets A & B be defined as follows, Let $A = \{(a, 0), (b, 0.6), (c, 0.8)\}$, $B = \{(a, 0.9), (b, 0.2), (c, 0.7)\}$, $1 - B = \{(a, 1), (b, 0.8), (c, 0.3)\}$. Let $T = \{0, 1, A\}$ in a fts , then (X, T) is a fts . Thus the fuzzy set B is g^{**} - open Fuzzy set but not a $*g$ -open fuzzy set in $fts(X, T)$.

Theorem 3.81: If $int(A) \leq B \leq A$ & if A is $*g$ -open fuzzy set. Then B is $*g$ -open fuzzy set in a $fts(X, T)$.

Proof: Here $int(A) \leq B \leq A$ Then $(1 - A) \leq (1 - B) \leq cl(1 - A)$. Since $(1 - A)$ is $*g$ -closed fuzzy set. Here we know that $(1 - B)$ is $*g$ -closed fuzzy set in $fts X$. Hence B is $*g$ -open fuzzy set in $fts(X, T)$.

Theorem 3.82: If $A \leq B \leq X$, where A is $*g$ -open fuzzy relative to B and B is $*g$ -open fuzzy relative to X . Then A is $*g$ -open fuzzy relative to $fts(X, T)$.

Proof: Let F be g -closed fuzzy set and $F \leq A$. Then F is closed fuzzy relative to B and hence $F \leq int_B(A)$ by the

theorem 3.46, hence $F \leq \text{int}(A) \wedge B$, which implies that $F \leq \text{int}(A)$. Again by the theorem 3.46, A is *g -open fuzzy set in X .

Theorem 3.83: The finite intersection of *g -open fuzzy set is *g -open fuzzy set.

Proof: Let A & B be *g -open fuzzy set in $fts X$. To show that $A \wedge B$ is *g -open fuzzy set. Let $F \leq A \wedge B$, when F be closed fuzzy set. Then $F \leq A, F \leq B$ and $F \leq \text{int}(A), F \leq \text{int}(B)$ as A & B are *g -open fuzzy set. Then $F \leq \text{int}(A) \wedge \text{int}(B) = \text{int}(A \wedge B)$ that is $F \leq \text{int}(A \wedge B)$

Hence $A \wedge B$ is *g -open fuzzy set. Therefore. Finite intersection of *g -open fuzzy set is a *g -open fuzzy set.

Theorem 3.84: If a fuzzy set A is *g -closed fuzzy set and $cl(A) \wedge (1 - cl(A)) = 0$. Then $cl(A) \wedge (1 - A)$ is *g -open fuzzy set.

Proof: Let A be *g -closed fuzzy set in a $fts X$. Let $F \leq cl(A) \wedge (1 - A)$, F is closed fuzzy set in X . Since F is zero and $F \leq \text{int}(cl(A) \wedge (1 - A))$ and $cl(A) \wedge (1 - A)$ is *g -open fuzzy set in $fts X$.

Remark 3.85: The above discussions are summarized interrelation between fuzzy sets in the following diagram.

Where $A \rightarrow B$ ($A \leftrightarrow B$) represents A implies B but not conversely. (A and B are independent).

4. CONCLUSION

In this paper, we introduced and studied a new class of fuzzy closed set namely, f^*g -closed set in Fuzzy topological spaces. This new class properly lies between the classes of g -closed and g^* -closed fuzzy sets. We also compared the fuzzy *g -closed set with various existing fuzzy closed sets in fts . Further research work may lead to study and analyses of the concept like, fuzzy continuous, fuzzy irresolute mapping, fuzzy homeomorphism mapping, and Compactness, Connectedness, Regular and Normal spaces in Fuzzy topological spaces.

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